Hydrology

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To the memory of

DR. ROBERT E. HORTON,

whose untiring efforts throughout more than a quarter of a century contributed immeasurably toward the development of the science of hydrology, this book is respectfully dedicated.

Preface

Hydrology is one of the newest of the natural sciences. As a result of intensive researches, new theories have been advanced during the past twenty years that have almost completely revolutionized our previous concepts of the subject and have accordingly changed many of our techniques. Although scores of articles in current literature have recorded these new developments, no single volume has appeared during all this time that has served to correlate them and bind them together into a cohesive unit. Therefore, nearly ten years ago, Dr. Robert E. Horton and the authors undertook this task. First as a result of World War II and then because of the untimely death of Dr. Horton, the preparation of the manuscript was delayed. Another factor contributing to its delay was the rapid accumulation of new data and new concepts that have caused the science to be in a state of flux. In fact, before the final chapter was finished some of the earlier ones had to be rewritten.

When Professor H. W. King first offered a course in hydrology at the University of Michigan in 1912 it was more or less an experiment. No textbook and but few references were then available. As a matter of fact, at that time hydrology was not generally recognized as a science. Nevertheless, the need for knowledge in this field was so apparent that the course was a success from the beginning and has been continued ever since. Because of the ever-increasing demands made upon our water resources, this subject becomes of greater and greater importance in the fields of civil engineering, forestry, agriculture, and allied sciences. Indeed, no important hydraulic structure and no utilization of our water resources can be properly planned without a thorough knowledge of this subject. Surely the time is not far distant when every college and university will include in its curriculum a course in hydrology.

Because of the evolution through which this science has passed since the earlier books on the subject were published, the authors have had practically no guideposts to follow. Perhaps not everyone

will agree with the order of presentation that has been followed herein. Although it is hoped that the book will be valuable to all who are interested in hydrology, it is primarily intended to fill the need for a textbook for college and university use. It is therefore expected that many of its readers will not have had previous contact with this subject. Inasmuch as the central theme is stream flow, its fluctuations and the causes thereof, it was considered desirable first to acquaint the student with the characteristics and peculiarities of the hydrograph and with the nature and diversity of the problems connected therewith. It is believed that after he has been presented with this general preview of what is in store for him he will better appreciate the need for a knowledge of precipitation, evaporation, infiltration, and related subjects which might otherwise be of but little interest to him.

The authors are deeply indebted to Mr. John G. Ferris for his splendid cooperation in writing the chapter on ground water. They also wish to express their gratitude to W. W. Horner, S. W. Jens, Professor M. L. Albertson, Dr. C. R. Hursh, LeRoy K. Sherman, Walter G. Wilson, and Don M. Corbett for the assistance that was so generously given.

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CHAPTER I

INTRODUCTION

The Hydrologic Cycle

Water constitutes one of our most valuable natural resources. Without it no form of life is possible. It not only supplies both the animal and vegetable kingdoms with daily sustenance but also provides highways of transportation, is a source of power, and serves many other useful purposes. At times, however, this normally helpful servant becomes temporarily transformed into a most destructive agent, through the medium of storms and floods, laying waste valuable property, taking a heavy toll of life, and eroding and carrying to the sea millions of tons of rich and fertile soil annually.

In one form or another, water occurs practically everywhere, varying in quantity from an almost unlimited supply in the oceans to nearly none in desert regions. It occurs in the atmosphere as water vapor, clouds, and precipitation. On the earth's surface it is found principally in streams, in lakes, and in the oceans, and beneath the ground surface it occurs under various classifications, as will be explained later.

Although at any instant by far the largest portion of the total water supply is stored in the oceans, a constant circulation is taking place. Evaporation from the ocean's surface is continuous. Although most of the moisture so evaporated condenses and returns at once as rain, a considerable portion is carried by the winds over the land areas where it is precipitated as rain, hail, sleet, or snow or condenses as dew or frost on the surface of vegetation and other objects. Nearly all the moisture in the form of dew and frost either is evaporated directly or is consumed by vegetation and then transpired through the vegetal pores. That which falls as precipitation, however, has a much more varied experience. Some is re-evaporated before it reaches the earth. Another part is intercepted by vegetation, buildings, and other objects, and part of this is re-evaporated directly. Another portion

runs off from the ground surface into the streams and is returned to the sea. Still another portion percolates into the ground. For this, there are numerous outlets: part of it is held by capillary action at or near the surface and is evaporated therefrom; another

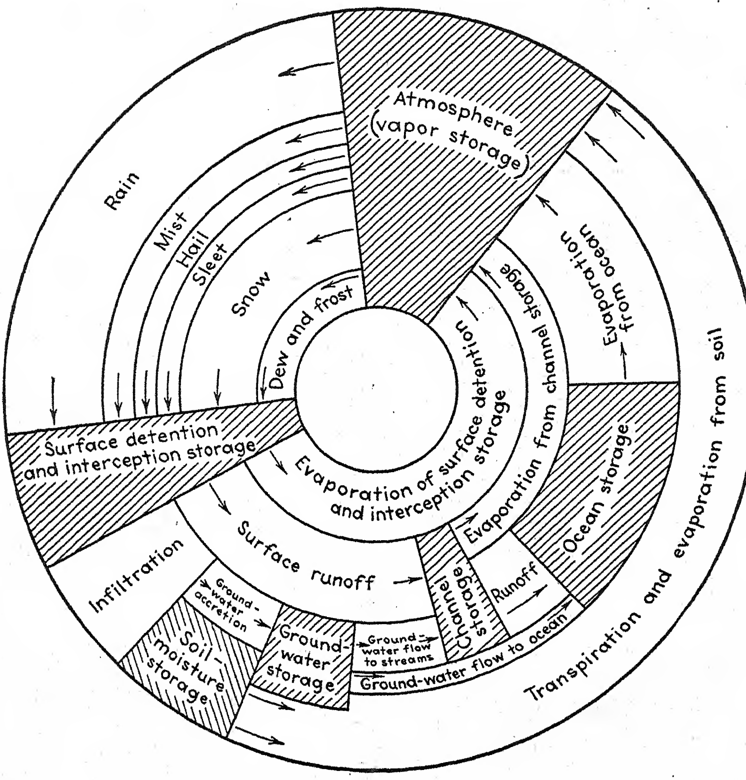


FIG. 1. The hydrologic cycle. Shaded areas represent storage; arrows indicate flow. Read diagram counterclockwise.

part is used by vegetation and returned to the air through the process of transpiration; still another portion joins the ground water and slowly finds its way to the streams, appearing after days, months, and sometimes much longer periods as ground-water flow; and finally, an amount that is usually insignificant but in a few drainage basins is of considerable importance, percolates to great depths and appears after long intervals, often at far distant points, as springs, artesian wells, and geysers.

Of the water that reaches the streams comprising the headwaters of the large drainage systems perhaps only a small portion flows directly to the sea. The remainder is evaporated from the surface of streams and lakes through which the streams flow, is used and transpired by vegetation growing along their margins, or seeps into the ground along the water courses where the groundwater table is lower than the surface of the streams. This last portion may later return to the same channel at points downstream; it may through underground channels find an outlet in distant springs, other river channels, lakes, or the sea; it may be reached and utilized by deep-rooted vegetation; or, finally, it may join the more or less permanent ground waters, appearing perhaps years later as springs and geysers.

This sequence of events, which is represented graphically in Fig. 1, is called the *hydrologic cycle*. It provides the groundwork upon which the science of hydrology is constructed.

Hydrology Defined

Hydrology is the science that deals with the processes governing the depletion and replenishment of the water resources of the land areas of the earth. It is concerned with the transportation of water through the air, over the ground surface, and through the strata of the earth. In other words, it is the science that treats of the various phases of the hydrologic cycle. A knowledge of hydrology is of basic importance in practically all problems that involve the use and supply of water for any purpose whatsoever. It is, therefore, of value not only in the field of engineering but also in forestry, agriculture, and other branches of natural science.

The questions that the hydrologist is called upon to answer are extremely varied. Sometimes he is most concerned with a determination of the maximum flood flow that may be expected every few years, as in certain drainage problems; at other times, as in the design of the spillway for an important dam, his problem is to determine the maximum flow that will occur once in a thousand years or more. The best procedures in solving these two problems may be entirely different as will be explained later. Again, the problem, often encountered by the municipal or sanitary engineer, may involve a determination of the minimum average low water flow for a day, a month, or a longer period. Or it may be a determination of the long-term average yield. In some instances the

manner in which that yield varies throughout the year or from year to year is of little or no importance; in others, its variation is of major importance. And so the hydrologist is presented with a great variety of problems, no two of which require the same data or involve the same methods of procedure for solution. For instance, in determining the maximum flood that may be expected he will be but little concerned with evaporation and transpiration losses, but if he wishes to determine the low water flow or the long-term average yield he will be very much interested in those losses. As a result, a broad general knowledge of the basic principles of hydrology is essential for the proper solution of these various problems.

History of Hydrology

Hydrology is a relatively new branch of the natural sciences. Although in ancient ruins unmistakable evidence has been unearthed that advanced knowledge in many of the sciences was held by man thousands of years ago, it appears that no such evidence of an early knowledge of the principles of hydrology has ever been found. In fact, one need not go back many years to find a time when there was practically no literature on the subject. It is believed that greater advancement has been made in the development of this science during the present century than was made during all previous history.

As an illustration of the tremendous changes that have occurred in this science recently, one need but recall that it was only a few years ago when runoff was generally considered and expressed as a percentage of rainfall. We now know that runoff is that which is left from the total rainfall after evaporation, transpiration, and various other factors have taken their toll; in other words, stream flow equals rainfall minus losses, not rainfall times a percentage factor. In a similar manner, it was not long ago that engineering literature was replete with discussions of the use of statistical and probability methods for determining maximum flood flows that could be anticipated with any given frequency. It can now be shown conclusively that such methods often produce results that are grossly misleading when applied to short-term discharge records for determining the maximum flood that may be expected to occur with a frequency of, say, once in a thousand years. And so these and many other of the earlier concepts have only recently been replaced by better and sounder theories.

Two milestones mark this progress. The first, the concept of the unit hydrograph, stands as a monument to LeRoy K. Sherman. The second, the theory of infiltration capacity, is one of the many contributions of Robert E. Horton. These, along with the work of W. W. Horner, Merrill Bernard, and a great many others, have so changed our knowledge of this subject that we may refer to the present century and, in fact, even to the period since 1930 as representing the dawn of the science of hydrology. Other advances are unquestionably in the making, but sufficient progress has already been made so that the student may rest assured that this science now provides him with a most useful tool for determining the answer to almost any problem that he may encounter in this field.

Practical Value of Hydrology

To the beginning student of hydrology, the natural question that first arises is: Why are we concerned with knowing all the complicated relationships between precipitation and runoff? Inasmuch as it is easier to measure the runoff from a drainage basin than to determine the average precipitation on that basin, since the latter requires measurement at a number of places, why not measure the runoff in the first place and be done with it? Why bother about the precipitation?

The difficulty of course lies in the fact that a river is not like a tract of land which, once surveyed, forever retains these same dimensional characteristics. Instead, the volume of water flowing in any given stream varies from day to day and from year to year. It is never absolutely constant even for a day. Frequently the magnitude of these changes is slight, but occasionally it is very large. For some streams the maximum flow is many thousand times the minimum, but for others this ratio is relatively small. Likewise for any stream the maximum flow for any one year will bear a certain ratio to the minimum flow, but for any other year that ratio will be entirely different. Furthermore, for any given stream the maximum, minimum, and average flows for any short period, for example five years, may be and often are radically different from those for any other, similar period.

Therefore, in order to determine the regimen of a stream over a long period of time, as must be done in the solution of a wide variety of engineering problems, it is necessary either to have discharge records covering such a long period or to have other

data and a knowledge of the relationship between the known data and the stream flow so that the flow may be determined with a satisfactory degree of accuracy. Rainfall and general climatic conditions affect our daily lives more directly than does stream flow. This statement is true at least for the average layman, even though it may be questioned by the engineer. As a result, records of rainfall, temperature, humidity, barometric pressure, and the like were initiated long before stream-flow records were even considered. Furthermore, rainfall and general climatic records require but little skill and training on the part of the observer, whereas reliable stream-flow records demand the services of an engineer or at least of a skilled technician.

Consequently, rainfall and climatic records are available for almost any drainage basin in the entire United States, oftentimes covering periods of fifty years and sometimes a hundred years or more. On the other hand, stream-flow records are comparatively few and far between. On only a few streams are there good, reliable records that are continuous for a period of fifty years or more. Many records are brief or intermittent and usually missing for the period for which they are most urgently needed. Sometimes good records are available on the main stream, whereas the problem at hand calls for a knowledge of the yield of a tributary far removed from the site of the available records, or vice versa. Often a close examination will reveal the fact that some of the existing records were obtained by methods or under circumstances that subject their accuracy and reliability to serious question.

Practically never does the engineer have the experience of finding available all the necessary stream-flow records at the proper site on the stream in question. Nearly always it is necessary either to use the records obtained at a more or less distant point or to extend the records to cover a longer period. In any case, the selection of the proper procedure is dependent upon a thorough understanding of the principles of hydrology.

Increasing Importance of Hydrology

In the early stages of development of our country, the water resources did not possess the same importance that they now have, nor do they now play the prominent role that they seem destined to assume in the future. In the early days those resources were entirely adequate to meet all the needs that then existed. They

could be had merely for the asking. Seldom were the resources developed to the limit of their possibilities but only to the extent of meeting the then existing requirement. As a result, but little concern was felt regarding the need for data and knowledge of the ultimate capacities of our rivers and underground sources to meet the many demands to which they now are and in the future will be subjected.

With the advance of civilization and a steadily increasing population, rivalry and competition for the use and control of our water resources have developed and are becoming more and more intense. In the pioneer days that use was practically restricted to logging, fishing, navigation, and small commercial power plants where the power was used in saw mills, flour mills, and small factories. Seldom was any attempt made to utilize all the power that was available at any site. How different is the picture today! On streams of any size practically all power plants are now hydroelectric and are installed right up to the limit of economic feasibility — in fact, many contain installations that are well beyond that limit. Storage reservoirs are in demand whereby the flood waters may be conserved and utilized during periods of low flow. For the benefit and protection of wildlife, however, the recreational interests insist that those reservoirs be maintained as nearly as possible at a constant level, a point which is at variance with the wishes of the power interests.

An increasing number of municipalities and industries are obtaining their water supplies from rivers. Their need for a supply that is as free as possible from contamination is in direct conflict with the interests of other cities and industries, located upstream, that wish to discharge their sewage and industrial wastes into that same river. Where the rainfall during the growing season is insufficient to meet the needs of vegetation, the withdrawal of water from the streams for irrigation purposes is in conflict with practically all other needs.

A careful examination of the manner in which each different utilization of our water resources affects the availability of those same waters for other purposes reveals the fact that each different use may be in conflict with most of the others. Although it is true that occasionally two uses may be found that on the surface appear to operate in harmony, more often than not that harmony will be found to be more apparent than real. As an illustration, the

storage of flood waters in reservoirs to supplement the low flows for power development, irrigation, water supply, or other purpose appears to be in perfect accord with flood prevention. Upon second thought, however, it is seen that storage for any of the former purposes demands that the reservoir be kept filled as much of the time as possible so that the water will be available when needed, whereas for flood prevention the reservoir should be emptied as quickly as practicable so that its capacity for storage will be available when the next flood arrives.

Furthermore, the competition for our water resources is certain to become more and more keen in the years to come. This increase will occur not only among the various types of uses that exist today but among the new uses that will develop. For instance, the fact is now well established that farm irrigation is profitable not only in the western part of the United States, to which area this practice was long confined, but also in the Midwest and even in certain sections of the East. When this truth becomes fully realized, a new impetus will be given to the demands for water in those areas.

Likewise the field of air conditioning is only in its infancy at the present time. This practice is almost certain to enjoy a tremendous growth. Inasmuch as some of the methods of air conditioning require large amounts of water, usually obtained from underground supplies, serious problems are almost certain to be created. With the advent of other at present unforeseen developments of similar nature and with a steadily increasing congestion of population, the competition for our water resources is certain to increase as time progresses.

In the adjustment of these conflicts and in the proper solution of the many problems arising in connection with them, complete data on our water resources and a full understanding of the principles of hydrology become a vital necessity.

Need for New Laws

To meet the new conditions and the ever-expanding demands, new laws will have to be enacted. Although literally deluged with a superfluity of legislation governing most of our daily activities, by way of strange contrast we are left almost entirely in the dark on the important question of our rights in connection with our natural water resources. At least that is the situation in most of the states. For example, in Michigan there are a few legislative

enactments relating to matters such as fish ladders in dams and the right of the public to fish in waters in which fish have been planted at public expense, a law restricting the pollution of streams, and a few others. All other matters are governed by judicial decree, and these decisions are frequently conflicting. In general, they find root in the old common law of England and are based upon priority of use, a doctrine which under present-day conditions is woefully inadequate. On such an important point as the extent and limitations of the right of the public and the riparian owner on any stream, one will search in vain for a clean-cut definition either by legislative act or by court decree. Those acts and decrees frequently contain references to "navigable" and "non-navigable" waters, to "the water's edge," to "the low-water mark" and "the high-water mark," etc., but specific definitions of these terms are glaringly absent.

A few states, principally in the West, have good up-to-date laws regulating the use of our water resources, but in most states a complete new water code is urgently needed. In the preparation of that code, the hydrologist should take an important part, for only he fully understands the proper solution of the many intricate problems that are arising and will continue to arise with increasing frequency in connection with the use and control of our natural water resources.

Need for More Basic Data

The most serious obstacle that always confronts the engineer in his study of problems dealing with stream flow is invariably the lack of data. These data include the following:

1. Stream-flow records.
2. Precipitation records.
3. Topographic maps.
4. Ground-water data.
5. Evaporation and transpiration data.
6. Data on the quality of the available supply.

It may be observed that these data can be divided into two general classifications. The first class includes records of variable factors showing the variations in either quantity or quality of the supply from time to time. Stream-flow and precipitation records are of this type. The other class includes data of a more or less

permanent character, such for instance as topographic maps of the drainage basin. To obtain the first kind requires a long period of time, and, other things being equal, the value of the records increases directly with the length of period covered. On the other hand, the value of the other class of data has no relation whatever to the length of time required for collection. It would be possible, for instance, to obtain the topography of an entire drainage basin in a month or even less with a sufficiently large staff, but not even an army of the most highly trained engineers could determine the regimen of a stream in so short a time.

Throughout the entire United States, the federal government maintains about 2900 stream-gaging stations. A number of other stations are maintained by private agencies, but the records obtained are not always available to the public. Large areas oftentimes covering important drainage basins can be found that are practically without any discharge records. On many others the records are so short and intermittent as to be of little value. It is a conservative statement to say that few, if any, government expenditures are more urgently needed or yield greater dividends than money for stream-flow records.

The U. S. Weather Bureau is at present maintaining about 8000 precipitation stations throughout the country. Except for the records that are being obtained by the Forest Service, the Soil Conservation Service, and a few other agencies from their experimental investigations, these are practically the only precipitation records available. Throughout the entire United States the average distance between adjacent precipitation stations is between 20 and 30 miles. Taking into consideration the large differences in rainfall that are often shown by the records of any individual storm at two stations less than 20 miles apart, it at once becomes apparent that, with the present number of precipitation stations in operation, it is impossible to determine to a satisfactory degree of accuracy the amount of precipitation that falls on any drainage basin during one of the more intense storms. For the more general storms of less intensity but covering large areas, the present records may provide a satisfactory basis for such determination, but unquestionably there are many intense storms covering relatively small areas that fall between stations and are unrecorded. This fact constitutes one of the serious obstacles in the path of those who attempt to determine the relationship

between precipitation and runoff. Consequently a great many additional precipitation stations, judiciously located, are needed.

Furthermore, of the above total number of precipitation stations now being maintained by the Weather Bureau, only about 2100 have recording rain gages. For many hydrologic studies, continuous records showing the varying intensities of rainfall are essential. The number of recording stations should therefore be increased as rapidly as possible.

Topographic maps and also soil and geologic maps provide a most valuable aid in the study of problems relating to stream flow and ground-water supply. From them may be obtained data on the character of the terrain, rock outcrops, area of basin, length of stream channel, stream density, and a vast amount of other valuable information. The U. S. Geological Survey is engaged in making a topographic map of the entire United States. At present that work is less than half completed. The map is published in sections, each section being about 16½ inches by 20 inches and usually covering either 15 minutes or 30 minutes of latitude and the same amount of longitude, although either larger or smaller scales are sometimes used depending upon the importance and character of the terrain covered. This work is carried on by the Survey in cooperation with the individual states. The progress in any state, therefore, depends upon the extent of the cooperation accorded. In about ten states, most of them in the eastern part of the country, the work is completed; in the remainder, varying degrees of progress have been made, and in many states the areas covered are so scattered that the available maps are of but little value in hydrologic studies.

Most of the data that have been collected on evaporation, transpiration, and ground water have resulted from investigations conducted by the U. S. Weather Bureau, the U. S. Geological Survey, the Bureau of Plant Industry, the Bureau of Soils, the Forest Service, the U. S. Corps of Engineers, and other government agencies. Valuable contributions have been made by various universities, scientific organizations, and private individuals. An enormous amount of further investigation and study is needed along these lines, however, before these data can be properly correlated and used in the solution of hydrologic problems.

The duty of collecting these basic data is primarily a governmental function. No private individual or organization can be

expected to finance and carry on the long, laborious, and expensive observations and experiments that are required for their collection. Especially is this true inasmuch as these data are used for the benefit of the general public. In view of the vast current government expenditures, it is unfortunate that the federal departments in charge of the important function of collecting these basic data continually find themselves seriously handicapped through lack of funds so that their work is either curtailed or completely stopped, thus greatly reducing the final value of the results.

Opportunities for Research

Here is a field in which the opportunities for research are almost unlimited. There are so many factors that affect stream flow, precipitation, and their interrelationship that to prepare a complete list of all the subjects that are in need of investigation would be a long and difficult task. However, the vast extent of the work that remains to be done in this field should deter no one from engaging in it. Although the main problem taken in its entirety is much too large for any one person to solve single handed, it naturally divides itself into a large number of smaller fields, any one of which provides abundant opportunity for research. Just as tiny raindrops slowly wear away the rock, so also will small contributions toward our general store of knowledge on these subsidiary questions eventually build up a chain of evidence that will solve the many current mysteries in the field of hydrology.

Hydrologic Failures

It is an unfortunate trait of human nature that all professions alike hesitate to advertise their failures. Notable successes are broadcast for all the world to hear, but failures are spoken of only in muffled tones. Professional pride and ethics are the principal reasons for this situation. It is nevertheless true that a full knowledge of the failures and their causes provides some of the most valuable information that can possibly serve to guide the engineer or other professional practitioner.

No attempt will be made here to present a list of the almost countless failures that have resulted from a faulty understanding of the principles of hydrology. The history of hydraulic structures is literally filled with examples of such failures. Beyond question a very large majority, perhaps over 90 per cent, of all failures of

hydraulic structures are directly due to hydrologic reasons rather than to structural weaknesses. This may be due in part to the fact that the principles of structural design have been more completely formulated and have been better understood, but it is also due in part to the fact that a greater safety factor is used in structural work than will ever be permissible in hydrologic computations. In the former, a factor of 3 or 4 is not uncommon, whereas in the latter case the requirements of economic design do not permit such high factors of safety.

Examples of hydrologic failures include the failure of dams resulting from inadequate spillway capacity, causing overtopping and erosion of embankments; the economic failure of water-power developments, storage reservoirs, and water-supply systems resulting from an overestimate of the available supply; the failure of a sewerage or drainage system to function as planned due to the occurrence of more intense storms than were anticipated; the failure of highway and railway bridges and culverts resulting from inadequate waterway openings; and so on for every type of hydraulic structure. At this point, it should, however, be explained that practically never should a structure be so designed that it could accommodate any possible flood to which it might be subjected. The only possible exception to this general rule is where a failure would result in great human suffering, loss of life, and tremendous property damage. In other cases, the problem is purely economic. The question is simply one of determining to what extent expenditures are justifiable from an economic viewpoint. In other words, it may oftentimes be true (paradoxical as it may seem) that the best-designed structure would have insufficient capacity for the very largest floods, whereas a poorly designed structure might have adequate capacity.

It is not uncommon to hear a learned judge, in rendering a decision in a case involving damages resulting from an unprecedented flood or other unusual natural phenomenon, refer to such occurrence as "an act of God." Such expressions are often misleading. It would be equally appropriate to refer in the same way to every rainfall or to every wind that blows. Every natural phenomenon springs from natural causes and occurs in exact obedience to definite natural laws. When those laws are once fully understood it will in all probability become possible to predict the occurrence of storms, floods, and all other natural phenomena far in

advance and with nearly the same degree of certainty as that with which it is now possible to predict the exact hour of sunrise on any day of the year.

Hydrologic Data

Many types of hydrologic data are collected and published by agencies of the federal government. Other data are collected by such organizations as the Tennessee Valley Authority, the Miami Conservancy District, and agencies of state governments. It is frequently possible to obtain valuable information from the engineering staffs of municipalities or power companies, from consulting engineering firms, and sometimes from amateur observers. More detailed information than that which is published may often be obtained from the organization that made the observations.

A complete description of information available from federal agencies is given in "Principal Federal Sources of Hydrologic Data," *Tech. Paper* 10, Water Resources Committee of the National Resources Planning Board. Another useful general reference is "Inventory of Unpublished Hydrologic Data," *U. S. Geological Survey Water-Supply Paper* 837.

In the following list are given some of the more important sources of hydrologic data.

Precipitation

Climatological Data, U. S. Weather Bureau (hourly).

Hydrologic Bulletin, Daily and Hourly Precipitation, U. S. Weather Bureau and U. S. Corps of Engineers. (These bulletins may be found at regional offices located in Albany, N. Y.; Macon, Ga.; Chicago, Ill.; Cincinnati, Ohio; Kansas City, Mo.; Fort Worth, Tex.; Albuquerque, N. Mex.; Portland, Oreg.; and San Francisco, Calif.)

"Storm Rainfall of Eastern United States," *Tech. Reports*, Part V, Miami Conservancy District. (Intense storms.)

Storm Rainfall in the United States, U. S. Corps of Engineers. (Intense storms.)

"Rainfall Frequency-Intensity Data," *U. S. Department of Agriculture Misc. Pub.* 204.

Other sources of information on precipitation are the Tennessee Valley Authority, the U. S. Soil Conservation Service, and the U. S. Forest Service.

Stream Flow

U. S. Geological Survey Water-Supply Papers.

Stream-flow data are also obtained by the U. S. Corps of Engineers, the

Tennessee Valley Authority, the U. S. Forest Service, and the U. S. Soil Conservation Service.

Evaporation from Water Surfaces; Temperature; Wind Velocity; Humidity

Climatological Data, U. S. Weather Bureau.

Information may also be obtained from the U. S. Bureau of Reclamation, the U. S. Soil Conservation Service, the U. S. Forest Service, and the Tennessee Valley Authority.

Ground Water

U. S. Geological Survey Water-Supply Papers.

Ground-water records are also obtained by many state agencies, the U. S. Corps of Engineers, the Tennessee Valley Authority, the U. S. Soil Conservation Service, and the U. S. Forest Service.

CHAPTER II

THE HYDROGRAPH

Definition

A hydrograph of a stream is a graphical representation of its fluctuations in flow arranged in chronological order (see Fig. 2). A complete hydrograph shows every minor variation in flow and can therefore be obtained only from an instrument that continuously records those changes, for no stream is constant in flow even for short periods of time. Frequently, however, such continuous records are not available, but instead instantaneous readings are obtained, usually one or two each day, and from these readings hydrographs are plotted. For streams that fluctuate but slowly, such results are ordinarily satisfactory, but for flashy streams they are inadequate for many purposes. In Fig. 2 is shown in solid line the hydrograph of Fall Creek near Ithaca, New York, from July 6 to July 16, 1935, as obtained from an automatic recording gage. The broken line shows the graph that would have been obtained from daily readings taken at 6 P.M. These graphs illustrate the inaccuracies that often result when hydrographs are plotted from too infrequent gage readings.

Instead of hydrographs, stage graphs are sometimes plotted to show the variation in the elevation of water surface as it occurs from time to time. As an indication of the danger from overflow of banks in the immediate vicinity, this procedure is perfectly satisfactory. There are certain objections, however, to its general use. The relationship between stage and discharge is seldom permanent. The channel is usually either silting or eroding. As a result, the stage at one point provides no reliable index either of the volume of water flowing past that point or of the stage that is later to be expected at other points downstream. For most engineering purposes, the important matter is the determination of the rate of discharge expressed either in cubic feet per second or in some similar unit. For this reason, discharge in cubic feet per second, rather than stage above a fixed datum plane, is more commonly plotted against time.

Another advantage in the use of discharge as the unit, instead of stage, arises from the frequent desirability of comparing the flow of one stream with that of another. With the discharge unit such comparisons are easily made, but with the stage unit they are impossible. For comparing the yield of one drainage basin

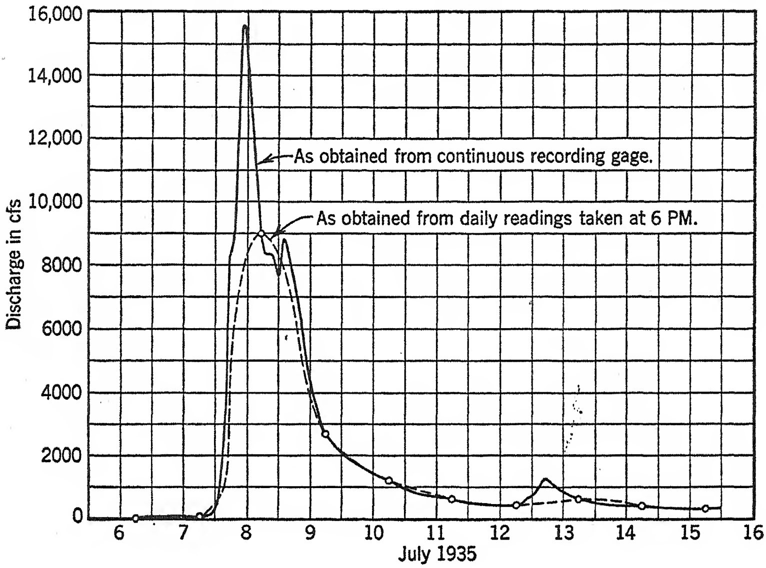


FIG. 2.

with that of another by reducing the discharges to cubic feet per second per square mile, the element of size of basin is eliminated and the comparisons are direct.

Variability of Stream Flow

Perhaps the most common misconception of stream flow existent among those who are not familiar with the subject is the idea that the yield of any drainage basin is more or less fixed, definite, and easily determinable; that a few measurements are all that are needed for determining the answer to any question or the solution of any problem that may arise in connection with that particular stream. Nothing could be farther from the truth.

The flow of every stream is constantly changing. Perhaps never has the hydrograph of any stream, even for a 24-hr period, been *exactly* duplicated on any other day. For longer periods the

differences increase, and seldom do the hydrographs of a stream for different years bear any great resemblance to each other.

Not only do the hydrographs differ from year to year but also the maximum, minimum, and average flows for any year differ from those of every other year. Nor are these differences ordinarily of a minor character. It is not unusual for the maximum flood flow for one year to be many times as great as the maximum flow for the preceding year or perhaps for several years preceding. In 1921 the Arkansas River at Pueblo, Colorado, experienced a flood that was over ten times as great as had ever occurred in the entire preceding period of record covering 29 yr. In a similar manner the minimum and average annual flows are likely to differ greatly from year to year. With these facts clearly in mind, the engineer recognizes the difficulties that confront him when he attempts to determine the maximum flood or minimum flow that may be expected in any given period or the average flow that can be depended upon for any specific purpose, and he is warned of the dangers that beset him if he jumps too quickly to a conclusion based upon only a few years of records.

Terms and Units

Although slightly different shades of meaning are sometimes given to these terms by various writers, throughout this book the terms *stream flow*, *runoff*, *discharge*, and *yield of drainage basin* are used practically synonymously. However, yield is usually considered in terms of total volume per year or as average flow for long periods of time, whereas these other terms ordinarily are applied to instantaneous rates or to average rates for shorter periods. Attention is here called to the fact that *runoff* is by no means the same as *surface runoff*. Runoff includes all the water flowing in the stream channel past any given section, whereas surface runoff includes only the water that reaches the stream channel without first percolating down to the water table. The units in which these quantities are expressed are always volume per unit of time. Many different units of volume and time are used, however. The following are the most common:

1. Cubic feet per second (cfs).
2. Cubic feet per second per square mile (csm).
3. Acre feet per day, month, or year.

4. Inches depth on drainage basin per day, month, or year.
5. Million gallons per day (mgd).

The first two of these terms are self-explanatory. An acre-foot per day is the rate of flow of that stream which, if it discharges into a reservoir having an area of 1 acre, will fill it to a depth of 1 ft in 1 day. It is, therefore, a rate of 43,560 cu ft per day. Since there are 86,400 sec in a day, for most practical purposes it is sufficiently accurate to consider a cubic foot per second as being equivalent to 2 acre feet per day.

Rainfall is usually expressed as inches depth on the drainage basin. For comparison, it is convenient to express runoff in the same units. If we let T_d represent the number of days in the period during which Q is the average discharge in cubic feet per second, then 86,400 T_dQ is the total runoff in cubic feet. Also if we let A represent the area in square miles from which Q is the runoff, then 5280^2A is the total area in square feet, and the depth in inches is

$$D_i = \frac{86{,}400T_dQ \times 12}{5280^2A} = \frac{T_dQ}{26.9A}$$

In some branches of engineering the commonly used unit of discharge is a million gallons per day. For this conversion

$$\frac{\text{cfs} \times 7.48 \times 86{,}400}{1{,}000{,}000} = \frac{\text{cfs}}{1.547} = \text{mgd} \quad \text{or} \quad \text{cfs} = 1.547 \text{ mgd}$$

Direct Sources of Runoff

The water flowing in a stream may have found its way into the stream channel from one or more of several different sources, namely:

1. Precipitation falling directly on the surface of the stream and its tributaries.

2. Surface runoff, that is, water that falls as precipitation on the ground surface and finds its way into the stream channel without infiltrating into the soil and percolating down to the water table.

3. Ground-water flow or water that had its origin in precipitation but infiltrated into the soil, joined the ground water, and then, after days, weeks, or even much longer periods, found its way through the soil into the stream.

In the above classification, the second source should, for certain areas, be divided into (1) water that flows directly over the ground surface and (2) water that infiltrates and then percolates, usually through a thin layer of loosely textured surface soil, until it encounters a relatively impervious substratum, after which a part of it may continue its downward journey, whereas the remainder moves laterally toward the stream channel and never penetrates to the water table. This latter quantity will be called *subsurface storm flow*. It behaves more nearly like surface runoff than like ground-water flow because it reaches the stream so quickly that it is usually difficult to distinguish it from true surface runoff. On the other hand, ground-water flow is oftentimes long delayed before it reaches the stream. For this reason, subsurface storm flow will, throughout this book, be treated as though it were a part of surface runoff.

For streams draining most basins except those having a large percentage of lake area such as the St. Lawrence, the first of these sources, direct precipitation, provides a relatively small portion of the total flow. Even in such exceptional areas as above noted, the evaporation from those water surfaces may nearly or more than balance the precipitation on them. This factor will therefore be ignored in the present discussion.

Except for some glacier-fed streams or streams with a large amount of lake storage, surface runoff from drainage basins whose area does not exceed a few thousand square miles is intermittent, occurring only during or immediately following periods of precipitation or of the melting of accumulated snow and ice. It provides the vast bulk of the water that produces floods. Drainage basins that are so pervious as to permit little or no surface runoff are seldom if ever subject to disastrous floods.

Classification of Streams

All streams may be divided into three general classes, each having a characteristic type of runoff depending upon the physical characteristics and climatic conditions of the drainage basin, namely:

1. Ephemeral.
2. Intermittent.
3. Perennial.

Ephemeral streams carry only surface runoff and hence flow only during and immediately after periods of precipitation or the melting of accumulated snow. They have no permanent or well-defined channels but follow slight depressions in the natural contour of the ground surface. The drainage basin is either impervious or the ground-water table is always below the bed of the ephemeral stream throughout its entire length; otherwise at times the flow would be sustained by ground water.

Intermittent streams, in general, flow during wet seasons and are dry during dry seasons. The ground-water table lies above the bed of the stream during the wet season but drops below the bed during dry seasons. Hence the flow is derived principally from surface runoff but during wet seasons receives a contribution from ground water. However, in the arid southwestern part of the United States there are many drainage basins, some having large areas, in which the stream channels are always above the water table and therefore carry only surface runoff. At times these basins are subjected to brief but intense rainfalls. Following such storms, gulches that are normally dry may carry raging torrents for brief periods, only to return to their dry state a few hours later. Another type of intermittent stream is sometimes found in northern latitudes where in the winter the flow is interrupted by the freezing of the ground water to some depth below the stream bed. This phenomenon usually occurs only in the smaller streams.

Perennial streams flow at all times. In such streams, even during the most severe droughts, the ground-water table never drops below the bed of the stream and therefore maintains a continuous supply.

It should be understood that the above classification applies only to a section or reach of a stream and ordinarily not to the entire drainage system. Perhaps only streams that have springs as their origin are perennial throughout their entire length, and few if any of importance are intermittent in their lower reaches.

The Runoff Process

When rain starts falling on a more or less pervious area, there is an initial period during which (1) the rainfall is intercepted by buildings, trees, shrubs, grasses, or other objects and thus prevented from reaching the ground; (2) it infiltrates into the ground; or (3) it finds its way to innumerable small and large depressions,

filling them to their overflow level. The first of these quantities, I,[1] is termed *rainfall interception*. Although not usually of major importance, it is oftentimes the means of disposal of the greater portion of the lighter rains. The second quantity is called *infiltration*, F. The maximum rate at which a soil, when in a given condition, can absorb water is its *infiltration capacity*, f. The last quantity is termed *depression storage*, S_d. All this storage is either evaporated or used by vegetation, or it infiltrates into the soil—none of it appears as surface runoff. The difference between the *total rainfall*, P, and that which is intercepted is called *ground rainfall*, P_g.

If, after the depression storage is filled, the rain intensity exceeds the infiltration capacity of the soil, the difference is called *rainfall excess*, p_e, or supply. Hence $p_e = p - f$. This excess first accumulates on the ground as *surface detention*, D, and then flows overland toward the stream channels. This movement is called *overland flow*, and the water that thus reaches the stream channels is *surface runoff*. Surface runoff can occur only as a result of storms having a rainfall excess. All water contained at any instant within the permanent stream channels is called *channel storage*, S_c. Surface runoff is said to be occurring at the basin outlet throughout the entire period of passage of water that reached the stream channels through overland flow.

The rain that falls in the beginning of a storm before the depression storage is completely filled is called the *initial rain*, and that falling near the end at a rate less than infiltration capacity is called *residual rain*. The intervening period is the *net supply interval*. The total surface runoff resulting from any storm is equal to the total rainfall excess minus the difference between the total residual rain and the total infiltration during overland flow after the end of the net supply interval.

Beneath the surface of most drainage basins whose overburden extends to any considerable depth is a *water table* below which the voids are completely filled with water. Only the water that is below the water table is called *ground water*; that which is above is called *field moisture*. The region above the water table is divided

[1] Throughout this discussion capital letters, with or without subscripts, are used to designate quantities, whereas lower-case letters symbolize rates. Both quantities and rates are usually expressed in terms of inches depth on the basin.

into three zones: (1) capillary zone, (2) intermediate zone, and (3) soil zone. Extending above the water table a distance usually ranging from about 1 ft to 8 or 10 ft, depending principally upon texture, is a zone called the capillary fringe throughout which the moisture content is maintained practically constant by capillarity. Extending down from the ground surface is the *soil zone,* which is defined as being the depth of overburden that is penetrated by the roots of vegetation. Throughout this zone the moisture content varies tremendously, ranging from a partly saturated state during and immediately following periods of protracted rainfall to a minimum content after a long-continued drought.

The region between the capillary fringe and the soil zone is called the intermediate zone. Throughout this zone the amount of water contained within any given volume of space is practically constant year in and year out. In some places the capillary fringe extends up into the soil zone, and where this occurs there is, of course, no intermediate zone.

Soon after a rain when all the gravity water has drained down to the water table a certain amount of water is retained on the surfaces of the soil grains by molecular attraction. This is called *pellicular water.* The maximum depth of this water that any soil can retain indefinitely against the action of gravity is called its *field capacity.* That portion of the pellicular water that is easily abstracted by the root action of vegetation is called *available moisture;* the remainder is *unavailable moisture* and by some writers has been termed *hygroscopic* water. The depth of water required to bring the soil moisture content up to field capacity is called the *field moisture deficiency.* During a rain any existing deficiency occurring at a given point must first be supplied before there can be any ground-water accretion. However, because of the varying amounts of soil moisture deficiency and the varying rates of replenishment at different points in a drainage basin, it is not at all uncommon for ground-water accretion to be occurring throughout certain portions of the basin although soil moisture deficiencies still exist in the remainder.

The water table normally slopes more or less gently toward its outlet which may be a stream, a lake, or the sea. The movement of ground water is usually extremely slow. Its velocity depends upon the gradient of the water table. Since that gradient is affected but little by ground-water accretion, the velocity itself varies only

to a minor extent, and, as a result, the fluctuations in the ground-water contributions to stream flow are slow. Hence streams which drain pervious areas and which are dependent largely upon ground water for their supply are relatively steady in their yield.

When there is no surface runoff from rainfall or melting snow, the stream flow is derived entirely from ground water. This results in a steady lowering of the water table and a constantly

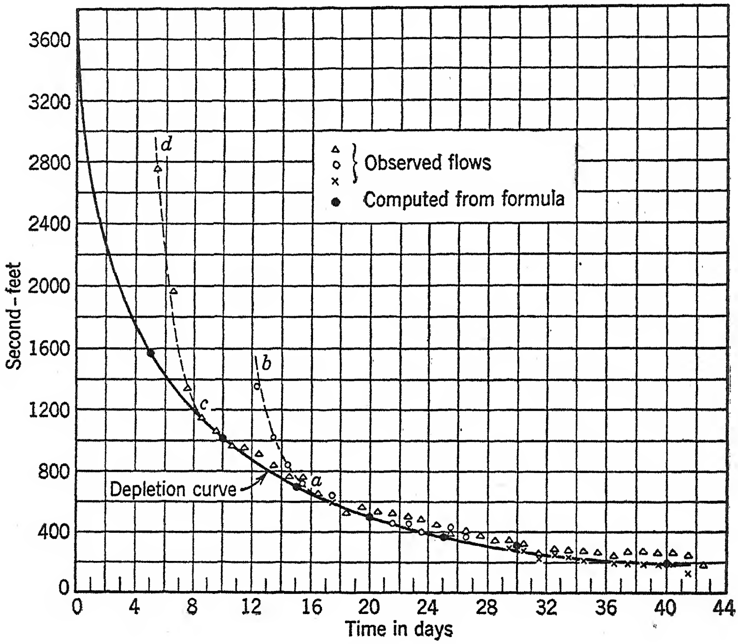


FIG. 3.

diminishing stream flow until a rain occurs of sufficient magnitude to produce either surface runoff or ground-water accretion. If the ground-water level were at its maximum height at the end of a period of surface runoff and no further precipitation should occur until stream flow ceased entirely, the resulting hydrograph during this period would represent a *ground-water depletion curve*. In a region of moderate or high precipitation, rarely if ever is there a rainless period of sufficient duration to permit the continuous development of a complete depletion curve. However, such a curve can usually be constructed from a number of segments of hydrographs each connecting successive periods of surface runoff.

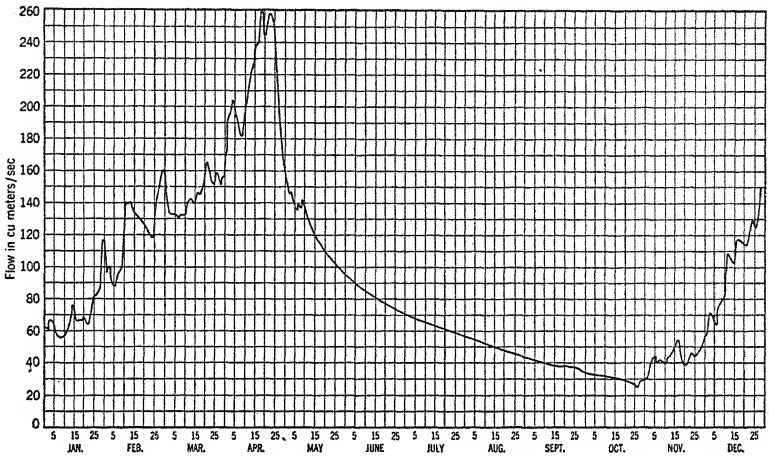


Fig. 4.

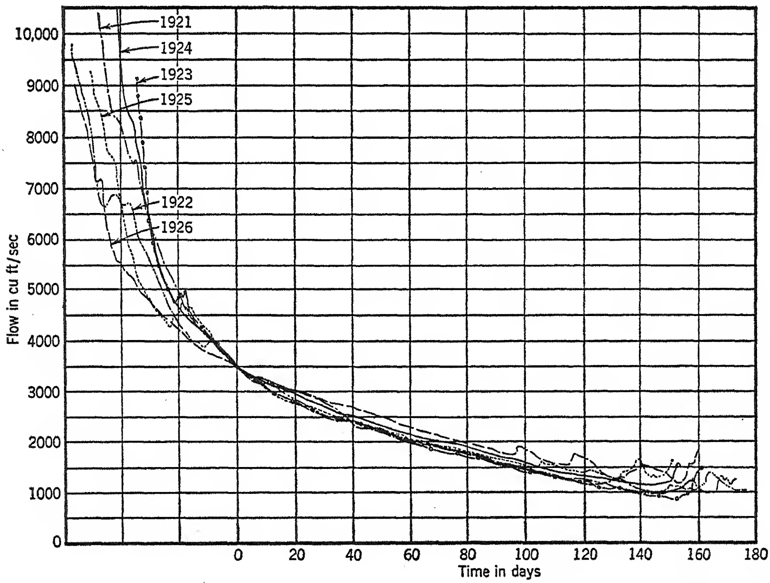


Fig. 5. Annual depletion curves for Lualaba River at N'Zilo.

In Fig. 3 is shown a depletion curve for Iowa River at Iowa City as derived by Horton.[1]

[1] Robert E. Horton, *Surface Runoff Phenomena*, Edwards Brothers, Inc., Ann Arbor, Michigan, 1935, p. 43.

The hydrograph of the Lualaba River (Fig. 4) in the Belgian Congo presents a striking example of a ground-water depletion curve. This is a tropical stream with extensive ground-water storage, no surface storage, and marked seasonal rainfall with light or no rainfall in the months of May to September inclusive. During this period there is a uniform depletion or recession curve, representing outflow from ground-water storage, whereas during the remainder of the year the rainfall is relatively heavy and surface runoff predominates. In Fig. 5 are shown depletion curves of Lualaba River for the years 1921 to 1926 inclusive. These are plotted so as to coincide when the flow is 3500 cfs. It will be noted that, from then on until the fall rains set in several months later, the curves are remarkably similar.

Stream Rises and Floods

Surface runoff invariably produces a stream rise but does not necessarily cause a flood, the difference being in magnitude only. It is impossible to differentiate rigidly between these two phenomena or to say that one particular stream rise is a flood and that another nearly as great is not a flood. A flood is commonly defined as being an unusually or abnormally high stage of the river. It is sometimes further described as being a stage so high as to overflow the banks and inundate the adjacent lands. Although it is true that the latter condition usually accompanies floods, it is not an essential characteristic, for, if it were, streams flowing through deep ravines, gorges, or canyons would never be subject to floods. However, streams are commonly recognized as being in flood when their stage is unusually high. Increases in flow of a lesser magnitude such as normally occur many times each year are called stream rises.

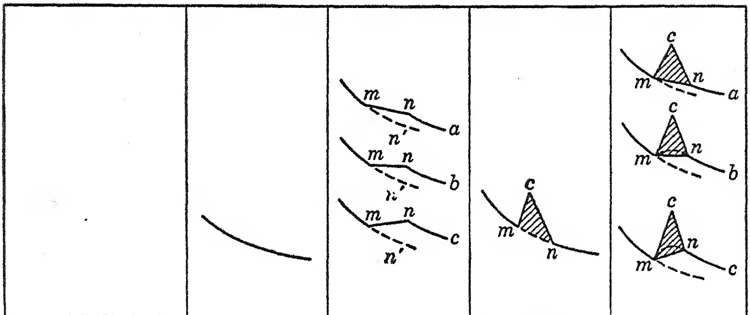
Classification of Stream Rises

The effect that a storm has upon the subsequent stream flow depends both upon the nature of the storm and upon the physical characteristics of the drainage basin. Horton gives the following classification of stream rises, a summary of which is shown in Fig. 6.[1]

Type 0 is so designated because nothing happens as far as the stream is concerned. For this type the rain intensity is less than the infiltration

[1] *Ibid.*, pp. 46 and 47.

capacity. There is, therefore, no surface runoff. The total infiltration is less than the field moisture deficiency and there is therefore no accretion to ground-water. The normal depletion curve continues its downward course uninterrupted. There is therefore no rise in the stream. These phenomena are characteristic of light rains occurring during generally dry weather, particularly after long droughts when the soil has the maximum infiltration capacity and large field moisture deficiency.



Type	0	1	2	3
Rain intensity	$<f$	$<f$	$>f$	$>f$
Field-moisture deficiency	$>P$	$<P$	$>F$	$<F$
Surface runoff	None	None	$Q_s = P_e$	$Q_s = P_e$
Ground-water accretion	None	P − fmd	None	F − fmd
Flow increase	None	Ground-water flow only	Surface runoff only	Surface and ground-water runoff

FIG. 6. Classification of stream rises. (Surface runoff is cross sectioned). After Horton.

Type 0 is, however, something more than a gesture since soil moisture accretion takes place. Soil moisture accretion effects are cumulative and the occurrence of conditions of Type 0 may hasten the time when a real rise in the stream will occur.

Type 1. Again the rain intensity is less than infiltration capacity and no surface runoff occurs. The total infiltration is greater than the field moisture deficiency and some accretion to the water table takes place, accompanied either by an increase in ground-water flow or a slowing down of the ground-water depletion rate. These small irregularities in a hydrograph look like the effects of observational errors or barometric

changes but they frequently result from rain. They are typical effects of light rain in the spring and of somewhat heavier rain of low intensity in the summer and fall.

Three different cases occur under Type 1. In each case accretion to the water table takes place during the interval denoted by *mn*. Normal ground-water depletion interrupted at *m* is resumed at *n*, while *n'* shows the corresponding stage had there been no ground-water accretion. In case (*a*) the rate of accretion is less than the rate of normal ground-water depletion. The depletion therefore continues but at a reduced rate. In case (*b*) the accretion and depletion rates are equal and the ground-water flow rate remains constant for a time. In case (*c*) the rate of ground-water accretion exceeds the rate of normal depletion and there is a rise of the water table and an increase in the ground-water outflow rate.

Type 2. Here the rain intensity exceeds the infiltration capacity and surface runoff occurs, but the total infiltration is less than the initial field moisture deficiency and there is no accretion to the ground-water and hence no change in ground-water flow. The normal depletion continues during the rise and the ground-water regimen is resumed at *n*. The stream falls after the rise to a lower stage than pertained when the rise began. This is a growing season or midsummer type and commonly occurs when the field moisture deficiency is large enough so that the field moisture capacity is not fully restored by infiltration. Such rises are typical of the effect of short, sharp showers of the thunderstorm type.

Type 3. Again the rain intensity exceeds the infiltration capacity and surface runoff occurs. In this type of rise, the total infiltration exceeds the field moisture deficiency and accretion to the water-table takes place. The point *n* at which the rise ends is the point at which the recession side *cn* of the discharge graph coincides with the normal depletion curve.

There are three cases under Type 3 identical with those for Type 1 rises, each dependent on the rate of ground-water accretion. Normal depletion flow is resumed at the end of a rise of Type 3 at a higher stage than for a rise of Type 2, other things equal, but the stage at the end of the rise may or may not be higher than the initial stage. In cases (*a*) or (*b*), Type 3, the stage at which the normal depletion flow is resumed will not be higher than the initial stage; in case (*c*) it will be higher. Whether a given rise is of Type 2 or Type 3 can be determined by extending the normal depletion curve underneath the rise. If the recession side of the graph returns to this curve as extended, the rise is of Type 2. If the normal depletion curve at the end of the rise is at a higher level than the extended curve under the graph, the rise is of Type 3.

Having a hydrograph of a rise plotted on a suitable scale, together with the rain graph which produced it, it is possible by inspection to

determine with considerable certainty to which of the above-described classes the rise belongs.

Hydrograph Analysis

The analysis of a hydrograph involves a separation of the various component contributions to stream flow with respect to their sources, which combined produce the total flow at the outlet of the drainage basin. As explained in the preceding paragraphs these sources consist of (1) precipitation received directly on the surfaces of the contributing waters, (2) surface runoff, (3) subsurface flow, and (4) strictly ground-water flow or, in other words, water draining into the stream from beneath the water table.

At this point let us consider the subsurface conditions during a period of rainfall and the subsequent stream rise. In Fig. 7 is shown

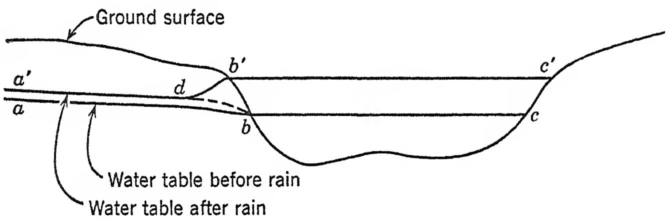


FIG. 7.

a vertical cross section of a stream channel and the adjacent banks, together with a profile of the water table, *ab*, as it existed at the end of a rainless period. Later, after a period of rainfall of sufficient duration and intensity to permit the infiltration to replenish the field moisture deficiency and provide ground-water accretion and after a period of surface runoff, this water table becomes *a′db′*. Had the water surface in the stream remained at *bc* the water table would have become *a′db*. Except for the smallest stream rises, the rise in the stage of the river occurs more quickly and is much greater in magnitude than the corresponding rise of the water table. This is evident when one considers the fact that, although an inch of infiltration can raise the water table only a few inches, an inch of surface runoff can easily produce a stream rise of several feet. Consequently as quickly as the water surface in the stream rises higher than the adjacent water table, thus creating at any given elevation a greater hydrostatic pressure in

the stream than in the banks, ground-water inflow into the stream channel ceases temporarily and the direction of flow reverses, creating bank storage represented in Fig. 7 by *db′b*. The volume of this bank storage continues to increase as long as there is a depression in the water table at *d* or until after the stream has passed its peak stage. As soon as the stage starts to fall the direction of flow again reverses, and for a time, because of the accumulated bank storage, the ground-water contribution to the stream is considerably increased. As soon as the bank storage is drained out, the ground-water flow again follows the normal depletion curve.

The manner in which the ground-water contribution to the stream fluctuates during this rise is, therefore, represented by the

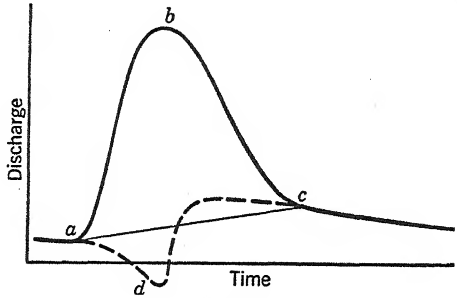


FIG. 8.

ordinates to the dashed line *adc*, Fig. 8. The portion falling below the horizontal axis represents outflow from the stream or bank storage. Inasmuch as it is impractical to determine the actual amount of ground-water flow occurring at any time during a stream rise and because it ordinarily represents but a small portion of the total runoff, the most common method of separating ground-water flow from surface runoff is by drawing a straight line, such as *ac*, Fig. 8. The exact location of *c* usually cannot be determined, but this is not of great importance as long as one always follows a consistent procedure. It may, however, be taken as the point of greatest curvature near the lower end of the recession side of the hydrograph. This point can usually be determined with the greatest assurance for a hydrograph of a single sharp stream rise resulting from a relatively short but intense rain. The location of *c* on such a graph may then be used as a guide for selecting a similar point on a

more complex graph, by making the duration of surface runoff following the end of rainfall excess the same in all cases.

Another method which may assist in selecting a consistent location of c is to use as an index the ratio of the discharge at any time to the discharge a short time, such as an hour, earlier. For many streams the value of this ratio will increase steadily along the recession side of the hydrograph until channel storage is depleted and will then become nearly constant for the ground-water depletion curve. Sometimes the location of c is based on an arbitrary value of this ratio.

The Unit Hydrograph

In 1932 LeRoy K. Sherman[1] first presented his now almost universally accepted theory of the unit hydrograph. This new concept of surface runoff is one of the most important contributions ever made to the science of hydrology. It provides a most useful tool for the determination of the hydrograph of surface runoff that will result from any given storm.

Briefly stated, the theory of the unit hydrograph is based upon two fundamental principles, viz.:[2]

1. For all unit storms regardless of their intensity the period of surface runoff is approximately the same. A unit storm may be defined as a storm of such duration that the period of surface runoff is not appreciably less for any storm of shorter duration.

2. If the total period of surface runoff is divided into any given number of equal time intervals the percentage of the total that occurs during each of these periods will be approximately the same for all unit hydrographs regardless of the magnitude of the total runoff.

In other words, suppose that two unit storms should occur on a given drainage basin, each of them lasting 24 hr, the first having a total rainfall of 3 in. and the second 5 in. If the time required for the surface runoff to pass a point at the outlet of this basin is 8 days for the first storm, it will also be approximately 8 days for the second storm or for any other storm lasting 24 hr or less. Also if the total runoff during the day of peak flow is, for example,

[1] L. K. Sherman, Streamflow from Rainfall by the Unit-Graph Method, *Eng. News-Record*, **108**, 501.

[2] Sherman's definitions of a unit hydrograph are given in Chapter VIII, p. 308. See also pp. 290 and 343 for a further discussion of unit hydrographs.

20 per cent of the total surface runoff for the first storm, it will also be 20 per cent of the total for the second storm or for any other storm of this same duration regardless of its intensity.

Although it can be easily proved theoretically that the above relationships cannot possibly hold true rigidly, the error is so trivial that from a practical viewpoint they may be considered as being correct. This theory in conjunction with the infiltration theory which will be discussed in a subsequent chapter provides the basis for the best method known at the present time for determining the hydrograph of surface runoff that may be expected from any given storm pattern.

Factors Affecting Hydrograph

The nature of the hydrograph of any stream is determined by two entirely different sets of factors, the one depending upon the nature of the precipitation and the other upon the physical characteristics of the drainage basin. The influence of the first group depends upon:

1. Type of precipitation.
2. Rainfall intensity.
3. Duration of rainfall.
4. Distribution of rainfall on basin.
5. Direction of storm movement.

Consideration will now be given to the effects produced by each of these factors upon stream flow. Throughout the discussion of the influence exerted by each it will be assumed that all other variables remain constant.

From the relatively steady flow of streams draining large flat pervious basins such as shown in Fig. 9 to the erratic yield of small impervious mountainous areas as shown in Fig. 10, there is every possible gradation in flow characteristics. These differences are the result of the combined influences of the various physical characteristics of the drainage basins. So important and extensive are these influences that a subsequent chapter will be devoted exclusively to this subject.

Effect of Type of Precipitation. In considering the influence of precipitation upon the hydrograph, the type of precipitation is of first importance. For instance, if precipitation falls in the summer in the form of rain, its influence is felt almost immediately pro-

vided only that its intensity and magnitude are great enough to affect runoff. On the other hand, if the basin lies in a northern latitude and the precipitation during a given period is entirely in the form of snow with no thawing temperatures throughout the

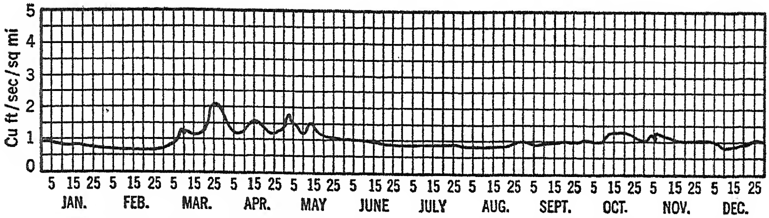


FIG. 9. Hydrograph of Manistee River near Sherman, Michigan, 1936.

period, the hydrograph will be unaffected except for the slight influence of the snowfall that is received directly on the water surface of the stream.

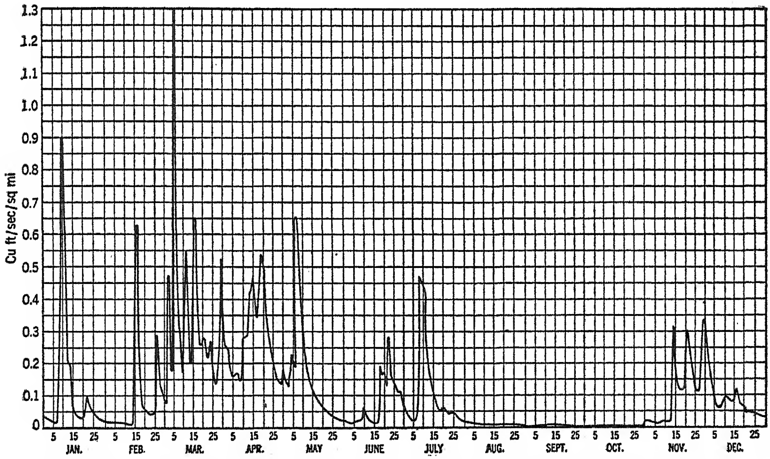


FIG. 10. Hydrograph of Catskill Creek at Oak Hill, New York, 1935.

The effect upon stream flow of snowfall when it is finally removed from the ground surface by melting at a later date is extremely difficult to predict. Because of the high rate of evaporation from snow, the entire snowfall unless followed by further precipitation or melting may be returned directly to the air in the form of vapor, thus producing no increase in runoff. If, however,

the snow falls on a saturated frozen surface and is soon followed by a warm period perhaps accompanied by rain, it is quite possible that practically the entire snowfall may appear directly as surface runoff.

Or, again, if the snow falls on a loosely textured but relatively dry ground surface, it may upon melting be entirely absorbed by the ground. Since at this time vegetation is inert, practically the entire amount is added to the ground water and most of it may later appear as stream flow.

Effect of Rain Intensity. Rain intensity has a direct bearing upon the resulting hydrograph, as has already been shown under the classification of stream rises (page 26).

When the intensity is great enough to exceed the infiltration capacity, f, and to produce surface runoff, the height of the stream rise increases rapidly with any further increase in the intensity. For instance, if the storm has been in progress long enough to permit the infiltration rate to become practically constant and equal for example to ½ in. per hr at the time when the storm has reached its maximum intensity, p_m, then the rate at which the rainfall excess, p_e, accumulates on the ground surface prior to overland flow to the stream channels is equal to $p_m - f$. If p_m is equal to 0.6 in. per hr, p_e will accumulate at the rate of 0.1 in. per hr, whereas if p_m is 1.0 in. per hr the accumulation is five times as rapid or 0.5 in. per hr. It follows therefore that, after infiltration capacity is exceeded, surface runoff will increase rapidly with an increase in rainfall intensity. However, the increase in stream flow is not at the same rate as the increase in rainfall excess because of the lag effect resulting from storage.

It should be observed that on any drainage basin for all storms of a given duration the principal difference produced in the hydrograph by storms of different intensities is in the height of the resulting stream rise. The width of base or, in other words, the time during which surface runoff is taking place is not greatly affected.

Effect of Duration of Rainfall. As already explained, for every drainage basin there is a certain unit storm period such that, for all storms of that duration or less, regardless of intensity, the period of surface runoff will be the same. Also the period of rise, or the time that elapses from the beginning of surface runoff until

the hydrograph reaches its peak, is approximately the same for all unit storms, regardless of their duration or intensity.

Let us now consider the effect of duration of rainfall excess upon the hydrograph of surface runoff when that duration exceeds the period of a unit storm. In doing this it will be necessary to consider all other factors such as intensity and distribution as remaining constant and the same in all cases. In Fig. 11, A represents an

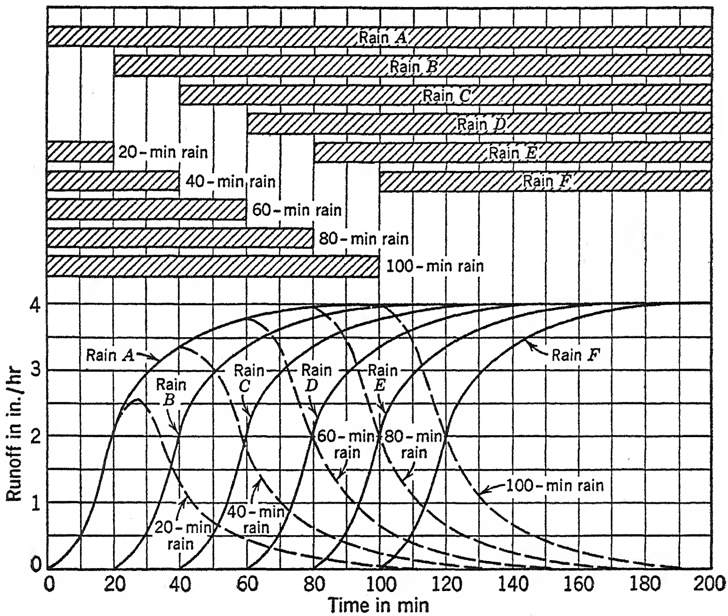


FIG. 11.

assumed hydrograph of surface runoff resulting from a storm in which the rainfall excess or supply rate as it is often called is 4 in. per hr and continues at that rate indefinitely. In this same figure hydrographs B, C, D, and E represent the rates of surface runoff resulting from storms of the same intensity, but each starting 20 min later than the preceding one. It follows that, if a storm of this same intensity were to continue for only 20 min and then stop, the hydrograph of the resulting runoff could be obtained by subtracting the ordinates of curve B from those of curve A, as shown by $A - B$. In a similar manner if this same storm had continued

for 40 min the resulting hydrograph would be represented by the differences between hydrographs A and C or by $A - C$, and so on for the same supply rates continuing for 60 and 80 min. On drainage basins of any considerable size, storms do not continue at this maximum rate long enough to permit this peak runoff rate to be reached. However, in connection with airport, storm sewer, and small culvert drainage problems, this relationship does become significant.

Perhaps it should be noted that throughout the above discussion it is assumed that the rainfall excess or supply rate remains constant. This means that because of the gradual reduction in infiltration capacity it is assumed that the rainfall intensity diminishes at the same rate. Only in this way can the effect of duration be shown.

Effect of Distribution of Rainfall on Basin. In the foregoing discussion it has been tacitly assumed that the various rainfalls considered were uniformly distributed over the drainage basin with respect both to area and to time of occurrence. If also the topography, soil, and other conditions are uniform throughout the basin, then, for all storms in which the total volume of rainfall is the same, the minimum peak runoff will be produced by that rain that is uniformly distributed. For any given total amount of rain falling on the basin, the more nonuniform the distribution is, the greater will be the peak runoff.

For drainage basins of appreciable size, large flood-producing storms are very seldom uniformly distributed. For small drainage basins high peak flows are the result of intense thunderstorms that cover only small areas. For large basins the high peak flows are produced by general storms of less intensity but covering much larger areas. In neither case are they uniformly distributed.

Because the runoff resulting from any rain depends to a considerable extent upon rainfall distribution, it is desirable to have a means of measuring this characteristic. This is provided by the *distribution coefficient,* which for any storm is obtained by dividing the maximum rainfall at any point by the mean on the basin. Hence for any given total rainfall, all other conditions being the same, the greater the distribution coefficient the greater will be the peak runoff.

Effect of Direction of Storm Movement. Rarely if ever does a rain begin or end simultaneously over the entire drainage basin. The

center of disturbance usually has a definite direction of movement. The direction in which the storm moves across the basin with respect to the direction of flow of the drainage system has a

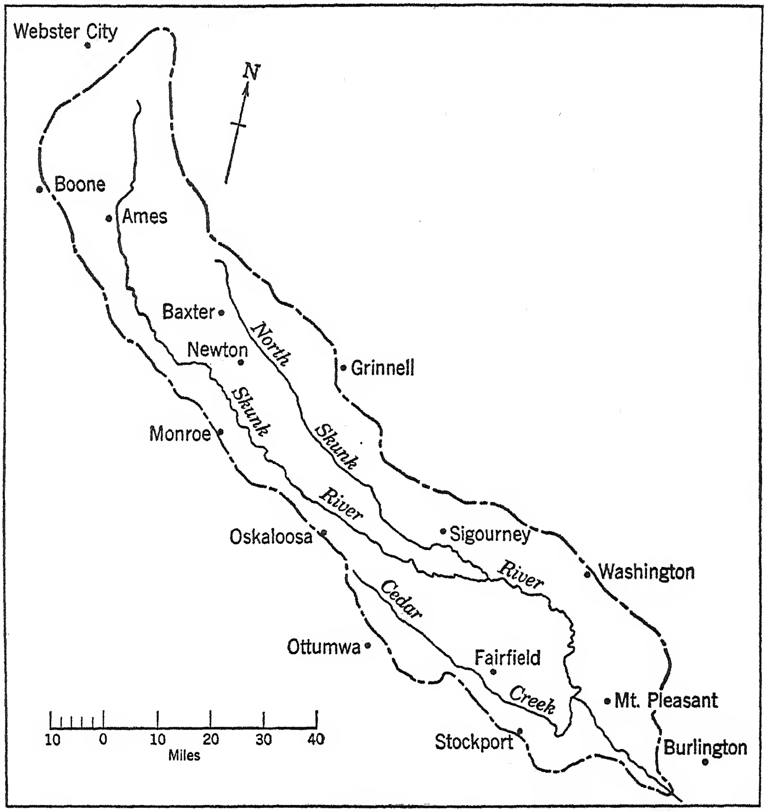


FIG. 12. Skunk River basin above Augusta, Iowa. Drainage area 4290 square miles. From *U. S. Geological Survey Water-Supply Paper* 772.

decided influence upon the resulting peak flow and also upon the duration of surface runoff. In other words it affects both height and width of base of the hydrograph.

Consider for a moment the Skunk River basin above Augusta, Iowa (Fig. 12). A storm striking this basin from the west and traveling with a velocity of 20 miles per hour in the direction of flow would reach Augusta about 8 or 9 hr later. Surface runoff from the upper part of the basin would have reached the stream

channels and would have flowed toward the outlet during that time interval before any surface runoff from the lower part of the basin reached the stream. When this water from the upper portion of the basin reached Augusta, a congestion would occur producing a higher peak and a shorter period of surface runoff than would otherwise occur. On the other hand a storm from the southeast, striking Augusta first and traveling upstream, would have the opposite effect. In this case, surface runoff from the lower part of the basin would have been flowing past the gaging station at Augusta for 8 or 9 hr before any surface runoff from the upper portion of the basin reached the stream channel. This would result in a lower peak flow and a period of surface runoff longer by 16 or 18 hr than for a storm from the opposite direction.

A storm from the northeast or southwest that would cross this basin transversely would produce a stream rise whose height and period covered would be a mean between the values resulting from the two cases described above.

CHAPTER III

THE DRAINAGE BASIN

The rainfall-runoff relation on any particular drainage basin is the composite result of the effects produced by the many and diverse climatic, geologic, and physiographic characteristics of the basin. The effects of the climatic factors have been discussed briefly in the preceding chapter. They will be treated in greater detail in Chapters IV and V. The geologic factors will be discussed in Chapters VI and VII. In this chapter the effect of the physiographic factors will be considered. As a first step it becomes necessary to devise methods for measuring and expressing quantitatively each of these various characteristics. It is not sufficient to say that a basin is large, hilly, oval shaped, and well drained. More definite descriptions are required.

The use of numerical indices for measuring and describing these characteristics is not new. For instance, area and temperature have long been so treated. Methods of measuring certain other factors have been developed in Europe, such as "Belgrand's ratio," the "form factor" of Gravelius, and others which will be described later. Years ago Horton developed and used methods for measuring many of these characteristics. They will also be described.

Even though the effect of these characteristics upon the rainfall-runoff relationship cannot be quantitatively determined at the present time, the nature of these influences is quite well known. Therefore, with methods for measuring each characteristic, direct comparisons between different drainage basins become possible, and more definite conclusions can be drawn of their comparative yield. The combined effect of all the factors to be discussed in this chapter might be aptly called the drainage efficiency of the basin.

Area of Basin

Every drainage basin is surrounded by a *divide,* so called because it is a line of separation that divides the precipitation that falls on two adjoining basins and directs the ensuing runoff into one river system or into the other.

It has already been shown that the total volume of water carried in the stream channels is made up of surface runoff and ground-water flow. Seldom are these two quantities drawn from the identically same areas. In other words, surrounding every drainage basin is a surface or *topographic* divide that demarks the area from which the surface runoff is derived. Determined usually by the geological structure, although sometimes influenced by the topography, there is an underground or *phreatic* divide that fixes the boundary of the area that contributes ground water to each stream system.

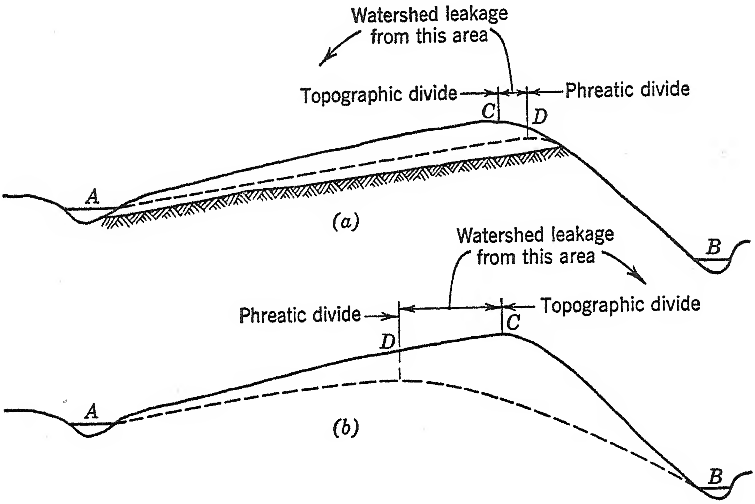


FIG. 13.

Where these two divides are not coincident, *watershed leakage* is said to occur and is equal to the ground-water flow from the area between them. This ground-water flow or watershed leakage always flows across the topographic divide. The area of the drainage basin is considered as being the area that contributes the surface runoff and is bounded by the topographic divide. The exact location of the phreatic divide is usually unknown.

Figure 13*a* shows how watershed leakage may be caused by geological formation. Because of the dip of the impervious stratum toward Stream *A*, the ground-water flow from the area between *C* and *D* is diverted from Stream *B* to Stream *A*. In Fig. 13*b* is shown a cross section through the divide between two drainage

basins which are exactly similar topographically to that shown in Fig. 13*a*, but in which there is no impervious stratum such as shown in the previous case. In Fig. 13*b* the diversion of ground-water flow is from Stream *A* to Stream *B*.

The location of the phreatic divide usually is not fixed and permanent but shifts with the changes in ground-water stage. The higher the stage of the ground water, the more nearly do the phreatic and topographic divides coincide. As the stage lowers, the two divides may become more and more widely separated.

In general, if two adjacent streams flow more or less parallel, watershed leakage is likely to occur from the higher to the lower basin. It may also occur at the head of a basin where a stream on the opposite side of the divide heads at a lower level. Watershed leakage through artesian aquifers occurs under a variety of conditions. Inasmuch as the profile of the free water table generally follows roughly that of the ground surface, the surface and ground-water divides are likely to differ less for steep impervious areas than for flat areas with permeable soils. If information to the contrary is lacking, it is generally assumed that the two divides are coincident. This assumption, however, may be greatly in error, especially in small drainage basins in highly permeable deposits.

Although of all the many drainage-basin characteristics that affect the rainfall-runoff relationship the effect of area or size is perhaps the easiest to analyze, widely divergent opinions have been expressed on this subject. The following statements illustrate this disagreement. "So pronounced is the effect of watershed area on flood flow, that widely scattered watersheds of equal area but of dissimilar topographical characteristics experience quite similar flood flows."[1] "Based on analysis of many major floods, especially those in the north-central, northeastern, middle Atlantic, and some in the North Pacific States and California, it appears that the amount of direct run-off that has occurred during single flood-rises of record has a fairly definite range for certain regions and that, except possibly for small drainage basins, the total direct run-off has been about the same for the various sizes of drainage-areas in a particular region."[2] Between these extreme viewpoints

[1] A. F. Meyer, *Elements of Hydrology*, John Wiley, second edition, p. 331.

[2] W. G. Hoyt and W. B. Langbein, Some General Observations of Physiographic and Climatic Influences on Floods, *Trans. Am. Geophys. Union*, 1939, Part II, p. 172.

it appears that the size of basin affects the runoff characteristics only in the following ways.

Effect upon Flood Flow. If all other factors including the depth and intensity of rainfall remain constant in all instances, the total runoff expressed in inches depth on the drainage basin will be the same regardless of the size of basin. However, the base of the hydrograph of flood flow will broaden out as the area of basin increases; in other words, the larger the basin the longer it takes for the total flood flow to pass a given station. Inasmuch as under the above assumptions the total runoff per square mile remains the same, it necessarily follows that the peak flow must decrease as the area of basin increases.

One other factor affects this relationship, however. It was assumed above that the depth of rainfall is the same in all instances. Actually for any locality the maximum intensity of rain that is likely to occur with any given frequency varies inversely with the area covered by the storm. Consequently, the larger the basin the less will be the intensity of the storm and therefore the lower will be the flood peak. This results from the fact that surface runoff is equal to rainfall minus infiltration, neglecting other losses. As an illustration, suppose that the maximum flood on a certain small basin results from a 6-in. rain in 1 day. On a nearby large basin the maximum flood is produced by a 12-in. rain in 4 days. Now if the average infiltration capacity is the same in both cases and is equal to 2 in. per day, the net supply rate for the small basin is 4 in. per day as compared with 1 in. per day for the large basin. Hence, although the rainfall rate on the small basin was only twice that on the large basin, the supply rate was four times as great.

Although the above-mentioned factors tend to cause flood flows, expressed in cubic feet per second per square mile, to be more intense for small basins, this effect may easily be obscured by the effect of the other basin characteristics. As a result, maximum flood flows may differ greatly even for watersheds of the same size. To illustrate this point the basins and pertinent data shown in Table 1 were selected more or less at random from the records of the U. S. Geological Survey.

These streams were chosen primarily for the purpose of comparing the flood flows to be expected from drainage basins of approximately 10,000 sq miles area located in various sections of the

United States. This table shows the enormous variation in the maximum flood flow to be expected from different drainage basins of approximately the same size. Longer records will undoubtedly show increased flood flows for all these streams, but probably their relative magnitude will not be materially changed. It will be

TABLE 1

Basin	Station	Area, sq miles	Period, yrs	Maximum Q cfs	Maximum Q cfs /sq mile
Souris River	Minot, N. Dak.	10,270	31	12,000	1.17
Deschutes Riv.	Moody, Oreg.	10,500	39	43,600	4.27
Gila River	Coolidge Dam, Ariz.	12,890	30	130,000	10.1
Cumberland Riv.	Carthage, Tenn.	10,700	21	186,000	17.4
Susquehanna Riv.	Wilkes-Barre, Pa.	9,960	45	232,000	23.3
Potomac River	Point of Rocks, Md.	9,650	43	480,000	49.7
Little River	Cameron, Tex.	7,030	27	647,000	92.0

observed that the actual maximum flood flow of the Little River at Cameron, Texas, was over fifty times as great as that of the Souris River at Minot, North Dakota, despite the fact that the basins have about the same area. Correspondingly large variations in flood flows can be found for other streams of any given size located in different sections of the United States.

On the other hand, if one would compute the average of the peak flood flows that have occurred on all the streams of each different size throughout the country and then plot average peak flow against size of basin, he would find a certain degree of correlation between these two quantities for the reasons explained (see Fig. 108). So, although the magnitude of flood to be expected from any drainage basin, expressed in cubic feet per second per square mile, varies inversely with the size of basin as long as all the other characteristics remain the same, it is by no means a dominant factor, and as a result much greater variations in flood flow are to be expected among the different basins of any given size because of these other factors.

Effect upon Minimum Flow. After surface runoff ceases, the entire flow of the stream is drawn from ground-water storage. Consequently, as this storage is depleted more and more, the stream flow becomes less and less until either the stream goes dry or the supply is replenished by precipitation. These replenishing rains are often local, some covering an area of only a few square miles. Scores of such rains may fall on various portions of a large drainage basin during a given drought although many of the small

component basins may be left untouched. Because each of these local rains contributes to the discharge of the main stream, larger basins are likely to provide a more sustained flow than smaller ones.

Effect upon Average Flow. A study of a great many drainage basins throughout the United States reveals the fact that seldom does the average unit yield remain constant throughout the length of stream channel. Progressing downstream, in some cases it increases and in others it decreases. The reasons for these changes are usually, however, attributable to surface conditions; in other words, the character of basin seldom if ever is the same throughout a large drainage system, and this factor exerts a dominant influence on the unit yield at various points on the stream. These variations are, therefore, not directly attributable to size but to other factors which will be discussed presently.

Shape

The shape of a drainage basin mainly governs the rate at which water is supplied to the main stream as it proceeds along its course from the source to the mouth. This has an important bearing on the economic utility of the stream as well as its profile and channel dimensions.

The outlines of large drainage systems are, as a rule, fixed at least in part by major geologic structures, folds, and mountain ranges. Such structures commonly fix the position of the watershed line across the head of the more important drainage basins, whereas the lateral boundaries may be fixed either by geologic structures or by competitive erosion. For the smaller basins erosion is usually the dominant factor.

Although the form of a drainage basin may lie anywhere between a long, narrow rectangle with its axis parallel to the stream, and a rectangle with its long axis at right angles to the stream having the outlet near the middle of one side of the basin, the majority of drainage basins are ovoid or pear shaped, with the outlet at the narrow end.

Drainage basins of triangular form with vertex upward receive most of their water in the lower reaches and are of least value economically, whereas pear-shaped basins or composites having narrow rectangular basin outlets with most of the runoff concentrated in the middle reaches provide the greatest available power.

Although it is difficult to express satisfactorily by means of a numerical index the shape of a drainage basin as that characteristic affects the hydrology of the stream, several indices have been suggested which are of value. Gravelius[1] proposed the use of the term "form factor" to express the ratio of the average width to the axial length of basin. The axial length is measured from the outlet to the most remote point on the basin. The average width is obtained by dividing the area by the axial length. For basins with side outlets the width may exceed the axial length, giving a ratio greater than unity. The form factor gives some indication of the tendency toward floods, because a basin with a low form factor is less likely to have an intense rainfall simultaneously over its entire extent than an area of equal size with a larger form factor.

Another index of the form of a drainage basin as suggested by Gravelius[1] is the ratio of the perimeter of the watershed to the circumference of a circle whose area is equal to that of the drainage basin. This ratio may be termed the *compactness coefficient*. If D is the diameter of a circle whose area is the area of the basin,

$$A = \frac{\pi D^2}{4}$$

or

$$D = \sqrt{\frac{4A}{\pi}}$$

Also, if C is the circumference of this circle,

$$C = \pi D$$

or

$$D = \frac{C}{\pi}$$

Equating

$$\frac{C}{\pi} = \sqrt{\frac{4A}{\pi}}$$

and

$$C = 2\sqrt{\pi A}$$

If P_l is the length of the perimeter of the basin in miles, then the

[1] Gravelius, *Flusshunde*, Berlin and Leipzig, 1914.

compactness coefficient,

$$K_c = \frac{P_l}{2\sqrt{\pi A}} = 0.28 \frac{P_l}{\sqrt{A}}$$

This coefficient is an abstract number, independent of the size and dependent only upon the shape. It has a minimum value of unity for a circular basin. The larger its value the greater the irregularity of boundary of the drainage basin and the greater the departure of its form from that of a circle. Hence, for two basins of the same size, the maximum flood should be expected from that one which has the smaller compactness coefficient, other characteristics being the same.

Snyder[1] has found that shape, as it affects the runoff characteristics of a watershed, is related to the distance along the main stream from the outlet to a point adjacent to the geographical center of the basin.[2]

Elevation

The variation in elevation and also the mean elevation of a drainage basin are important factors in relation to temperature and to precipitation, particularly as to the fraction of the total amount which falls as snow. Not only does elevation, because of the resulting differences in temperature, have a profound effect upon water losses, which are all evaporative in nature, but it is an important factor in determining the extent to which the available water supply in winter is impounded as frozen assets in the form of snow storage, ice in lakes and rivers, and soil moisture within the zone of frost penetration.

For large basins the mean elevation can be most easily determined by the intersection method. A topographic map of the basin is subdivided into squares of equal size by enough lines so that at least 100 intersections fall within the area. The mean elevation of the basin is then taken as the average of the elevation at all the intersections.

A more complete analysis of the elevation characteristics of a basin may be made by measuring on a suitable map the area lying between successive pairs of contours. The percentages that each

[1] Franklin F. Snyder, Synthetic Unit-Graphs, *Trans. Am. Geophys. Union*, 1938, Part I, p. 447.

[2] See also p. 314 of this book.

TABLE 2

Limiting Contour Elevations	Area between Contours, acres	Per Cent of Total	Per Cent of Total over Given Lower Limit
1	2	3	4
170– 300	500	2.4	100.0
300– 400	1700	8.2	97.6
400– 500	1900	9.2	89.4
500– 600	2400	11.6	80.2
600– 700	3000	14.5	68.6
700– 800	2970	14.3	54.1
800– 900	2270	11.0	39.8
900–1000	2180	10.5	28.8
1000–1100	1500	7.2	18.3
1100–1200	640	3.1	11.1
1200–1300	610	3.0	8.0
1300–1400	410	2.0	5.0
1400–1800	620	3.0	3.0

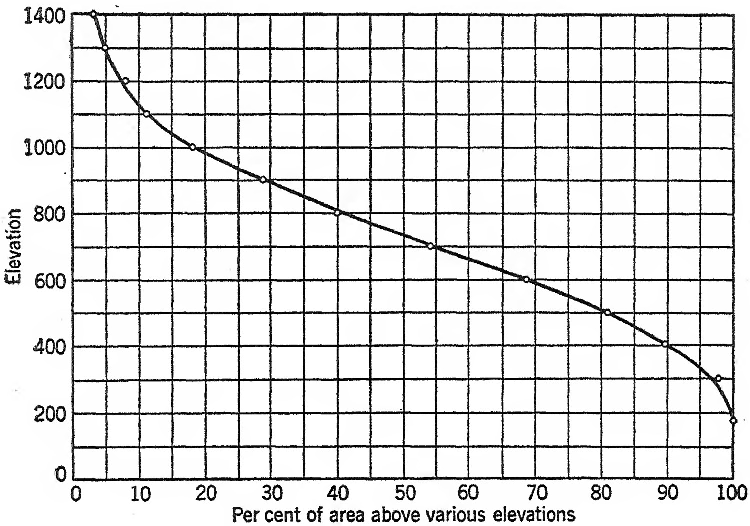


FIG. 14. Hypsometric curve for San Pablo drainage basin near Richmond, Calif.

of these areas are of the total are then computed and the percentage of the total area lying above or below each different contour is obtained by summation, as shown for the San Pablo drainage basin, near Richmond, California, in Table 2.

If a is the area between any given pair of contours of which e is the mean elevation, the mean elevation of the basin is

$$E = \frac{\Sigma ae}{A} \tag{1}$$

in which A is the area of basin. Substituting the data contained in Table 2 in equation 1, we find that the mean elevation of the San Pablo basin is 758.

If the data contained in Table 2 are shown graphically, by plotting column 4 against the lower elevations in column 1 a typical hypsometric curve will be obtained (Fig. 14). This curve shows that 50 per cent of the drainage area lies above elevation 732, which is called the median elevation and is more representative of the effect of elevation in relation to hydrology than the mean elevation. As a usual thing the mean elevation is higher than the median, but the difference is generally unimportant.

Slope

The slope of a drainage basin has an important but rather complex relation to infiltration, surface runoff, soil moisture, and ground-water contribution to stream flow. It is one of the major factors controlling the time of overland flow and concentration of rainfall in stream channels and is of direct importance in relation to flood magnitude. Alvord[1] suggested a method of estimating slope based upon the area between different contours within the basin. Such a method, used by Horton,[2] is as follows.

In Fig. 15 is shown a drainage basin crossed by a number of contours having equal differences in elevation. Lines ab and cd are drawn midway between contours 400 and 410 and between 410 and 420 respectively. Now let

a_1 = area of strip *abcd*.
w_1 = average width of strip *abcd*.
l_1 = length of contour 410.
s_1 = average slope of strip *abcd*.
S = average slope of basin.

[1] John W. Alvord and others, Tables of Excessive Precipitations of Rain at Chicago, Ill., from 1889 to 1897 inclusive, *J. Western Soc. Engrs.*, April 1899, p. 157.

[2] R. E. Horton, Derivation of Runoff from Rainfall Data, Discussion, *Trans. A.S.C.E.*, 1914, **77**, 369–375.

D = contour interval.
A = area of basin.
L = total length of contours.

Then

$$s_1 = \frac{D}{w_1} = \frac{Dl_1}{a_1}$$

and by weighting the slope of each strip in accordance with its area,

$$S = \frac{Dl_1}{a_1} \cdot \frac{a_1}{A} + \frac{Dl_2}{a_2} \cdot \frac{a_2}{A} \cdots \frac{Dl_n}{a_n} \cdot \frac{a_n}{A} \tag{2}$$

From which

$$S = \frac{D}{A}(l_1 + l_2 + \cdots l_n) = \frac{DL}{A} \tag{3}$$

In other words the average slope of the basin is equal to the total length of contours multiplied by the contour interval and

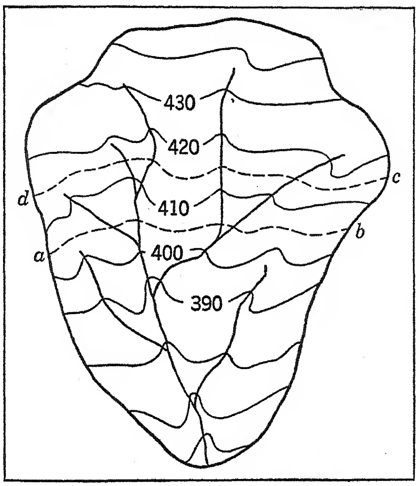


FIG. 15.

divided by the area of basin. The work involved in measuring the length of all contours on large and rugged basins is likely to be time consuming and tedious. Fortunately, however, satisfactory results can usually be obtained by measuring with an opisometer the length of contours at 20-ft or 40-ft intervals for small or flat

areas and the length of contours at 100-ft to 500-ft intervals for large or steep areas.

Another method of slope determination that is especially applicable to large areas has been suggested by Horton. It is called the intersection line method. The area whose slope is to be determined is subdivided by a gridwork of lines into a number of squares of equal size. The number of contours crossed by each subdividing line is counted and the lengths of the grid lines are scaled. Then the average distance between contour crossings on the subdivision lines is

$$l' = \frac{L'}{N}$$

where L' is the total length of the subdividing lines and N is the number of contours crossed.

If these crossings were all at right angles the average slope would be D/l', where D is again the contour interval. However, these crossings are likely to be at any angle from 0° to 90°. If x is the horizontal angle at which each of two parallel contours crosses a subdividing line, then $l' \sin x$ is the horizontal distance between and normal to the two contours. The mean value of $\sin x$ for all angles from 0° to 90° is equal to $2/\pi$ or 0.637. If the same number of crossings occur at all different angles, the average slope of the basin is

$$S = \frac{D}{0.637l'} = 1.571\frac{D}{l'} = 1.571\frac{DN}{L'} \tag{4}$$

If instead of occurring at all possible angles the intersections occur at the most probable angle, 45°, the average horizontal distance between contours will be $l' \sin 45° = 0.707l'$.

The mean slope of the basin would then be

$$S = \frac{D}{0.707l'} = 1.414\frac{D}{l'} = 1.414\frac{DN}{L'} \tag{5}$$

The decision as to which of these two formulas will give the more nearly correct results in any particular case will depend upon whether the crossings are uniformly distributed at all angles or whether most of them occur at approximately 45°. Inasmuch as the greatest likelihood is that some will occur at all angles but more will intersect at 45° than at any other angle it seems reason-

able to conclude that for all practical purposes

$$S = 1.5 \frac{DN}{L'} \tag{6}$$

In the application of this method it is assumed that each contour crossed represents a difference in elevation along the subdivisional line equal to the contour interval. Of course, it may happen that two adjacent contours are at the same elevation and are separated by land that is only slightly higher or lower. On the average, however, the elevations of summits or depressions between equal contours will differ from those of the adjacent contours by an amount equal to nearly half those of the contour intervals. It can readily be seen, therefore, that the average slope between a pair of contours of equal elevation is nearly the same as if the contours were separated by the contour interval, D, and that the method gives nearly correct results even where the subdivision lines cross adjacent contours of equal elevation, as in the case of summits and depressions. Results obtained from this method, when compared with those obtained from the measured total length of contours, as previously described, are usually found to be in good agreement.

Orientation

Although slope affects the rainfall-runoff relationship principally because of a speeding up of the velocity of overland flow, thereby shortening the period of infiltration and producing a greater concentration of surface runoff in the stream channels, a secondary influence resulting from the general direction of the resultant slope, or orientation of basin, must not be overlooked. This factor affects the transpiration and evaporation losses because of its influence on the amount of heat received from the sun. Also the direction of the resultant slope to the north or the south affects the time of melting of accumulated snows. If the general slope is to the south, each successive snowfall may soon melt and either infiltrate into the ground or produce surface runoff. On the other hand, if the slope is to the north, these snows may accumulate throughout the winter and remain on the ground until late spring when they may be removed by a heavy rain, thus producing a high flood peak and a low minimum flow. The effect of the orientation of the basin with respect to the direction of storm movement

was discussed on page 36; also the effect of the orientation of a hilly or mountainous watershed with respect to the direction of the prevailing winds is discussed on page 68.

The Drainage Net

Another important characteristic of any drainage basin is the pattern or the arrangement of the natural stream channels which through past ages has been developed by nature within the area. The reasons for this importance are twofold. In the first place, the efficiency of the drainage system and therefore the characteristics of the resulting hydrograph are dependent upon this factor. For instance, if the basin is well drained the length of overland flow is short, the surface runoff concentrates quickly, the flood peaks are high, and in all probability the minimum flow is correspondingly low. In other words, the more efficient the drainage the more flashy is the stream flow, and vice versa.

Perhaps of equal if not greater importance, however, is the indication that this factor gives to the hydrologist of the nature of the soil and surface conditions existing in the drainage basin, for there can be no question but that the character and extent of nature's carving of stream channels through erosive processes is definitely related to and restricted by the type of materials from which these channels are carved. It is entirely conceivable that, when this subject has been fully explored, analyzed, and understood, it may be found that an ordinary map of the drainage system provides a reliable index of the permeability of the basin and will give some indication of the yield.

The characteristics of the drainage net may be fairly well described by (1) order of streams, (2) length of tributaries, (3) stream density, and (4) drainage density and length of overland flow.

Order of Streams. Every large stream has its important tributaries, each of which has its own tributaries, and so on until at last the ultimate branches or finger tips of the drainage net are reached which have no branches. As a rule, the larger the stream the greater is the number of branchings or bifurcations. It is convenient to classify streams according to the number of bifurcations of the tributaries. In German practice the main stream is always of the first order, its direct tributaries are of the second order, their tributaries are of the third order, and so on. In this system, Goose

Creek, a mere trickle less than a mile in length but emptying directly into the sea, is a stream of the first order just the same as the Mississippi River. Furthermore, Deer Creek, comparable in size to Goose Creek but located in the extreme headwaters of a large system, may be a stream of the tenth order. In other words, under this system the order of any particular stream is no indication of its size or importance.

Another common procedure is to designate all nonbranching tributaries, regardless of whether they enter the main stream or its branches, as of the first order. Streams which receive only nonbranching tributaries are of the second order. Streams of the third order are formed by the junction of two streams of the second order, and so on. In accordance with this system, the order number of the main stream indicates at once the extent of bifurcation of its tributaries and, as a rule, is a direct indication of the size and extent of the drainage net. Streams of the same order have the same system of bifurcation, regardless of whether they enter the main stream, enter another tributary, or flow directly into the sea.

For almost every stream there are some undivided feeders that flow directly into the main stream or into the larger tributaries. The relative numbers of divided and undivided tributaries afford an indication of the character of the drainage. For example, in watersheds in the Middle West that are covered with a heavy blanket of permeable deposit, there are, as a rule, relatively few minor tributaries, whereas, for streams draining precipitous slopes, such as those tributary to the Finger Lakes in central New York, there are usually comparatively few major or branching tributaries.

To determine correctly the order of a stream and make a complete analysis of the drainage net it is necessary to have a map of the basin showing all tributaries. This should include both perennial and intermittent tributaries, but it cannot well include ephemeral rain gullies that have not developed definite stream channels. On this map each different stream and tributary should be numbered according to its order. In Fig. 16 is shown a map of the Thunder Bay River drainage net which has been so numbered. This map shows that the Thunder Bay River is a stream of the fourth order and that it has two tributaries of the third order, eight of the second, and thirty-three of the first.

The number of tributary streams of a given order per square mile of drainage basin is a function of the order of the tributaries.

For this particular purpose, it is convenient to use the *reverse order*, in which the main stream is given the order of *one*, tributaries of the next lower order have the reverse order of *two*, and so on, in accordance with German practice. If each order of tributaries had a given number of branches of the next higher order and no others, then the number of tributaries of a given

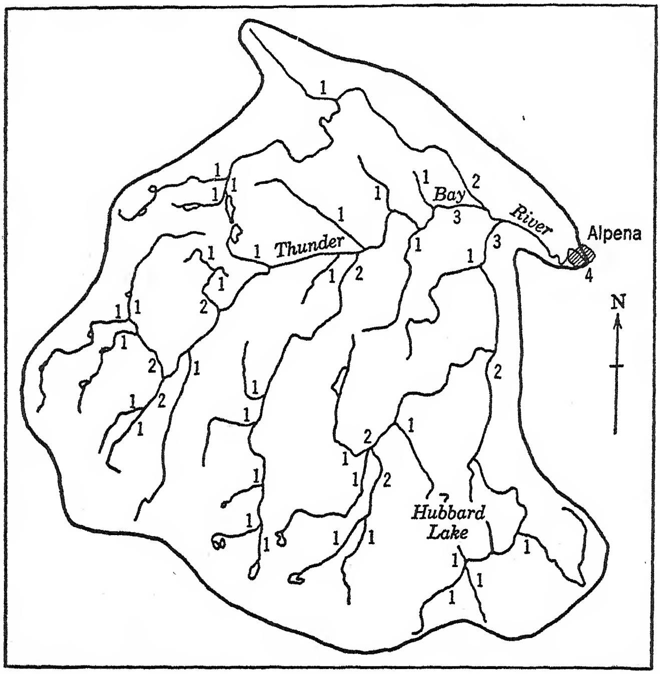


FIG. 16.

order would decrease in a geometrical progression as the reverse order decreases in an arithmetical progression. Actually this does not occur. If it did, a fifth-order stream would receive, let us say, two fourth-order streams and no others; each of these fourth-order streams would receive two third-order streams and no others; and so on. As a matter of fact, the fifth-order stream may receive directly from first-, second-, and third-order streams, in addition to the two fourth-order streams. However, the geometrical ratio is usually well sustained in actual experience, as is illustrated in Fig. 17, which shows for eight different streams the number of

tributaries of different orders, plotted semi-logarithmically. Horton found that the equation of the resulting graph is of the form,

$$\log N = KO_R - C \tag{7}$$

in which N is the number of streams of reverse order, O_R, per 100 sq miles of drainage basin; K is the slope of the graph with respect

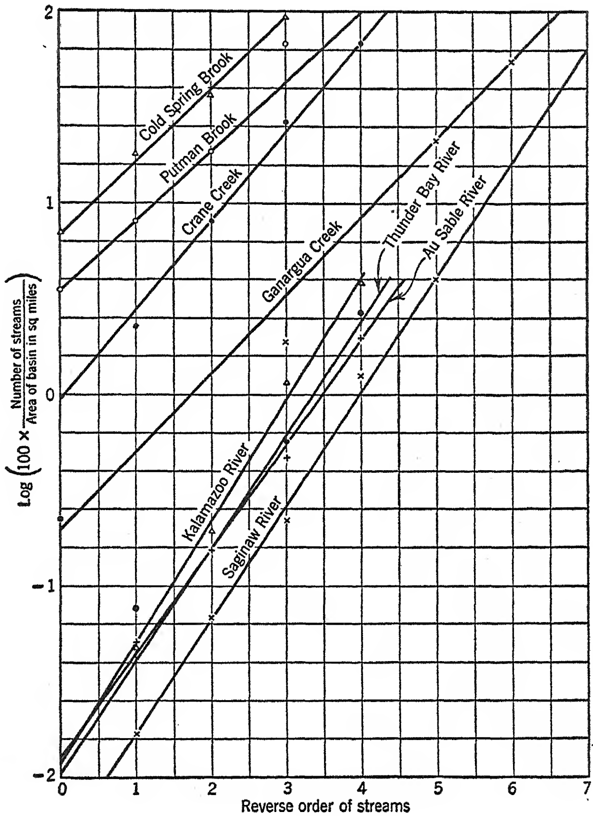


FIG. 17.

to the log N axis; and C is the negative value of log N when O_R is equal to zero. For Ganargua Creek in New York, the equation becomes

$$\log N = 0.406O_R - 0.70$$

For the Saginaw River in Michigan, the equation is

$$\log N = 0.594O_R - 2.37$$

Usually for a given stream the actual number of tributaries falls very close to a straight line. In Fig. 17 it is interesting to note that the graphs for most of the streams in New York are approximately parallel with each other; in other words, they have approximately the same values of K but different values of C. Those for the Michigan streams are also nearly parallel with each other but have steeper slopes or higher values of K than those for the New York streams. From this, it appears that certain broad general characteristics of basin more or less common to any particular region determine the slope of these graphs or the value of K in equation 7. On the other hand, certain minor variations that are more or less common to all basins regardless of location seem to determine the value of C. As to what particular characteristics of basin affect the values of K and C, little is at present known. However, there can be but little question that there is a close relationship between these constants and certain drainage-basin characteristics, which in turn are closely related to stream flow. Here in all probability lies a fertile field for research. No one can predict the extent of the possibilities of correlating these factors and determining thereby the stream-flow characteristics based entirely upon the type of drainage net.

Length of Tributaries. The length of tributaries is an indication of the steepness of the drainage basin as well as of the degree of drainage. Steep well-drained areas usually have numerous small tributaries, whereas in plains regions where the soils are deep and permeable only relatively long tributaries are, as a rule, maintained as perennial streams. In different basins it is better to compare the average length of tributaries of the same order, especially of the first order, than to compare the average length of all tributaries.

The lengths of tributaries increase as a function of their order. This is also approximately a geometrical law of progression, although the reason for its being so is not quite as evident as in the case of the numbers of tributaries. The relation does not hold closely for individual streams. Comparison has been made of the average lengths of tributaries of a number of streams of different orders in central New York and central Michigan. It was found that for any particular order the average length of the streams in

Michigan is considerably greater than that of the New York streams. For instance, the average length of first-order streams in New York is less than a mile, whereas in Michigan it is over 6 miles; in New York second-order streams average about 2 miles in length, and in Michigan, 12 miles, and so on.

In measuring stream lengths by opisometer on topographic maps the course of the stream can, in general, be followed quite closely. The measured length of all except meandering streams and the length along the axis of the valley are, as a rule, nearly the same.

For meandering streams the length is sometimes measured along the valley axis, the measured length consisting of a series of linear segments joining at various angles. Sinuosities due to oxbows and the general tortuosity of the stream are neglected, and the resulting length may be materially less than the actual distance through which the water flows in its course down the valley. The relations between the air-line length and the meander length of a number of midwestern streams is shown in Table 3. These stream valleys were fairly straight throughout the lengths of most of the reaches, so that the air-line length represents approximately the length as measured along the axis of the stream valley. In the last two columns of this table are shown the slopes

TABLE 3

Stream	State	Length, meander miles	Length, air-line miles	Ratio of Measured Length to Air-Line Length	Average Fall by Meander	Average Fall by Air Line
Cottonwood R.	Kans.	120.0	62.0	1.94	1.89	3.66
Maria de Cygnes	Kans.	138.0	88.0	1.57	1.43	2.24
Salt Creek	Nebr.	78.0	36.0	2.17	1.78	3.86
Nemaha R.	Nebr.	42.4	24.9	1.70	3.07	5.22
Elkhorn R.	Nebr.	21.8	13.6	1.60	3.88	6.25
Deep Fork	Okla.	62.0	41.0	1.51	1.90	2.90
Des Moines R.	Iowa	68.0	37.0	1.84	1.54	2.84
Big Sioux R.	Iowa	36.6	17.0	2.15	1.37	2.94
Locust Cr.	Mo.	59.0	25.0	2.36	1.59	3.76
Saline Cr.	Mo.	38.0	18.5	2.04	1.47	3.00

of these streams computed by means of the air line and also the meander length of the streams.

Stream Density. The stream density or stream frequency of a drainage basin may be expressed by relating the number of streams to the area drained. If N_s is the number of streams in the basin and A is the total area, the stream density, D_s, may be expressed as

$$D_s = \frac{N_s}{A}$$

i.e., the number of streams per square mile. The inverse form, namely the area per stream, might also be used as a measure of stream density.

In determining the total number of streams, only the perennial and intermittent streams are included. The main stream, extending

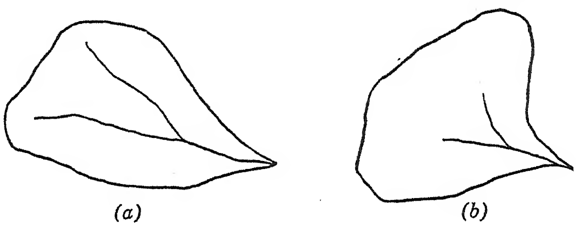


FIG. 18.

from its source to the mouth, is counted as one; there are then n_1 tributaries of the next lower order, each extending from its source to its junction with the main stream; n_2 tributaries of the next lower order, each extending from its source to its junction with a stream of the next higher order; and so on down to the first order of tributaries.

That the relationship between the number of streams and the area drained does not provide a true measure of drainage efficiency is shown in Fig. 18. In this figure (a) and (b) represent two basins of equal size, each having the same numbers of streams. However, it is quite evident that (a) is better drained than (b).

Drainage Density and Length of Overland Flow. Drainage density is expressed as the length of stream per unit of area. Let D_d represent the drainage density, L the total length of perennial and intermittent streams in the basin, and A the area; then

$$D_d = \frac{L}{A}$$

Length of overland flow is here defined as being the average distance the water would have to flow overland if it flowed in a straight line from the point where it fell as rain to the nearest point on a permanent stream channel. This distance is usually quite different from the length of sheet flow. Where rain intensity exceeds infiltration capacity, the excess starts flowing overland as a sheet. This type of flow, which is often laminar, does not continue for any great distance. Very soon, depending upon the nature of the ground surface, slope, and cover, this water collects in tiny rivulets which flow for short distances. These join with other, similar rivulets, still flowing in ephemeral channels until finally a permanent stream channel is reached.

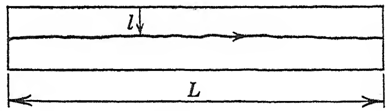


FIG. 19.

Length of overland flow, l, is related to drainage density. If a watershed is assumed to be a rectangle of length L, having a permanent stream channel passing through its center (Fig. 19),

$$A = 2lL$$

and

$$l = \frac{A}{2L}$$

From this it may be seen that D_d is one half of the reciprocal of l. The actual distance that the water flows overland and the length of sheet flow may differ considerably from the values determined in this idealized manner but these several quantities are more or less closely related.

Indirect Drainage

Throughout the preceding discussion it has been assumed that all parts of the drainage basin are tributary through direct overland flow to some surface stream. In two important cases this condition may not exist, viz., in areas characterized by (1) karst topography and (2) a very pervious surface. In neither of these cases does any precipitation find its way through overland flow directly into the stream channels; instead it infiltrates into the soil and later finds its way through underground channels either into adjacent or far distant streams, lakes, or the sea.

Regions underlain by soluble rock formations, especially limestone, oftentimes have characteristic undulating surfaces with conical knolls and circular sinks. Such areas are said to have a *karst* topography. The runoff usually enters the ground through sinkholes and fissures and pursues its course to an outlet through a system of underground passages or solution channels. The resulting underground drainage net is often complex but is somewhat similar to a surface drainage net in its characteristics. The surface is pockmarked with depressions, some dry, some containing pools; streams flow into some and out of others.

Extensive karst regions are to be found in France and in other parts of continental Europe and in various localities in the United States, especially in the cave regions of Kentucky and Virginia. In the Helderberg region in eastern New York there are considerable areas where the fissured limestone is at the surface and the runoff enters directly into widened joint openings and solution channels. In northern Florida and in the basin of the north branch of the Thunder Bay River in Michigan, there are hundreds of sinkholes usually from 20 ft to 80 ft deep and several hundred feet in diameter, caused by the dissolution of the underlying soft limestone and the subsequent collapse of the overlying thin, hard strata. Most of the runoff from these areas finds its way to these sinkholes and thence through underground channels to distant outlets.

In regions of low relief and high permeability, especially regions overlain with recent glacial deposits, numerous undrained depressions often occur. Most common among these are kettle holes, which are generally pockets left in the surface by the melting of stranded blocks of ice as the glaciers disappeared from the region. In such areas all rain passes directly into the soil, so that there is no surface runoff and no drainage net is developed.

In determining the runoff from basins containing karst topography, it is best to exclude all karst areas for it is seldom that they contribute either runoff or ground-water flow to the stream to which they are immediately tributary. For instance, it is doubtful that the karst areas of the Thunder Bay basin make any appreciable contribution to the flow of the Thunder Bay River. However, probably these areas contribute equally with other adjacent areas to the flow of the St. Lawrence, for the underground channels very likely find an outlet in Lake Huron.

On the other hand, in estimating the yield of drainage basins containing highly permeable depression areas, these areas should usually be excluded only in making estimates of surface runoff but should be included in making studies of ground-water flow, for usually the water table beneath these areas drains directly into the adjacent streams.

Artificial Drainage

The presence of artificial drains in a portion of a drainage basin lowers the water table in that region. Thus a temporary underground storage space is made available. The portion of the rainfall that becomes infiltration is delayed in its movement to the flowing stream by the presence of this storage. On the other hand, if surface drains are present, the portion of the rainfall that becomes surface runoff is likely to reach the stream channels in a shorter time than it would without the surface drains. In the lower portions of a drainage basin, speeding up the runoff process is likely to decrease flood flow, whereas slowing down the process may increase the flood peak. In the upper reaches, the effects may be just the opposite. It follows, therefore, that artificial drainage may have the effect of either increasing or decreasing flood flow, depending upon the type of drains, the location of the drained areas within the watershed, and the infiltration capacity.

Some drained regions are so low that dikes are necessary to keep them from being flooded during periods of high water. The loss of the natural storage space as the result of such drainage projects accentuates the flood peaks at downstream points.

Because of the lowered water table in drained regions, the evaporation and transpiration losses are likely to be decreased and, therefore, the total yield will be increased. However, this same lowering of the water table may decrease low water flow because a large amount of potential ground-water supply has been drained away.

CHAPTER IV

PRECIPITATION

Precipitation Defined

The term precipitation as used in hydrology includes all forms of water deposited on the earth's surface and derived from atmospheric vapor. The principal forms are mist, rain, hail, sleet, and snow. Unless otherwise specified, the terms precipitation and rainfall are often used indiscriminately to apply to any or all of the forms included in this group. Condensation on solids and water surfaces in the form of dew and frost are sometimes considered as forms of precipitation.

Water Vapor

Water in the form of a true gas is called steam. For a gas there is a critical temperature above which it cannot be condensed by pressure alone. When at a temperature below its critical temperature a gas is called a vapor. Vapors also differ from gases in the respect that they depart more widely from the ordinary gas laws. According to Boyle's law the volume of any given gas varies inversely as its pressure with no change in temperature; in other words, pressure times volume is equal to a constant. If, however, the temperature changes, according to Charles's law, the volume increases 1/273 for every degree centigrade increase in temperature with the pressure remaining constant. Within a moderate range of temperatures and pressures and not too close to the point of condensation, both air and water vapor obey the gas laws sufficiently well so that these laws are useful for many practical purposes.

The water vapor of the air is chiefly derived by evaporation from water surfaces, such as oceans, lakes, and streams, from evaporation from moist soil, and by transpiration from plants which is also an evaporative process. A small amount of water vapor is added to the air by artificial means, such as exhaust steam from engines and water vapor in the exhaust from automobiles.

The presence of any considerable amount of water vapor in the

atmosphere is characterized by a bluish haze. The Smoky Mountains and the Blue Ridge Mountains of eastern United States derived their names from this haze which results from the high vapor content of the atmosphere in this region. On the other hand, in the Rocky Mountain region of the West the relative humidity is low, and the air is remarkably clear and free from haze. As a result, objects at great distances appear to be near at hand.

Maximum Vapor Pressure

Any given space can hold only a certain amount of vapor in the presence of a solid or liquid surface. If more vapor is added, it will be immediately condensed on the surface. Where there is a mixture of gases or vapors, such as moist air, each component behaves physically as if it alone were present. The pressure exerted on a containing vessel by any one component of such a mixture is called the partial pressure. The partial pressure exerted by the vapor content of saturated air at a given temperature is called the *maximum vapor pressure.* Maximum vapor pressure is commonly expressed in terms of the height of a column of mercury which would produce an equivalent pressure. The maximum pressure of water vapor is independent of the barometric pressure and depends only on the temperature. Table 4 shows the maximum pressure of water vapor in inches of mercury.

Humidity—Absolute, Relative, and Specific

By *absolute humidity* is meant the actual weight of water vapor present in a unit volume at any instant. This is a constantly changing quantity, and it varies from practically zero to saturation. Inasmuch as a direct relationship exists between the amount of water vapor in a unit volume and the pressure exerted by it, absolute humidity can be expressed either in terms of the actual weight of the water vapor contained in unit volume or in terms of the pressure created by it on a surface of unit area. The actual weight can be expressed as grams per cubic meter, grains per cubic foot, or other, similar units. When expressed as pressure, it is usually given in terms of inches height of mercury column that will produce an equivalent pressure although it may be expressed directly as dynes per square centimeter or other units of force and area. A commonly used unit is the millibar (mb). One millibar is equivalent to 1000 dynes per sq cm.

TABLE 4

SHOWING MAXIMUM VAPOR PRESSURE IN INCHES OF MERCURY

Temperature, ° F	V_p	Temperature, ° F	V_p	Temperature, ° F	V_p	Temperature, ° F	V_p
−30	0.007	10	0.063	50	0.363	90	1.423
−29	.007	11	.067	51	0.376	91	1.469
−28	.008	12	.070	52	0.390	92	1.515
−27	.008	13	.074	53	0.405	93	1.563
−26	.009	14	.077	54	0.420	94	1.612
−25	.010	15	.081	55	0.436	95	1.662
−24	.010	16	.085	56	0.452	96	1.714
−23	.011	17	.089	57	0.469	97	1.767
−22	.011	18	.094	58	0.486	98	1.822
−21	.012	19	.099	59	0.504	99	1.878
−20	.013	20	.103	60	0.522	100	1.936
−19	.013	21	.108	61	0.541	101	1.994
−18	.014	22	.114	62	0.560	102	2.055
−17	.015	23	.119	63	0.580	103	2.117
−16	.016	24	.125	64	0.601	104	2.181
−15	.017	25	.131	65	0.623	105	2.246
−14	.018	26	.137	66	0.645	106	2.314
−13	.019	27	.143	67	0.668	107	2.382
−12	.020	28	.150	68	0.691	108	2.453
−11	.021	29	.157	69	0.715	109	2.525
−10	.022	30	.165	70	0.740	110	2.599
− 9	.024	31	.172	71	0.766	111	2.676
− 8	.025	32	.180	72	0.792	112	2.754
− 7	.026	33	.188	73	0.819	113	2.833
− 6	.028	34	.195	74	0.847	114	2.915
− 5	.029	35	.203	75	0.876	115	2.999
− 4	.031	36	.212	76	0.906	116	3.085
− 3	.033	37	.220	77	0.936	117	3.173
− 2	.034	38	.229	78	0.968	118	3.264
− 1	.036	39	.238	79	1.000	119	3.356
0	.038	40	.248	80	1.033	120	3.451
1	.040	41	.258	81	1.068	121	3.548
2	.042	42	.268	82	1.103	122	3.647
3	.044	43	.278	83	1.139	123	3.749
4	.047	44	.289	84	1.176	124	3.853
5	.049	45	.300	85	1.215	125	3.960
6	.052	46	.312	86	1.254	126	4.069
7	.055	47	.324	87	1.295	127	4.181
8	.057	48	.336	88	1.336	128	4.295
9	.060	49	.349	89	1.379	129	4.412

Relative humidity expresses the ratio between either (1) the amount of water vapor actually contained per unit volume and the amount that it can hold at the same temperature when saturated, or (2) the actual vapor pressure and the saturation vapor pressure at the same temperature. Inasmuch as these are ratios, they are dimensionless and are equal regardless of the units or methods used in their derivation.

If either the temperature or the barometric pressure of any given air mass is changed, the volume is changed and hence both the absolute humidity and the relative humidity are correspondingly affected. It is often desirable to express humidity in terms that are independent of temperature and barometric pressure. This can be done by means of *specific humidity*, which is defined as the number of grams of water vapor contained in 1 kg of natural air. Specific humidity can be determined by the equation[1]

$$h_s = \frac{623h_a}{P_b - 0.377h_a} \tag{1}$$

in which h_s is the specific humidity, h_a is the absolute humidity, and P_b is the barometric pressure, the last two quantities being expressed in the same units.

Dew Point

If, without any change in barometric pressure, the air is cooled until it becomes saturated or, in other words, until the relative humidity becomes 100 per cent, the corresponding temperature is called the *dew point*. Any further cooling would result in the condensation of moisture on any surfaces with which the air comes in contact. If the temperature of the contact surfaces is above 32° F, dew will form; if below 32°, the condensation will be in the form of frost.

Saturation Deficit

The difference between the actual vapor content of the air and its content when saturated at the same temperature and pressure determines the saturation deficit. If determined from relative humidity, it is equal to 100 per cent minus the relative humidity; if, however, it is determined from the absolute humidity, it is equal

[1] S. Petterssen, *Introduction to Meteorology*, McGraw-Hill, 1941.

to the difference between the actual vapor content or pressure and the corresponding quantity at saturation.

Measurement of Humidity

Humidity may be measured by any one of four methods. The most direct method consists of extracting the water vapor from a certain volume of air and weighing it. This is done by passing the moist air through a granular desiccant, the increase in weight of the drying agent being the weight of moisture contained in the air. The development of techniques to adapt this method to field use have been reported by Thornthwaite and Holzman.[1]

The dew-point method of measuring humidity utilizes the fact that, when water vapor is saturated, a reduction in temperature will produce condensation. The dew-point apparatus usually consists of a polished cup containing a volatile liquid such as ether. The surface of the cup is cooled by forcing air through the liquid. The water vapor in contact with the cup is also cooled and when the dew point is reached, condensation may be observed on the cup. The corresponding temperature is noted from a thermometer immersed in the liquid. The dew point is taken as the average between the temperature at which the condensation appears during the cooling process and the temperature at which it disappears when the liquid is allowed to warm again. The vapor content of the air is then determined by referring to tables giving the unit weight of saturated water vapor for various temperatures.[2]

The simplest method of measuring humidity is by means of a sling psychrometer such as that shown in Fig. 20. This instrument consists of two thermometers mounted side by side, one of which has its bulb covered with muslin. Before use, the muslin is wetted so that evaporational cooling will lower the temperature of that thermometer. The difference in the readings of the two thermometers may be converted to relative or absolute humidity by means of calibration tables.[2] It is recommended that the psychrometer be "whirled rapidly for 15 or 20 seconds; stopped and

[1] Measurement of Evaporation from Land and Water Surfaces, *U. S. Department of Agriculture Tech. Bul.* 817, May 1942.

[2] C. F. Marvin, Psychrometric Tables for Obtaining the Vapor Pressure, Relative Humidity and Temperature of the Dew Point, *U. S. Department of Agriculture, W. B.* 235, 1937.

quickly read, the wet bulb first. This reading is kept in mind, the psychrometer immediately whirled again and a second reading taken. This is repeated three or four times or more if necessary, until at least two successive readings of the wet bulb are found to agree very closely." Instead of whirling the psychrometer, the air may be blown over the thermometers. Such a procedure has been described by Rohwer.[1] During subfreezing temperatures the water in the muslin is likely to freeze, causing some difficulty in the use of this method. It is claimed, however, that the proper results may still be obtained by whirling the ice-covered bulb until its minimum temperature is reached.[2]

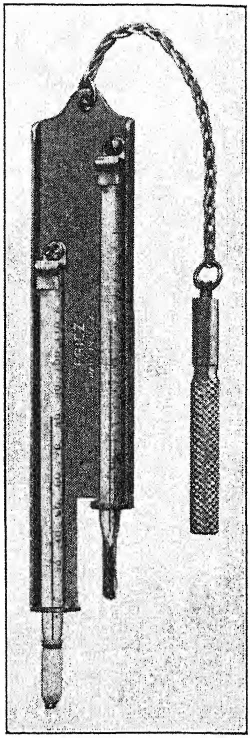


FIG. 20. Friez pocket sling psychrometer. Courtesy Bendix Aviation Corp.

The fourth method of measuring atmospheric moisture content is by means of hygrometers. Hygroscopic fibers such as hair increase in length when the relative humidity increases and shrink when it decreases. By careful calibration a group of such fibers attached to an indicator arm may be made to register relative humidity. Such instruments lend themselves very readily to obtaining automatic continuous records of relative humidity. They must, however, be recalibrated frequently since they tend in time to deviate from their original calibration.

Variations in Humidity

The amount of water vapor present in the atmosphere varies constantly with respect both to place and to time. The major controlling factors are temperature and source of supply. The source of all atmospheric vapor is evaporation,

[1] Evaporation from Free Water Surfaces, *U. S. Department of Agriculture Tech. Bul.* 271, December 1931.

[2] C. F. Marvin, Psychrometric Tables for Obtaining the Vapor Pressure, Relative Humidity and Temperature of the Dew Point, *U. S. Department of Agriculture W. B.* 235, 1937.

either from the oceans or from the lands, including evaporation of rainfall intercepted by vegetation and transpiration. Since the oceans provide the principal supply, the greatest concentration occurs near the ocean's surface in the tropics. It decreases with altitude, with latitude, and with distance inland from the seashore.

Absolute humidity decreases rapidly with altitude. Approximately one half of the total moisture content of the atmosphere occurs within 1 mile of the earth's surface. Vapor at higher altitudes must have been carried aloft by ascending air currents. The principal process by which such currents are produced is through convective action, and at the higher altitudes this action becomes feeble.

Inasmuch as temperature plays a major role in determining the moisture content of the air, influencing not only the rate of supply but in a more important way the ability of a given space to hold water vapor, it follows that the absolute humidity decreases with increase in latitude. It is also higher in summer than in winter, usually reaching a maximum in July or August and a minimum in January or February. Its daily variation is generally from a minimum in the early morning to a maximum in the middle or late afternoon. Its variation with the distance inland from the coast depends upon the direction of the wind and upon the character of the intervening topography. With the wind from the ocean, the absolute humidity will be high even for a considerable distance inland unless a coastal range forces the air to rise to high altitudes, where it expands, cools, and precipitates most of its moisture leaving the air relatively dry after passing over the mountains. On the other hand, with the wind toward the ocean, air that has traveled a long distance overland is relatively dry even near the coast.

Relative humidity varies quite differently from absolute, being higher at night than in the daytime, with a maximum in the early morning and a minimum in the afternoon. Although the annual variation follows no fixed pattern, it is usually greatest in the fall when the lowering temperatures reduce the maximum vapor pressure and lowest in the spring when the rising temperature increases the pressure. With distance from the source of supply, relative and absolute humidity normally vary in about the same manner. In the tropics the relative humidity is high, having an average value of 80 per cent or more. It decreases gradually

throughout the trade-wind belts and reaches an average minimum of about 70 per cent in the horse latitudes. Approaching the polar regions it again increases to a maximum of 80 to 90 per cent.

Amount of Vapor in Atmosphere

In connection with the study of precipitation it is interesting and of considerable importance, too, to know something about the actual amount of moisture contained in the atmosphere and also the capacity of space to hold moisture at various temperatures. Inasmuch as the first of these quantities varies with respect both to time and to place it is impossible to give either the average amount or the extreme variations in the total moisture content of the air at all points on the earth's surface. In fact, but few observational records are available from which these quantities may be determined.

W. H. Dines[1] gives the following data bearing on this subject.

1. Humidity records obtained from 250 registering balloons sent up over England and continental Europe show that the average moisture content of the atmosphere in winter, in terms of equivalent rainfall, varies between 0.25 in. and 0.80 in. with an average of about 0.40 in.; whereas in summer the mean is about 0.80 in. and ranges between 0.50 in. and 1.50 in. From this it appears that in summer the moisture content is just about twice as great as in winter.

2. The total depths of water contained in a column of saturated aqueous vapor at various temperatures are given in the following table. In this it is assumed that the maximum vapor pressure exists throughout and that the reduction in temperature is 10° F per kilometer of height, which is about the average normal rate prevailing in the lower strata.

Ground Temperature in ° F	Total Contents in in.
80	2.86
70	1.90
60	1.24
50	0.84
40	0.53
30	0.33
20	0.18

[1] *Symon's Meteorological Magazine*, October 1918, pp. 95–97.

It should be kept clearly in mind that the above figures represent, for the given ground temperatures and for the normal lapse rate, the total water content of the atmosphere when *saturated.* Alone, they provide no indication whatever of the amount of rainfall to be expected from such an air mass. As a matter of fact, there is no relationship between the amount of moisture in the air over any given area and the resulting precipitation. For instance, it is not uncommon for the atmosphere over the arid regions of the Southwest to contain a greater amount of moisture during any given period than is contained over an equal area in the North Central States; nevertheless there may be no precipitation in the first area and an abundance in the other. As an illustration, at El Paso, Texas, the average absolute humidity is nearly the same as at St. Paul, Minnesota. Nevertheless at El Paso the mean annual precipitation is less than 10 in., whereas at St. Paul it is nearly 30 in.

The above tabulation, however, brings out the fact that the amount of rain that can fall at any given place is definitely limited unless the supply is replenished from outside sources. For instance, with a ground temperature of 80° F, even though the air were completely saturated at the beginning and every drop of moisture in it were condensed and precipitated, the total resulting rainfall would be only 2.86 in. Actually greater rains often occur at temperatures of 80° or less. It thus appears that such rains must be supplied by inflowing winds from other areas.

Sources of Atmospheric Moisture

Although all water vapor that produces precipitation came originally from the sea, the direct source of that vapor has long been the subject of much debate. It may have been derived from any one or more of the following sources:

1. Evaporation from the ocean.
2. Evaporation from the land.
3. Evaporation from lakes and rivers.
4. Transpiration from vegetation.

A very wide difference of opinion exists as to the proportionate amount that is obtained from each of these sources.

For instance, Bruckner[1] estimated that only 7 per cent of the

[1] *Science*, **38**, 69.

land precipitation came directly from oceanic evaporation. Mead[1] thought that this figure should be between 25 and 35 per cent. Meyer[2] states that "only that portion of the rainfall which runs off through the streams represents water which has evaporated from the ocean." On the other hand, Holzman,[3] as a result of studies based upon data obtained at a large number of aerological stations located throughout the country, concluded that, although 75 per cent of the moisture precipitated over continents is returned to the atmosphere by evaporation, only 25 per cent is reprecipitated on the land. It would seem to follow that approximately 75 per cent of the precipitation on continents is derived from oceanic evaporation. Although these conclusions are believed to be applicable to the United States as a whole, there are undoubtedly drainage basins, especially those located in the North Central States, for which this figure is too high.

Although there may be a difference of opinion as to the exact percentage of the land precipitation that is derived directly from oceanic evaporation, there is no relationship between this quantity and the stream flow into the ocean. As far as the hydrologic balance is concerned, every drop of water that falls as rain in the United States might be derived directly from oceanic evaporation. Actually the portion of the land precipitation that is derived directly from oceanic evaporation is equal to the stream flow plus the direct underground flow into the ocean plus also that portion of the precipitation on the ocean that had been derived directly from transpiration and evaporation from the land, lakes, and rivers. The sum of the last two quantities far exceeds the total direct stream flow into the ocean.

According to Holzman's concept, dry polar air masses moving equatorward over the North American continent and the oceans absorb moisture derived from transpiration and evaporation and are thereby transformed into warm moist tropical air masses. During their return journey poleward these same air masses are cooled, their capacity for moisture is reduced, and they become the principal source of precipitation for this country. It thus appears that, in general, precipitation here is incidental to the vast

[1] D. W. Mead, *Hydrology*, McGraw-Hill, 1919, p. 123.

[2] A. F. Meyer, *Elements of Hydrology*, John Wiley, 1928, pp. 64–65.

[3] Benjamin Holzman, Use of Aerological Soundings in Determining the Sources of Moisture for Precipitation, *Trans. Am. Geophys. Union*, 1937, Part II, pp. 488–489.

world-wide system of atmospheric circulation and is but little influenced by local transpiration and evaporation, except perhaps in the central portion of the United States.

Condensation

Condensation of atmospheric vapor results, in general, in the formation of clouds, but not all clouds produce precipitation. Close observation of small clouds on a hot day often reveals the fact that they gradually grow smaller and smaller and finally disappear as a result of evaporation. It is, therefore, necessary to distinguish between the conditions that produce condensation and those that result in precipitation.

Condensation may result from any one or more of four principal causes, viz.:

1. Dynamic or adiabatic cooling.
2. Mixture of two air masses of different temperatures.
3. Contact cooling.
4. Radiational cooling.

Several other processes produce condensation but are of minor importance. Condensation resulting from mixing, contact, and radiational cooling occurs at such feeble rates that it seldom produces precipitation. Contact and radiational cooling cause the formation of dew, frost, and fog. The most important condensation process is adiabatic cooling.

Dynamic or Adiabatic Cooling

When unsaturated air at or near the earth's surface is carried to higher levels, either through convection or through other means, expansion will occur due to the reduction of pressure with altitude. Except near the earth's surface this expansion is adiabatic, meaning that no heat is added to the air from outside sources, and none is subtracted from it. However, its temperature is lowered because of the heat energy that is transformed into work in the process of expansion. This reduction in temperature is called *dynamic* or *adiabatic cooling*. It is the principal cause of condensation and is directly responsible for practically all rainfall.

The Formation of Raindrops

It has been previously mentioned that condensation does not necessarily cause precipitation. Condensation forms fog or clouds

which consist of small droplets of water having an average diameter according to Petterssen[1] of 40 microns. Raindrops on the other hand have diameters varying from 500 to 4000 microns. It follows that some process which will increase the size of the drops is necessary before precipitation will occur. Petterssen indicates that there are two principal methods by which this is thought to occur. The more important one, known as Bergeron's theory, requires that particles of ice and droplets of water, cooled below freezing temperature, be mixed in a cloud. For subfreezing temperatures the saturation vapor pressure is lower at an ice surface than at a water surface. The air in the cloud will have a vapor pressure somewhere between these two saturation pressures. As a result the water droplets will evaporate and at the same time condensation will occur on the ice particles. Large droplets are thus built up which begin to fall. In falling they collide and combine with other droplets to further increase their size.

A second process of forming raindrops which does not require subfreezing temperatures consists of mixing warm and cold droplets together. Here again the vapor pressure of the air would be between the saturation vapor pressures of the warm and cold droplets. As a result, the warm droplets will evaporate, and at the same time condensation will occur on the cold drops. Showers produced by this method are usually quite light.

Types of Precipitation

From the preceding it appears that a rising air column is a necessary antecedent to precipitation. It naturally follows that precipitation may be classified in accordance with the causes that produce such a rising column, of which there are three, viz.:

1. Convectional.
2. Orographic.
3. Cyclonic.

Convectional precipitation is most common in the tropics but frequently occurs at many places in the United States during the summer. On a hot day the ground surface becomes heated as does also the air in contact with it. This causes the air to rise, expand, and cool dynamically, causing condensation and precipitation.

If because of a topographic barrier, moisture-laden air is forced

[1] S. Petterssen, *Introduction to Meteorology*, McGraw-Hill, first edition, 1941.

to rise to higher levels, expansion, cooling, and precipitation follow. Where conditions are favorable for the production of this orographic rainfall, as it is called, the heaviest precipitation occurs. For instance, where the prevailing winds, heavily laden with moisture from the Pacific, strike the western slopes of the coast ranges in Washington and Oregon and are thereby forced to rise, the areas having the highest precipitation in the United States are to be found. Here is a region having a mean annual precipitation of over 100 in. Similar conditions are to be found at many other places such as the Philippines, the East Indies, and on the southern slope of the Himalayas near the head of the Bay of Bengal in India, where, at Cherrapunji, the average annual rainfall is nearly 500 in. As a matter of fact orographic precipitation may be produced without the aid of a mountain barrier. If in winter or at night, when the land is cooler than the water, moisture-laden air is carried over the land, two factors combine to produce precipitation: (1) the temperature of the air is lowered through contact with the cooler land and may be reduced below the dew point; (2) because of the greater roughness of the land surface, air turbulence and friction are increased, the velocity is retarded, and the depth of the air current is increased, forcing the upper air to rise and cool dynamically. Although relatively flat areas adjacent to large bodies of water oftentimes receive precipitation from these causes, these areas are not subject to such excessively high annual precipitation as above noted.

Cyclones

Especially throughout the central part of the United States the major portion of the precipitation is cyclonic in character. Cyclones are of two general classes, tropical and extratropical, so called depending upon whether they occur within or beyond the tropics. Inasmuch as all cyclones occurring in the United States are of the extratropical variety, this kind alone will be considered. When the word cyclone is used it will be understood to refer to the extratropical type. These cyclones are seldom destructive. A typical cyclone is a large whirling mass of air ranging from 500 miles to 1000 miles or more in diameter and normally having a velocity of about 30 miles per hr. At the center of this mass the barometric pressure is low; in the Northern Hemisphere the air approaches the center spirally in a counterclockwise direction with a vertical com-

ponent. The central portion acts as a chimney through which the air rises, expands, cools dynamically, and produces condensation and usually precipitation.

In the United States, cyclones have certain fairly well-defined paths of travel. The most frequented path is that of those originating in northwestern United States and southwestern Canada which travel eastward along the Canadian border, dipping somewhat to the south as they approach the Great Lakes, and then passing out through the St. Lawrence valley. Some others having the same origin are deflected to the south in the general direction of the Gulf of Mexico. Numerous other paths are less frequently followed, but practically all are characterized by having a decidedly easterly component to their general direction. It usually requires about 4 days for one of these storm centers to cross the United States, the average speed of travel being about 30 miles per hr.

The approach of a cyclone is heralded by increasing cloudiness, rising temperature, a falling barometer, and shifting winds. If the path of the storm lies to the north of the observer, the wind usually sets in from the southeast and then shifts around to the south, the west, and finally the northwest. If the path is south of the observer the wind at the beginning is from the northeast, then the north, the west, and finally from the southwest. Precipitation is usually confined to the southwest quadrant of the cyclonic area. With the passing of the storm center the temperature falls, the barometer rises, and the skies clear.

Cause of Cyclones

Numerous explanations of causes and conditions essential to the production of a cyclone have been advanced. The theory that has gained widest acceptance may be called the *air-mass concept.* Although it is not susceptible of definite proof and leaves unexplained certain features of cyclones, it appears to provide the most satisfactory and plausible explanation of these phenomena that has been proposed to date. For a clear understanding it is necessary first to review briefly the general atmospheric circulation.

At and near the thermal equator lies a belt perhaps 30° in width throughout which, because of the more direct rays of the sun, the air near the earth's surface is heated, rises, expands, cools dynamically, and releases a portion of its moisture in convectional precipitation. It is important to keep in mind, however, that only

that portion of the total moisture is precipitated that is in excess of its dew-point content after expansion. In other words, after this air has released its convective precipitation and has started its journey poleward, it is still relatively warm and is laden with moisture.

The lower atmosphere near the equator rotates about the earth's axis with nearly the same velocity as the earth's surface itself. Because of the diminishing diameter in the higher altitudes and, therefore, the lower linear velocity of rotation of the earth's surface, this air from the tropics flowing poleward has a greater easterly component of velocity than has the earth's surface, and hence to an observer on the ground appears to be moving in a northeasterly direction. The northerly limits of such an air mass is called a *warm front*. In a similar manner as the descending cold dry air in the polar regions spreads out and starts flowing toward the equator, it has an easterly velocity that is less than that of the earth's surface over which it flows, and, hence, to an observer it appears to be coming from the northeast. The southerly boundary of such an air mass is called a *cold front*. A warm front is, therefore, one in which warm air replaces colder air, whereas in a cold front the opposite occurs.

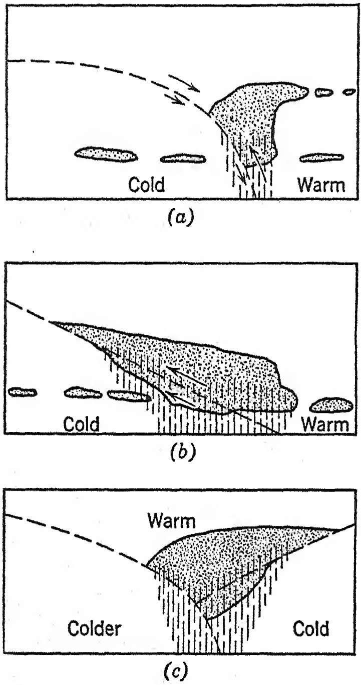


FIG. 21. After Petterssen. By permission from *Introduction to Meteorology*, by Sverre Petterssen, copyrighted 1941 by McGraw-Hill Book Co., Inc.

The surface separating the warm air mass and the cold air mass is called a frontal surface. In the Northern Hemisphere the frontal surface slopes upward toward the north, the air above the surface being warm, moist, and light, while that below is cold, dry, and heavy. For normal conditions in the middle latitudes, Petterssen gives the slope of this surface as being approximately 1 in 100. In

a cold front the moving cold air is in contact with the earth so that the layers of air nearer the ground are retarded by friction and turbulence causing a relatively steep frontal surface (Fig. 21*a*). Thus the warm moist air is forced to rise abruptly, producing intense rains over small areas. The lower strata of warm air in a warm front are also retarded, but in this instance the effect is to produce a flattened slope (Fig. 21*b*). As a result, the moist air rises relatively slowly, the rainfall being spread over a large area. Hence,

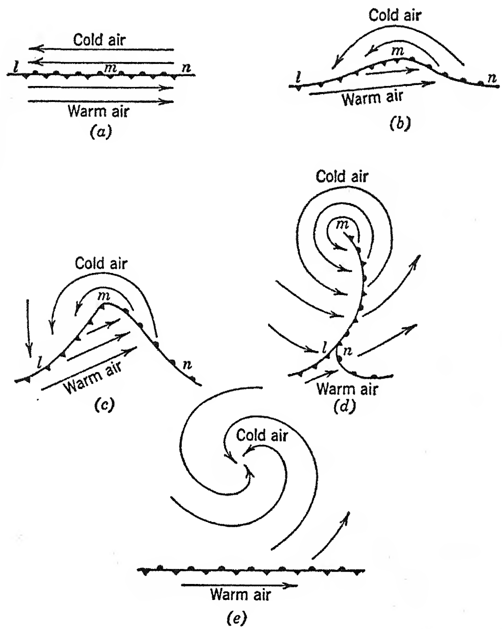


FIG. 22. After Petterssen. By permission from *Introduction to Meteorology*, by Sverre Petterssen, copyrighted 1941 by McGraw-Hill Book Co., Inc.

cold fronts produce intense storms that cause greatest floods on small drainage basins, whereas warm fronts are accompanied by more widespread storms that are productive of the maximum floods on large drainage basins.

Assume now that two air masses approach each other in the Northern Hemisphere, one from the tropics and the other from the polar regions. By the time they meet between latitudes 30° and 60° they will be moving nearly in easterly and westerly directions respectively. In Fig. 22*a* let *lmn* represent the intersection with the ground of the frontal surface separating the two air masses. It is

now seen that this surface is a surface of discontinuity not only with respect to temperature, density, and moisture content but also with respect to velocity. It is this last feature that explains the formation of cyclones. The shear stresses and turbulence occurring at the surface of discontinuity tend to cause waves as shown in Figs. 22*b* and 22*c*. The entire wave is moving to the right, *lm* being a cold front and *mn* a warm front. In its later stages the cold front gradually overtakes the warm front as shown in Fig. 22*d*, thus forcing upward the warm moist air that occupied the intervening space to form what is called an occluded front. A section through an occluded front is shown in Fig. 21*c*. When the wave has developed to the stage shown in Fig. 22*d* it is called a cyclone. Petterssen states that the time required for the cyclone to develop to the stage shown in Fig. 22*c* usually requires from 12 to 24 hr, while the remaining stages require from 2 to 3 days. In the final stages the cyclone is simply a large mass of whirling air (Fig. 22*e*) which gradually dissipates its energy.

The preceding discussion was presented for the purpose of giving the reader an elementary knowledge of cyclones. For a more complete and detailed presentation of this subject the reader is referred to Petterssen,[1] from whom this discussion was largely taken, or to any other standard textbook on meteorology.

Thunderstorms

Ordinarily the most intense rainfalls that occur on small areas are the result of thunderstorms. This type of storm occurs in all parts of the earth although its frequency decreases rapidly from the tropics, where there may be as many as 200 per yr, toward the polar regions, where the average occurrence is less than once a year. They are also more common over land and in mountainous regions than over water and in level country. In the United States, thunderstorms range in frequency from about 60 per yr in the Gulf States to about 15 to 20 per yr along the Canadian boundary except on the Pacific Coast where they occur on an average of only about twice a year.

When a thunderstorm first develops, it usually covers but a small area, perhaps not more than 3 or 4 sq miles. As it advances it spreads out so that the area covered is pear shaped. The average life of a thunderstorm is about 6 hr, at the end of which time the

[1] S. Petterssen, *Introduction to Meteorology*, McGraw-Hill, 1941.

frontal length is oftentimes between 50 and 100 miles with a depth about half as great. Their usual travel is from west to east and with an average velocity of about 30 miles per hr. They most frequently occur in the southern quadrants of a cyclonic low.

The most outstanding characteristic of a thunderstorm is a strong vertical air current supporting a cumulus cloud with a cauliflower-shaped head. Within this characteristic feature is contained a clue to the cause of the thunder and lightning and the heavy downpour of rain that accompanies these storms. It has been experimentally determined that raindrops, regardless of size, cannot fall through air of normal density at velocities greater than about 18 miles per hr. Large drops, upon attaining this velocity, break up into smaller drops whose speeds cannot, because of friction, exceed this limiting value. It has also been demonstrated by Simpson[1] that the breaking up of raindrops is productive of electrical energy, some of the drops becoming negatively charged and some positively. Suppose that in the turbulent air of a cumulonimbus cloud, raindrops form and start to descend. Caught in a strong vertical updraft they are broken into smaller drops and carried back to higher levels where they again coalesce and start to descend, only to repeat their previous experience. Throughout this time the cloud's water content is being constantly increased by the condensation from the rising air current. However, this process cannot go on indefinitely. Eventually, unless other processes may have developed in the meantime, the weight of the water being supported will retard the rising air current enough to permit the rain to descend. This descent is accompanied by thunder and lightning, usually, however, only after an abundant supply, sufficient to produce an intense downpour, has been stored up.

Measurement of Rainfall

In the United States daily measurements of rainfall are made at about 8000 Weather Bureau stations. At about 2100 of these stations automatic continuous recorders provide records not only of the total daily precipitation but also of the intensity variations throughout the day.[2] At the remainder, which are known as co-

[1] G. C. Simpson, Mem. Indian Meteorological Dept., Simla, India, *Physics of the Air*.

[2] Merrill Bernard, *Physics of the Earth IX, Hydrology*, Chapter II, Precipitation, McGraw-Hill, 1942.

operative stations, observers read and record daily only the total precipitation for the preceding 24 hr.

Unfortunately observations are not made at a certain fixed hour. Most of the measurements are made in the late afternoon, usually about sundown, and are recorded as the precipitation for that day, although actually they represent the amount that fell during the preceding 24 hr. The records obtained at all automatic recording stations, and at a few others usually located at power plants, represent the precipitation occurring from midnight to midnight. An additional few observers make their readings in the mornings and record them as the precipitation falling on the preceding day. It is seen from the above that at three adjacent stations a 24-hr rain that may have fallen, let us say, on June 2, might be recorded at one station as having fallen on June 1 and 2, at another on June 2 and 3, and at a third on June 2 only.

Because of this fact, in determining the mean depth of rainfall on an area for each day of a storm, it is necessary to adjust the precipitation records obtained at some of the stations so that the daily values for all the stations will be for the same 24-hr period. For instance, assume that the following records are available from southeastern Michigan.

	Detroit*	Eloise	Monroe†	Ypsilanti	Ann Arbor
June 1	0.	0.	1.08	0.	0.
June 2	3.20	2.35	1.46	1.72	1.60
June 3	0.	0.47	0.	0.33	0.22
Totals	3.20	2.82	2.54	2.05	1.82

* Recording gage; records are from midnight to midnight.

† Readings are taken at 8 AM and recorded for the day preceding. All other readings are made at about sundown and are for the preceding 24 hr.

In this case, as usual, most of the records cover the period from sundown to sundown. It is, therefore, easiest to adjust the other records to cover the same period. If the Detroit continuous record were available, from it the total catch prior to sundown could be determined and this would be taken as the rainfall at Detroit for June 2 and the remainder would be considered as having fallen on June 3. If the continuous record at Detroit were not at hand then Eloise is the nearest station and its records would be used for prorating the total Detroit catch between June 2 and June 3, giving 2.67 in. for June 2 and 0.53 in. for June 3. To determine the

Monroe rainfall for the same periods beginning and ending at sundown, the Eloise and Ypsilanti records are used because they are about equidistant from Monroe and both are nearer than Detroit and Ann Arbor. On the average, 84 per cent of the rain that fell during this storm at Eloise and Ypsilanti fell before sundown on June 2, and 16 per cent fell after that time. Prorating the

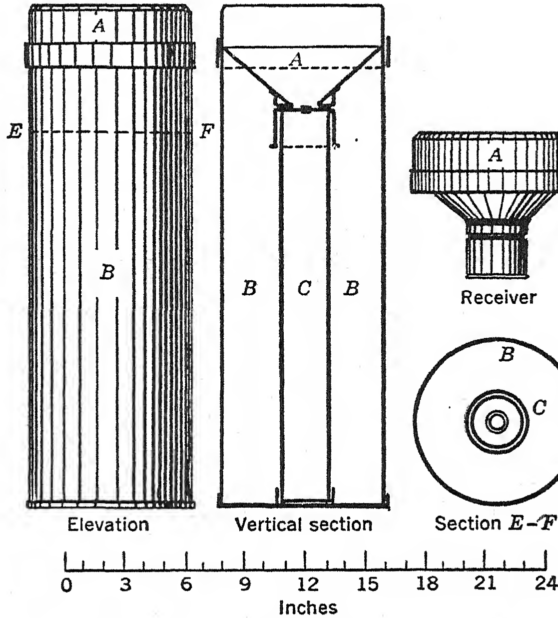


FIG. 23.

total Monroe catch on this same basis results in 2.12 in. for June 2 and 0.42 in. for June 3. Although of course values thus obtained will not be exactly correct, they will seldom be greatly in error.

Nonrecording Rain Gages. At the cooperative stations standard U. S. Weather Bureau rain gages such as those shown in Fig. 23 are used. This gage consists of a can, *B*, 8 in. in diameter and 24 in. deep. Fitted over the top is a brass receiver, *A*, whose top rim is a knife edge. The bottom consists of a funnel that carries the water into the brass measuring tube, *C*. A measuring stick 24 in. long by $\frac{3}{8}$ in. wide and about $\frac{1}{8}$ in. thick completes the equipment. The cross-sectional area of the measuring tube minus the cross-sectional area of the measuring stick is exactly one tenth of the area of the

opening of the receiver; in other words, the measured depth of water in the tube is ten times the actual depth of rainfall. The tube filled with water and containing the measuring stick therefore represents 2 in. of rainfall. If the day's rainfall exceeds 2 in., the excess overflows into the 8-in. can. In such cases, when the observer makes his reading he first submerges the measuring stick, empties the measuring tube, and then pours this excess into the tube for measurement.

Recording Rain Gages. Although there are a number of different types of recording rain gages, only three have gained widespread use, (1) tipping bucket, (2) weighing, and (3) float.

For years the tipping-bucket gage has been commonly used. This gage consists of a bucket that is divided into two compartments so arranged that when the one is filled the bucket tips, empties, and brings the other into position. When it in turn is filled, it tips back to its original position, and so on. The bucket is electrically connected with a recorder so that, inasmuch as 0.01 in. of rain on the opening of the receiver fills a compartment, each tipping records 0.01 in. of rainfall.

There are two principal objections to this type of gage. In the first place, if the buckets are designed to tip at exactly the right instant for any given intensity of rainfall, because of inertia they will tip either too soon or too late for other intensities. As a result, both the intensity and the total rainfall recorded will be in error except during the period when that one given intensity prevails. The total rainfall, as determined from the record at the end of the day, can be corrected by measuring the water that has been dumped by the buckets into the bottom of the gage. The intensities can be corrected by judiciously distributing the total error among the different periods when the intensity as recorded exceeded the intensity for which the gage was designed. The other objection to this type of gage is based upon the inconvenient form of the record obtained. When the intensity of rainfall is high the bucket tips so rapidly that the jogs in the record tend to overlap and blend into one broad solid line, making it difficult if not impossible to read. Furthermore the determination of the intensity for any given period necessitates the counting of the jogs in the record during that period.

Another type of recorder known as the weighing gage is widely used. In this instrument, illustrated in Fig. 24, the receiver rests

on a weighing scale which actuates a pen that draws a graph in the form of a mass diagram of the rainfall. Such a record is shown in

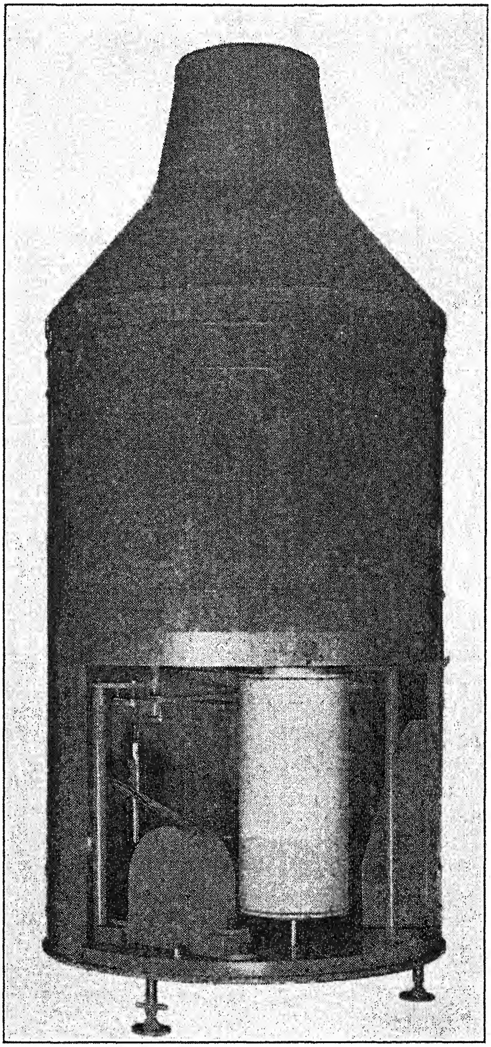


Fig. 24. Stevens snow-rain recorder. Courtesy Leupold & Stevens Instr.

Fig. 25. Inasmuch as in this graph the abscissas represent time and the ordinates represent inches depth of rainfall, the slope of the graph with respect to the horizontal axis gives the intensity of

the rainfall. It is, therefore, an easy matter to determine from the graph the average intensity of rainfall for any given period. As an illustration, in Fig. 25 the total rainfall up to 2:25 PM was 0.58 in.; at 2:35 PM it was 1.09 in.; hence, in this 10-min period a depth of 0.51 in. fell, and the average rate was therefore 3.06 in. per hr. This method of measuring both the intensity and the total rainfall is believed to give more accurate results than can be obtained by the tipping-bucket type of gage and its use is becoming more general.

The float gage is quite similar to the weighing gage. The pen is actuated by a float on the water surface in the receiver in the same

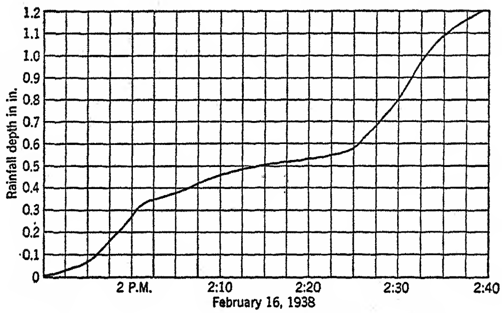


FIG. 25.

manner as for the water-stage recorders described in greater detail in Chapter X. The record produced by this gage is also in the form of a mass diagram as illustrated by Fig. 25.

Measurement of Snowfall

The equivalent depth of water contained in any snowfall may be found by melting a sample obtained at a point where no drifting has occurred. The 8-in. rain-gage cylinder is inverted and used as a cookie cutter in obtaining the sample. A measured quantity of hot water is poured into the can, the snow is melted, and the mixture is measured in the brass tube. The difference between these measured quantities is the depth of water precipitated.

At cooperative U. S. Weather Bureau stations, the usual practice is to melt the catch obtained in the standard 8-in. cylinder with the receiver and measuring tube removed. Usually this method gives results that are too low because of the effect of the wind in

MAP OF UNITED STATES SHOWING MEAN ANNUAL PRECIPITATION

Red lines and figures indicate average annual precipitation in depth in inches

By courtesy U. S. Geological Survey

Fig. 26

Prepared by Henry Gannett
mainly from data of the
United States Geological Survey
and United States Weather Bureau

deflecting snow over and around the gage. Several different types of shields have been devised for the purpose of eliminating this error, but up to the present time none of them has come into general use.

Annual Precipitation

In Fig. 26 a map is presented showing the variation in the mean annual precipitation in the United States as prepared by Gannett from data of the U. S. Geological Survey and the U. S. Weather Bureau. It may be seen that the highest precipitation occurs along the northern Pacific coast where values in excess of 100 in. are not unusual. Just east of the coastal mountain ranges there lies a belt of very low annual rainfall, varying from 5 to 10 in. From this region eastward through the Rocky Mountain area the precipitation is irregularly distributed, there being small areas of relatively high rainfall but for the most part less than 20 in. East of the Rockies, beginning at about the 101st meridian, the annual precipitation increases in an easterly and southerly direction culminating in values ranging from 40 to 50 in. along the Gulf coast, with some local mountainous areas having as much as 80 in. per yr. It is interesting to note that the 101st meridian, running from Texas to the Dakotas, corresponds approximately to the 20 in. rainfall line as well as to the western boundary of the corn belt.

Mean Rainfall on Basin

Three methods are commonly used for computing the mean rainfall on an area, viz., (1) arithmetic mean, (2) Thiessen mean, and (3) isohyetal method. As will presently appear, the first two are purely mechanical processes requiring no special skill or judgment; on the other hand, the results obtained by the third method, which perhaps should be the most accurate, depend for their accuracy upon the good judgment of the person making the computations. These methods may be used for determining the mean depth of rainfall on an area either during a storm or for any longer period such as a month, year, or period of record.

Arithmetic Mean. As the name implies this result is obtained by dividing the sum of the depths recorded at all the stations on the basin by the number of stations. If the stations are uniformly distributed over the basin and the rainfall varies in a regular manner the results obtained by this method will not differ appre-

ciably from those obtained by either of the other methods. On the other hand, if widely differing records are available at a relatively few irregularly spaced stations the arithmetic mean is likely to

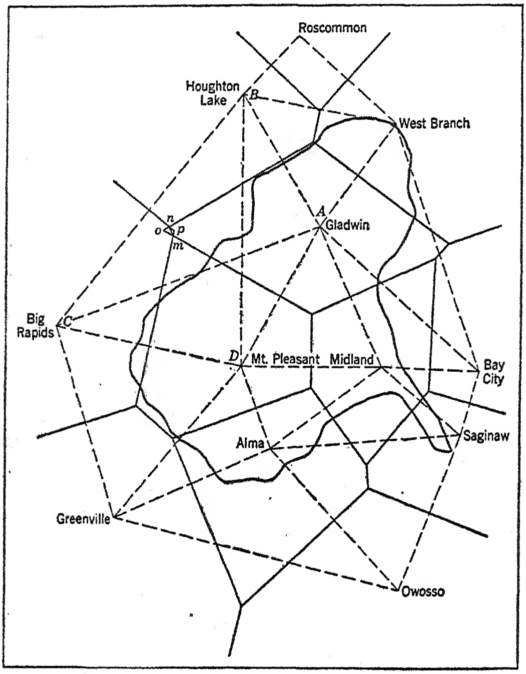


FIG. 27.

differ considerably from the results derived by either of the other methods.

Thiessen Mean.[1] In the application of this method, adjacent stations are joined by straight lines thus dividing the entire area into a series of triangles (Fig. 27). Perpendicular bisectors are erected on each of these lines thereby forming a series of polygons, each containing one and only one rainfall station. The entire area within any polygon is nearer to the rainfall station contained therein than to any other and it is therefore assumed that the rainfall recorded at that station should apply to that area. If P is the mean rainfall on the basin whose area is A, and $P_1, P_2 \cdots P_n$ repre-

[1] A. H. Thiessen, Precipitation Averages for Large Areas, *Monthly Weather Rev.*, July 1911, p. 1082.

sent the rainfall records at the stations whose surrounding polygons have areas $A_1, A_2 \cdots A_n$, then

$$P = \frac{A_1P_1 + A_2P_2 + \cdots A_nP_n}{A} \tag{2}$$

In constructing the triangles upon whose sides the perpendicular bisectors are erected sometimes a question will arise as to which

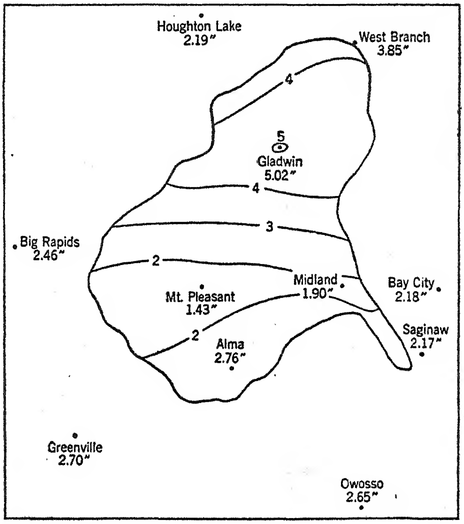


FIG. 28.

stations should form the vertices of a triangle. In Fig. 27, for instance, should the quadrilateral $ABCD$ be divided into triangles by the line AC or BD? Most frequently it is found that the correct line to be drawn is the shorter of the two diagonals, but this is not necessarily true as in the present case. By drawing BD, which is shorter than AC, it is found that the polygons enclosing A and C overlap, forming a quadrilateral *onpm*, in which *onm* is nearer to C than to A and the area *npm* is nearer A than C. By drawing the diagonal AC, however, this difficulty is eliminated. Such cases can be determined only by trial.

Isohyetal Method. In Fig. 28, isohyetals, or contours of equal rainfall, have been drawn. By planimetering the areas between adjacent isohyetals, the mean rainfall on the basin can be found

from equation 2, in which now $A_1, A_2 \cdots A_n$ are the areas between the successive isohyetals and $P_1, P_2 \cdots P_n$ represent the mean rainfalls on the respective areas.

To obtain best results from this method, however, good judgment is required both in drawing the isohyetals and in assigning the proper mean rainfall values to the areas between them. If the entire area is subject to the same rainfall no special judgment is necessary in drawing the isohyetals, but if, for instance, the upper part of the basin were a high plateau the isohyetals should

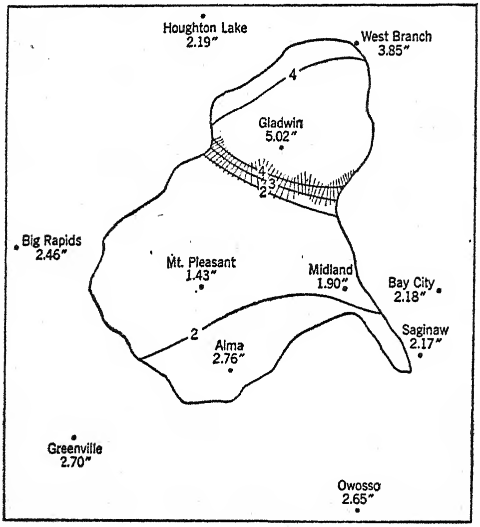


FIG. 29.

perhaps be drawn as shown in Fig. 29. To provide a comparison of the results obtained by the above methods, the mean rainfall for June 1946, as determined (1) by the arithmetic mean of the stations within the basin is 2.78 in.; (2) by the Thiessen mean, 2.97 in.; and (3) by the isohyetal method, 3.08 in. If the isohyetals were drawn as shown in Fig. 29, the mean would be 2.64 in.

As to a choice between the arithmetic mean and the Thiessen mean, much depends upon (1) the distribution and distance between stations and (2) the character of the rainfall distribution. If, for instance, the stations are distributed so uniformly that the polygons formed by the perpendicular bisectors of the Thiessen mean are of practically uniform size, or if the rain is the result of

a general storm varying uniformly in depth between stations, the results obtained by these methods will differ but slightly. As the conditions depart from those described above, it becomes more and more desirable that the Thiessen method be used.

A further advantage possessed by both the isohyetal and Thiessen methods arises from the fact that stations located a short distance beyond the boundary of a drainage basin are used in determining the mean rainfall on the basin but their influence diminishes as their distance from the boundary increases. This is as it should be. On the other hand, in the arithmetic-mean method every station has equal weight regardless of its location. If only

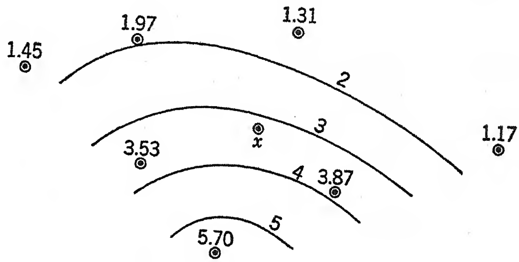


FIG. 30.

stations lying within the basin are used, a station lying just inside has the same weight as one lying near the center, whereas another lying just outside has no weight at all. This advantage is greater if there are only a few sparsely scattered stations within the basin, but it practically disappears if the number is large and the stations are close together.

Supplementing Precipitation Records

In a great many problems it is necessary to supplement certain rainfall records that are missing at one or more stations, as for instance if one wishes to compare the mean rainfall on two drainage basins for a certain period and finds that the records are complete and satisfactory except for one storm. Such a record can best be supplied by interpolation on an isohyetal map that has been prepared from records at adjacent stations. Such a map is shown in Fig. 30. By interpolating between the 3-in. and 4-in. isohyetals, the precipitation at x is found to be 3.23 in.

Records for any short periods may be supplied in this manner.

However, for longer periods, as for instance a year, it is best to take into consideration the variation in the mean annual rainfall at the different stations. This can be done as follows. Suppose that, for the period for which simultaneous records are available at A and B, the mean annual rainfall at A is 37.54 in. and at B it is 40.20 in. For the year in which the records are missing at A, the rainfall at B is 38.87 in. If we let X equal the rainfall at A for this year, then

$$\frac{X}{38.87} = \frac{37.54}{40.20}$$

and

$$X = 36.30 \text{ in.}$$

If there are two or three other stations whose distance from A is about the same as that of B, the same procedure may be followed

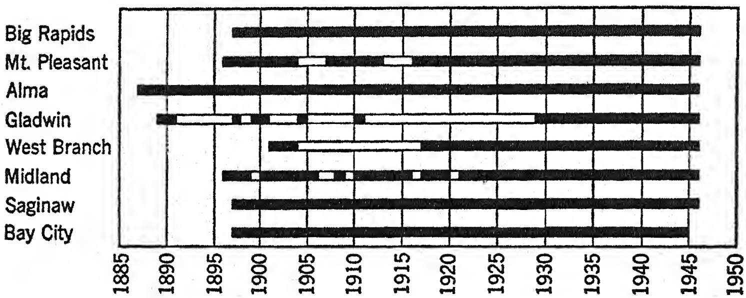


FIG. 31.

with them, thus obtaining three or four estimated values of X, the mean of which may be taken as the most probable value.

Areal Mean Annual Rainfall

Because of the periodical variations in the annual rainfall at any station (see page 91) certain definite restrictions must be placed upon the use of rainfall records obtained at different stations in determining the mean annual rainfall on any area.

Suppose for instance that we want to find the mean annual rainfall on the Tittabawassee River basin in Michigan. In Fig. 31 are shown graphically the periods covered by the various rainfall records available at the stations which are needed for determining the mean precipitation on this basin by the Thiessen method. From this figure it appears that, by the method described in the

preceding article, the records obtained at these stations can without difficulty be made to cover the 50-yr period ending with December 1946. As an illustration, the records at Gladwin for 1898, 1901, 1902, and 1903, would be supplemented by using those at Mt. Pleasant, Midland, and West Branch. In supplementing the Midland records for 1899, those at Bay City, Saginaw, Alma, and Mt. Pleasant would be used, and so on. Always the records of the nearest stations are prorated using the ratio that exists between the mean annual rainfall at the two stations for the entire period of simultaneous records.

With these extended records an isohyetal map of mean annual rainfall can be prepared or the mean annual rainfall on the Titta-

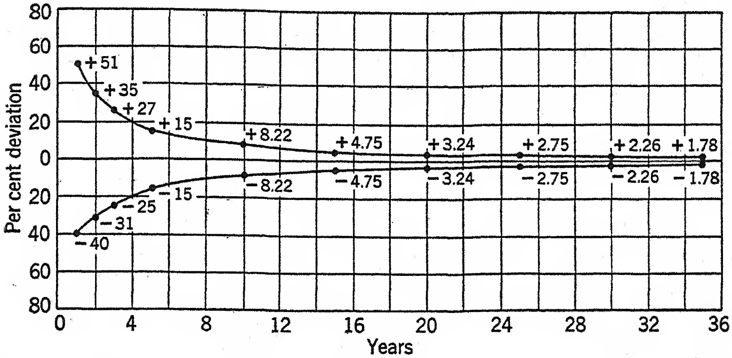


FIG. 32.

bawassee basin can be computed. Without these extended records, however, such a map or computed mean annual rainfall could be only for a period for which there were available continuous records at all the stations used.

Variations in Annual Rainfall

With rainfall records covering only a limited period at any station it is of course impossible to determine the true long-term mean. If the period covered by the records is more than 30 yr, however, the average does not depart greatly from the true mean. Alexander Binnie[1] made a study of the periodic variation in rainfall and his findings are shown in Fig. 32 in the form of a curve giving

[1] Alexander Binnie, The Variation in Rainfall, *Proc. Inst. Civil Engrs.* (*London*), Vol. 109.

the average percentage of deviation from the true mean for records whose lengths are shown as abscissas. For instance, according to this figure any record 5 yr in length is likely to be nearly 15 per cent in error; a record 10 yr in length is probably within 8.2 per cent of the true mean; one 20 yr in length should be within 3.3 per cent of the correct value and so on. Records 30 or 40 yr in length in all probability give the true long-term mean rainfall with an average error of about 2 per cent, which is ordinarily near enough for all practical purposes.

As a result of his studies, Parker[1] states the following conclusions.

1. The maximum rainfall during any year is between 25 and 70 per cent greater than the mean, with an average excess of about 46 per cent.

2. The minimum rainfall during any year is between 20 and 45 per cent less than the mean, with an average shortage of about 33 per cent.

3. Corresponding figures for the two consecutive driest years are 14 and 40 per cent less than the mean, with an average of 25 per cent; and for the three consecutive driest years, 13 and 36 per cent, with an average of 20 per cent.

4. Of the years of record 46 per cent exceed the average, and the rainfall for these years exceeds the average by 19 per cent; for the remaining 54 per cent of the years which are deficient in rainfall, the average deficiency is 17 per cent of the mean.

5. It is unlikely that at any station there will be more than 5 or possibly 6 consecutive years during which the total rainfall for each year will exceed the average, or during which the total rainfall for each year will be below the average. For such high periods the average excess is about 19 per cent, and for the low period the deficiency is about 18 per cent.

6. The above departures from the mean rainfall when expressed as percentages are greater for areas having a low mean annual rainfall and smaller for areas having a high rainfall.

As explained in the preceding paragraphs the total annual rainfall occurring at any station varies greatly from year to year. Furthermore these variations appear to be most irregular. The dotted line in Fig. 33*a* represents the fluctuations in the annual rainfall at Lansing, Michigan, over an 82-yr period. From this graph it is difficult to judge whether there is any tendency for the

[1] Philip A. Morley Parker, *Control of Water*, D. Van Nostrand, 1913, p. 178.

rainfall to increase and decrease in a cyclic manner. In only one 5-yr period, viz., from 1880 to 1885, was the annual rainfall for every year appreciably above the average and never were there 5 successive years below the average.

If, however, a curve is plotted representing progressive 5-yr means as shown by the solid line in Fig. 33*a*, a somewhat different

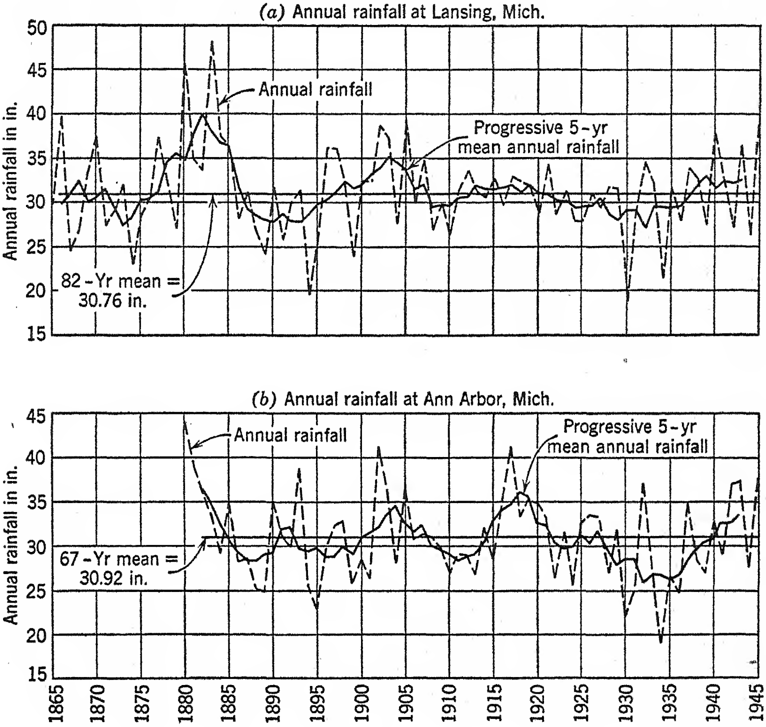


FIG. 33.

picture is presented. In the construction of this curve the value of 29.83 in. for 1866 is the average of the total annual rainfalls for 1866 and for the two preceding and the two succeeding years. In a similar manner the value of 30.78 in. for 1867 is the average for the 5 yr from 1865 to 1869 inclusive.

If one had only the records covering the period from 1870 to 1910 he might draw the conclusion that there is a 20-yr cycle in rainfall. The subsequent records, however, do not indicate such cyclic variation. For instance in 1922 and 1923, at the very time

that, if there are regular cycles, a peak should be occurring, the graph shows that there is actually less than average rainfall. Furthermore the progressive 5-yr mean rainfall at Ann Arbor, Michigan, shown in Fig. 33*b*, does not reveal any such 20-yr cycles. In this graph, peaks occur in 1882, 1892, 1904, and 1918, with intervening periods of 10, 12, and 14 yr. The 5-yr progressive mean at Ann Arbor for 1918 lacks only a fraction of an inch of being the highest on record, whereas at Lansing the corresponding value is about the average. If there are any regular cyclic variations in rainfall it seems but reasonable that the forces and causes controlling them must be vast and far-reaching in extent. Most assuredly they cannot be local. Therefore, all rainfall records, at least in any locality, would necessarily exhibit the same cyclic variations. The fact that the records at different stations located not far apart oftentimes show strikingly different characteristics seems to present strong evidence that the variations in annual rainfall are very largely governed by chance and not by any universal law.

In this connection attention might be called to the records obtained at Mt. Pleasant, Bay City, and adjacent cities in Michigan for 1911. At Mt. Pleasant the total recorded rainfall for this year was 16.31 in., the lowest in 40 yr of records. At Bay City, 45 miles from Mt. Pleasant, the corresponding rainfall was 46.67 in., the highest in 44 yr of records. At Midland, which is only 20 miles from Bay City, the total rainfall for 1911 was 17.05 in.; and at Alma, less than 20 miles from Mt. Pleasant, it was 35.31 in. These figures well illustrate the large differences that oftentimes occur in the amount of rainfall experienced at different stations not far apart and in an area throughout which the rainfall characteristics do not vary greatly.

Relation between Storm Frequency and Mean Annual Rainfall

If throughout any area the probability of occurrence of a storm of any given intensity is the same at every point, that area is said to be *meteorologically homogeneous*. Everywhere in such an area the rainfall expectancy is the same. Inasmuch as the annual rainfall is the total depth of the rains of all different intensities, it necessarily follows that if everywhere throughout a given area or in two different areas the frequency of storms of all different intensities is the same, the average annual rainfall must also be the same at

all points within such areas. The converse of this proposition is not necessarily true, however, because on two different areas having the same total annual rainfall those totals may be made up of an infinite number of combinations of depths of rains of different intensities. As an illustration, in southwestern United States and in north central Canada there are areas in each of which the mean annual rainfall is 20 in. In southwestern United States this total depth of 20 in. usually results from a few storms of short duration but of high intensity, whereas in north central Canada the corresponding total is usually the result of a much greater number of storms of longer duration but of lesser intensities. These two areas, therefore, have the same mean annual rainfall but are not meteorologically homogeneous.

The same factors affect and determine the mean annual rainfall of an area as affect its meteorological homogeneity. These factors are as follows.

1. Distance from the ocean.
2. Direction of prevailing winds.
3. Mean annual temperature.
4. Altitude.
5. Topography.

These influences are more or less interdependent. For instance, normally, with prevailing winds from the ocean, the mean annual rainfall will be high near the shore and, except when the topography is steep and precipitous, will decrease slowly toward the interior. However, if the prevailing winds are from the land to the sea the mean annual rainfall will not be excessive even near the shore, and the effect of the ocean's presence will not be noticeable for any considerable distance inland.

The condition most favorable for a high mean annual rainfall and likewise for a high frequency of intense storms would be found on the ocean side of a mountain range near the coast, in the tropics, at an altitude of 4000 to 5000 ft and with the prevailing winds from the sea. Without the mountain range or without the presence of the ocean, with lower mean annual temperature, with lower altitude, or with a different direction of prevailing winds, the frequency of intense storms and the mean annual rainfall would be correspondingly reduced.

Because of the interrelationship existing between these various

influences it is impossible with our present limited knowledge to express quantitatively the effect that each of these factors has upon mean annual rainfall and frequency of intense storms. The best that can be done at present is to recognize the existence and character of these various influences and thereby, with the aid of mean annual rainfall records, determine whether or not two different areas are meteorologically homogeneous. In all probability if two stations have the same average annual precipitation and the same average number of days of rainfall per year, they are meteorologically homogeneous.

Frequency of Intense Rainfalls

Storms of any given intensity occur with varying frequency in different localities. By frequency is here meant the *average* time interval that elapses between successive occurrences. There is, however, no uniformity to the lengths of the intervening time intervals. For instance, although the frequency of a 4-in. 1-day rainfall at a certain station may be once in 40 yr, two or more such storms may occur in the same or in successive years followed by a period of 50 yr or more before the next recurrence.

With only our present knowledge of the causes and physical processes involved in rainfall, the most practical method of determining the probable frequencies of storms of different intensities is through a study of past records. Usually, however, those records are of such short duration that they are entirely inadequate for this purpose if the frequency determinations are restricted to the actual period of time covered by the records obtained at any one station. To overcome this difficulty the station-year method[1] is generally used.

Storm frequency is closely related to flood frequency. For this reason the problem of determining the maximum intensity of storm that may be expected with a given frequency on any particular basin is of prime importance in all flood studies. The maximum storm that occurs with a frequency of, for instance, once in a hundred years is not the same in magnitude as the storm that causes the maximum flood that occurs with that same frequency. The latter is considerably smaller than the former for the reason that the infiltration capacity of the drainage basin is much greater

[1] Katharine Clarke-Hafstad, Reliability of Station-Year Rainfall-Frequency Determinations, *Trans. A.S.C.E.*, 1942, p. 633.

during some of these storms than during others. Nevertheless there is a relationship between these phenomena and in order to determine the magnitude of the flood that will occur with any given frequency it is necessary to know the frequency and magnitude of the storm that produces it. For this purpose all the rainfall records that are available at the various stations throughout the area are arranged in the decreasing order of their magnitude. The highest value is then taken as the maximum rainfall that may be expected to occur in the given storm period, not within the area, but at any particular station within the area, once in the total combined years of records at all the stations. The second highest value is considered to be the maximum depth of rain that may be expected to occur once in half the total combined number of years of record; the tenth highest value is the maximum to be expected in one tenth of the total years of record, and so on.

As an illustration suppose, for instance, that we wish to determine the maximum 1-day rainfall that is likely to occur on an average of once in 100 yr at Lansing, Michigan. It is believed that the area within 50 miles of Lansing is for all practical purposes meteorologically homogeneous. The rainfall records that are available at stations within this area, each covering a period of 30 yr or more, are as follows.

Station	Length of Record, years	Station	Length of Record, years
Alma	61	Howell	45
Greenville	35	Ann Arbor	68
Flint	59	Charlotte	44
Owosso	52	Olivet	32
Saranac	52	Battle Creek	64
Lansing	84	Albion	36
Hastings	55	Jackson	51
		Total	738

From the above it is seen that there is not a single record that covers a period of 100 yr. However, the total combined length of all the records at these fourteen stations is approximately 700 yr. It is believed that all these records are of sufficient length to include years of high and of low rainfall; in other words, each record represents average rainfall conditions fairly well.

Suppose now that the maximum 1-day storms selected from these records and arranged in decreasing order of their magnitude are as follows.

1	5.60	8	4.86
2	5.47	9	4.75
3	5.38	10	4.62
4	5.10	11	4.58
5	5.06	12	4.53
6	4.97	13	4.49
7	4.90	14	4.40

From the above the conclusion may be drawn that once in 100 yr a 1-day rainfall of 4.90 in. or more may be expected to occur at Lansing or at any other station within this area; also that once every 50 yr a 1-day rain of 4.40 in. or more may be expected, and so on. It might also be assumed that once in 700 yr a 1-day rainfall of 5.60 in. or more would occur. Because this last figure is based upon only one period of observation it is very likely to be greatly in error. On the other hand, the occurrence of a rain of 4.40 in. with an average frequency of once in 50 yr is subject to a much smaller error because it is based upon fourteen periods of observation. In general, it is true that, the greater the number of periods of observation of any natural phenomenon, the greater will be the accuracy of any predictions for the future that are based upon those data. Perhaps never should such data be extrapolated, and ordinarily at least ten periods of observation should be available to prevent an excessively large error.

The accuracy and reliability of the results obtained by the use of this method depend upon (1) the actual length of the individual records, (2) the meteorological homogeneity prevailing at the different rainfall stations, and (3) the absence of permanent climatic changes in the area.

For areas that are meteorologically homogeneous there is a close correlation between mean annual rainfall and frequency of storms of any given intensity. It necessarily follows that for any particular area whose meteorological homogeneity remains constant throughout the years the frequency of intense storms varies with the annual rainfall. Referring now to Fig. 33*a*, for the period extending from 1877 to 1886 during which the mean annual rainfall was considerably above the average, it is apparent that the frequency of intense storms was also greater than normal. Had this particular period of

the Lansing records been the only portion of those records available for determining the frequency of intense storms in this locality, it should have been excluded; otherwise the frequencies derived would have been too high. The same rule applies to all other records whose length is so short that the average rainfall for the period covered is likely to depart considerably from the true long-term mean. From Fig. 32 it is seen that for records 20 yr or more in length the mean annual rainfall is not likely to be more than about 3 per cent in error. It does not by any means follow, however, that records of such length will provide a measure of the frequency of intense storms with an error of only 3 per cent. As a result of some rather hasty studies the authors believe, however, that it should be permissible to use records covering periods of 20 yr or more. Some such records will undoubtedly give frequencies that are somewhat too high, but others will give results that are too low and the resulting errors should not be great.

Inasmuch as the objective in this study is to determine the frequency with which storms of any given intensity may be expected to occur at any point in a given area, and that frequency is dependent upon meteorological homogeneity, it follows that if the various stations are not meteorologically homogeneous then only an average frequency of storms will be obtained that will not accurately represent the frequency existing at any particular station. The magnitude of the error resulting from this cause will depend upon how widely the rainfall characteristics differ at the various stations. If the area is rugged and mountainous, it is quite impossible to determine any average frequencies that can be applied to the area as a whole. In such cases only those stations should be used that have about the same altitude and are similarly located with respect to topography, source of moisture, direction of prevailing winds, etc., and have about the same mean annual rainfall. On the other hand, if the entire area is at practically the same elevation and has about the same mean annual rainfall throughout, the error from this cause should be negligible.

Another source of error resulting from the use of the station-year method of determining storm frequencies has been claimed by some writers where the stations are so close together that the same storm is recorded at two or more stations. Such records are not independent as it is contended that they must be in order that the method give correct results. According to this viewpoint no

two stations should be spaced more closely than the diameter or equivalent dimension of the storm area of significant rainfall depth.

If the problem were that of determining the frequency with which storms of a given intensity occur, say, in Iowa, it would of course then be improper to combine the records of two or more stations at which the same storm was recorded. That, however, is not the problem under consideration. Instead we wish to determine the frequency with which a storm of a given intensity is likely to occur *at any station or point within a certain area.*

Suppose, for instance, that we wish to determine the frequency with which a 3-in. 1-day rainfall will occur at Lansing or at any other station within a 50-mile radius. Now storms of this intensity are likely to cover areas of any size from a few acres up to, for instance, 1000 sq miles. Undoubtedly many of the smaller storms occur between stations and are unrecorded. This fact, however, does not in the slightest degree affect the accuracy of the results obtained by the station-year method. Furthermore, one of the larger storms of this intensity may be recorded at only one of these fifteen stations, or it may be recorded at ten or more; nor does this fact affect the accuracy of the results. Inasmuch as there is an infinite number of points within any area, and it is assumed that the probability of occurrence of a storm of any given intensity is identically the same at all points, it seems logical that the longer the combined period of record may be and therefore the greater the number of observations the more accurate will be the frequency determination. Of course much is dependent upon the homogeneity of the area and also upon the requirement that the record at each station be long enough to represent the true mean rainfall within the allowable margin of error. If, however, the actual facts depart considerably from either of these specifications, the station-year method will not yield correct results nor will it be possible to determine the probable error because that error will depend upon the extent to which the rainfall characteristics at the different stations are nonhomogeneous and also to the extent to which, because of the shortness of record, they are abnormal.

Duration-Area-Intensity-Frequency Relationships

In most rainfall studies the objective is the determination of the relationship existing in some particular locality between duration

of storm, depth of rainfall, area covered, and frequency of occurrence. For instance, the question to be answered may be "What is the maximum depth of rainfall in any 24-hr period covering an area of 1000 sq miles that may be expected to occur with an average frequency of once every 100 yr?" Station-year methods provide a means for the determination of the frequency of intense storms that may be expected to occur at any given point but provide no information in regard to the area covered by such storms. Furthermore, sufficient data are not available in most localities to provide a satisfactory basis for the determination of the areas covered by our most intense storms. If, for instance, a rainfall station were located in every square mile of area, it would be possible to determine the areal extent of every storm, the location and maximum intensity at the center of the storm, and the mean rainfall on any given area. However, with only one rainfall station, approximately, to every 375 sq miles, as is the present distribution in the United States, there must of necessity be many intense rains covering small areas that are entirely unrecorded; others leave but a partial record with the area of most intense rainfall probably lying between widely separated stations. Only rarely is a station located within the area covered by the maximum intensity of rainfall. Furthermore, with the present station distribution, when an intense storm is recorded at only one station it is impossible to determine the actual area covered which may easily be 1000 sq miles or more.

Recently however the various agencies of the federal government have initiated studies that in time will at least partly remedy this difficulty. In certain selected areas throughout the country a large number of rain gages have been located relatively close together and fairly uniformly distributed. As an illustration, in the Muskingum basin in eastern Ohio the Soil Conservation Service has installed about 500 rain gages within an area of approximately 8000 sq miles or an average of 1 gage for every 16 sq miles. Although the distance between gages is variable and some gages represent areas considerably greater than 16 sq miles and others less, most storms cover areas great enough so that the average area represented is not far from that figure. It may, therefore, be assumed that if a certain storm is registered at twenty adjacent stations the area covered is 320 sq miles. If, however, greater

accuracy is desired, the storm area may be found by totaling the individual areas represented by each station at which rain was recorded.

When rainfall records such as are now becoming available in the Muskingum basin extend over a period sufficiently long to be truly representative of average conditions, it will become possible to apply a modification of the station-year method, which for want of a better name may be called the area-year method of determining the frequency with which storms of different intensities visit different-sized areas. When the present study was made only a few years of records were available at the many stations in the Muskingum basin. Although admittedly the period covered is entirely too short to yield reliable results, these records will be used to illustrate the application of the area-year method to the determination of the frequency with which 1-day storms of different intensities may be expected to occur on different-sized areas in the Muskingum basin. The procedure is as follows.

1. Tabulate each 1-day rainfall showing the number of adjacent stations at which was recorded 1 in. of rain or more; also the number of stations at which was recorded 2 in. or more; 3 in. or more; 4 in., and so on. Then total each of these columns, thus finding the total number of station-days of recorded rainfall of each different amount.

2. Next prepare a set of tables, one for each different depth of 1-day rainfall. Each of these will be similar to Table 5, which is for a 2-in. rainfall, and may be obtained as follows.

Determine from the table prepared under (1) the number of 2-in. rains recorded at only one station, the number recorded at two stations, at three stations, and so on as shown in Columns 1 and 2. In Column 3 are the areas in square miles, covered by each of these storms. In this table it is assumed that the area represented by each station is 16 sq miles. If greater accuracy is desired these areas can be found by the Thiessen method. In Column 4 is shown the number of different areas within the Muskingum basin on which storms of these various extents might have occurred. By multiplying each of these numbers of areas by the number of years of records, the number of area-years of records shown in Column 5 is obtained. Column 6 contains the number of station-occurrences or the number of stations at which a 2-in. 1-day rain was recorded on areas equal to or larger than the areas shown in Column 3. This

column is obtained by starting with the largest storms at the bottom of the table and working up to the smallest. In Column 7 is the number of area-occurrences or in other words the number of times when 2-in. rains fell in 1 day on areas as large as or larger than the areas shown in Column 3. It is obtained by dividing the values in Column 6 by the number of stations in Column 1 respectively. Column 8 is obtained by dividing the numbers of area-years of records in Column 5 by the numbers of area-occurrences in Column 7 respectively. The values in this column

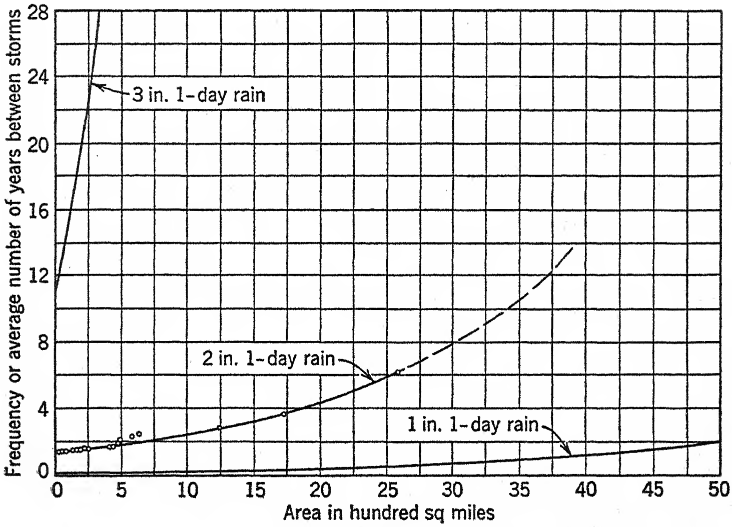


FIG. 34.

represent the average frequency, or the average number of years between successive occurrences, of 2-in. 1-day storms on areas shown in Column 3. The values in Column 8 are then plotted against the values in Column 3 respectively, giving the curve of relationship between frequency and area for a 2-in. 1-day rainfall as shown in Fig. 34. Similar tables are prepared and curves drawn for other depths of rain as shown in this same figure and also for other durations of storm and depths of rainfall as desired.

It should be recognized, however, that the values shown in Column 8 of Table 5 *do not* represent the frequency with which 2-in. 1-day storms will center over some particular drainage basin. Instead these values represent the average frequency with which

each drainage basin of a given size would experience a 2-in. 1-day rainfall provided that each such storm, covering an area equal to or greater than that of the drainage basin, would center over such a basin and would not partially cover two or more such basins without completely covering any. For instance, suppose there are ten adjacent drainage basins either circular or square, each having

TABLE 5

Number of Stations	Number of 2-in. Storms	Area, sq. miles	Number of Areas	Area-Years	Station-Occurrences	Area-Occurrences	Frequency, years
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	16	16	500	1000	716	716	1.40
2	7	32	250	500	700	350	1.43
3	3	48	167	333	686	229	1.46
4	2	64	125	250	677	169	1.48
5	2	80	100	200	669	134	1.49
6	1	96	83	166	659	110	1.51
7	1	112	71	142	653	93	1.53
9	2	144	56	112	646	72	1.56
11	1	176	45	90	628	57	1.58
12	1	192	42	84	617	51	1.64
13	1	208	38	76	605	46.5	1.64
16	1	256	31	62	592	37.0	1.68
26	1	416	19	38	576	22.1	1.72
27	1	432	18	36	550	20.4	1.76
30	2	480	17	34	523	17.4	1.95
31	1	496	16	32	463	14.9	2.15
36	1	576	13.9	27.8	432	12.0	2.32
39	1	624	10.2	20.4	396	8.1	2.52
78	1	1248	6.4	12.8	347	4.5	2.85
108	1	1728	4.6	9.2	269	2.5	3.68
161	1	2576	3.1	6.2	161	1.0	6.20

an area of 200 sq miles and all being meteorologically homogeneous. The odds are almost infinite that a storm covering an area of 200 sq miles would not exactly center over any one of these ten basins but would instead cover portions of two, three, or four basins. Therefore, to allow for practically complete coverage of one basin and the usual amount of overlapping on adjacent basins, the storm area should be taken approximately 50 per cent greater than the area of basin. As an illustration, to determine the frequency of occurrence of a 3-in. 1-day rainfall on a basin having an area of 200 sq miles, from Fig. 34 it is found that the frequency of such a

storm covering an area 1.5 × 200, or 300 sq miles, is once in 26 yr.

Briefly summarized, the suggested procedure for determining the probable frequency with which a rain of any given depth and duration may be expected to occur on any particular drainage basin is as follows.

1. For the given basin, or for some other area that is meteorologically homogeneous and similar and for which there are sufficient rainfall records available, construct area-frequency curves similar to those shown in Fig. 34.

2. From these curves determine the intensity of storms of any given frequency and duration occurring on an area 50 per cent greater than that of the given drainage basin.

In the above discussion it was assumed that the areas covered by storms were circular in shape and also that the drainage basins were either circular or square. Ordinarily the shapes of storm areas and of drainage basins are irregular and oftentimes extremely so. In fact, the variations are infinite and for this reason some standard shape has to be adopted. However, this generalization does not seriously affect the value of the proposed method. In certain cases where the shape of basin departs radically from that of a circle it may be found desirable to apply some multiplier other than 1.5 to the area of basin to find the area of storm whose frequency is desired. If, for instance, the basin is long and narrow and lies transversely to the prevailing direction of storms, it might be advisable to use a factor of 2, 3, or even 4.

Alternative Method

As already explained the use of the area-year method of determining the depth of rainfall in a given time that may be expected on any area with a specified frequency is at the present time restricted to comparatively few localities because of lack of sufficient data from which curves similar to those shown in Fig. 34 may be constructed. For the interim, until such data become available, the following method is suggested.

In Figs. 35*a* to 35*e* are shown curves that represent the variation in depth of rainfall versus area for a number of the most important storms on record lasting from 1 to 5 days inclusive and which occurred in the northern states east of the 103d meridian. In Fig. 36 are shown similar curves for some of the most important storms

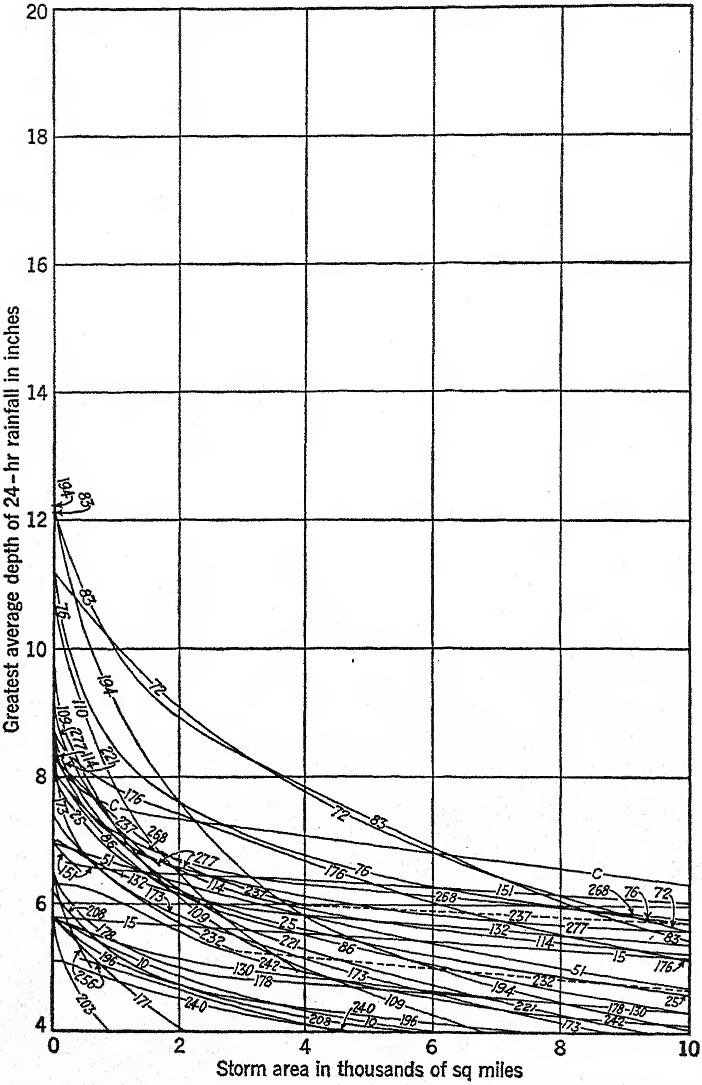
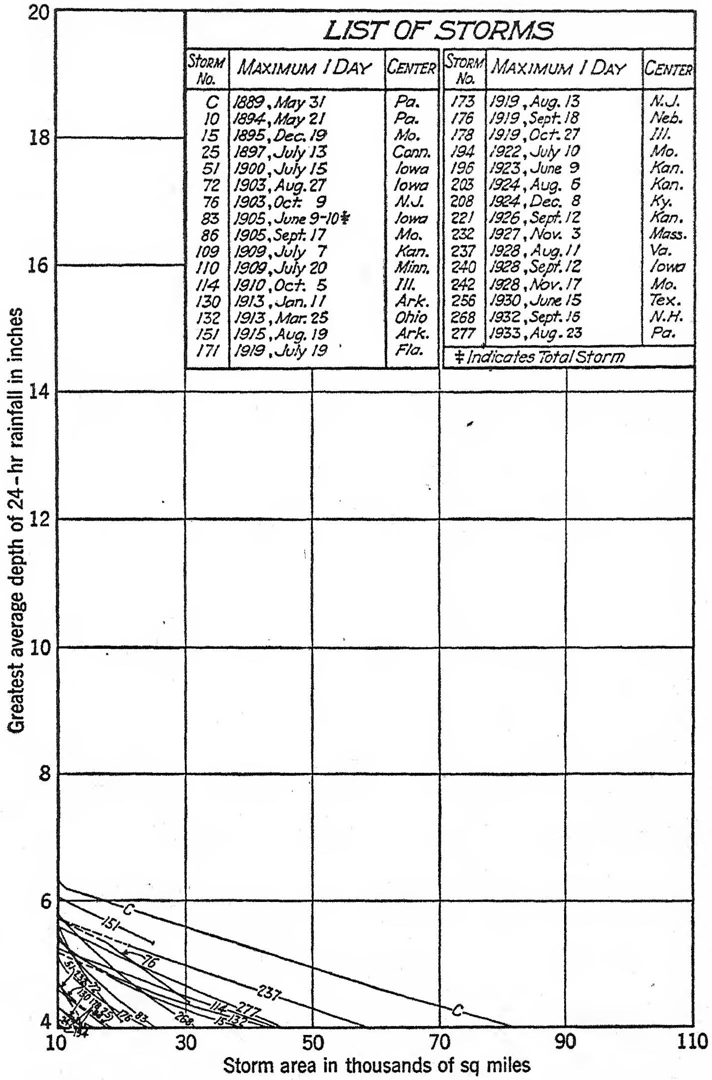


Fig. 35a. Time-area-depth curves for storms over northern states, showing greatest average depth of rainfall during 1 day.



LIST OF STORMS

Storm No.	Maximum 1 Day	Center	Storm No.	Maximum 1 Day	Center
C	1889, May 31	Pa.	173	1919, Aug. 13	N.J.
10	1894, May 21	Pa.	176	1919, Sept. 18	Neb.
15	1895, Dec. 19	Mo.	178	1919, Oct. 27	Ill.
25	1897, July 13	Conn.	194	1922, July 10	Mo.
51	1900, July 15	Iowa	196	1923, June 9	Kan.
72	1903, Aug. 27	Iowa	203	1924, Aug. 6	Kan.
76	1903, Oct. 9	N.J.	208	1924, Dec. 8	Ky.
83	1905, June 9–10‡	Iowa	221	1926, Sept. 12	Kan.
86	1905, Sept. 17	Mo.	232	1927, Nov. 3	Mass.
109	1909, July 7	Kan.	237	1928, Aug. 11	Va.
110	1909, July 20	Minn.	240	1928, Sept. 12	Iowa
114	1910, Oct. 5	Ill.	242	1928, Nov. 17	Mo.
130	1913, Jan. 11	Ark.	256	1930, June 15	Tex.
132	1913, Mar. 25	Ohio	268	1932, Sept. 16	N.H.
151	1915, Aug. 19	Ark.	277	1933, Aug. 23	Pa.
171	1919, July 19	Fla.			

‡ Indicates Total Storm

FIG. 35a. Continued. Note change in horizontal scale.

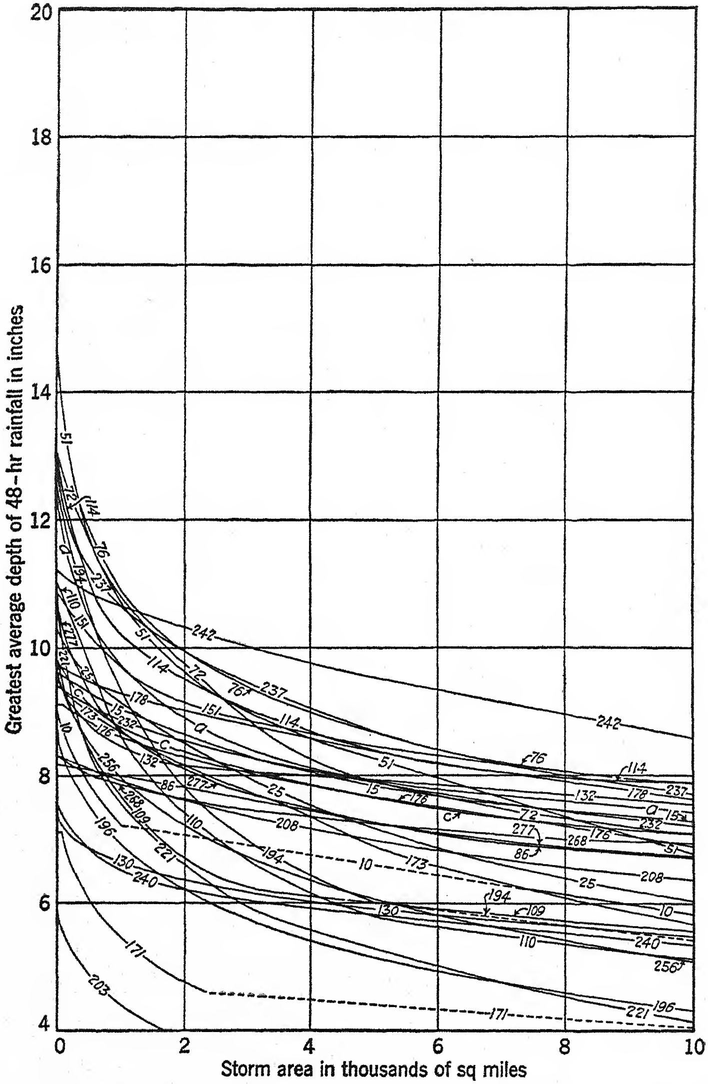


FIG. 35*b*. Time-area-depth curves for storms over northern states, showing greatest average depth of rainfall during 2 days.

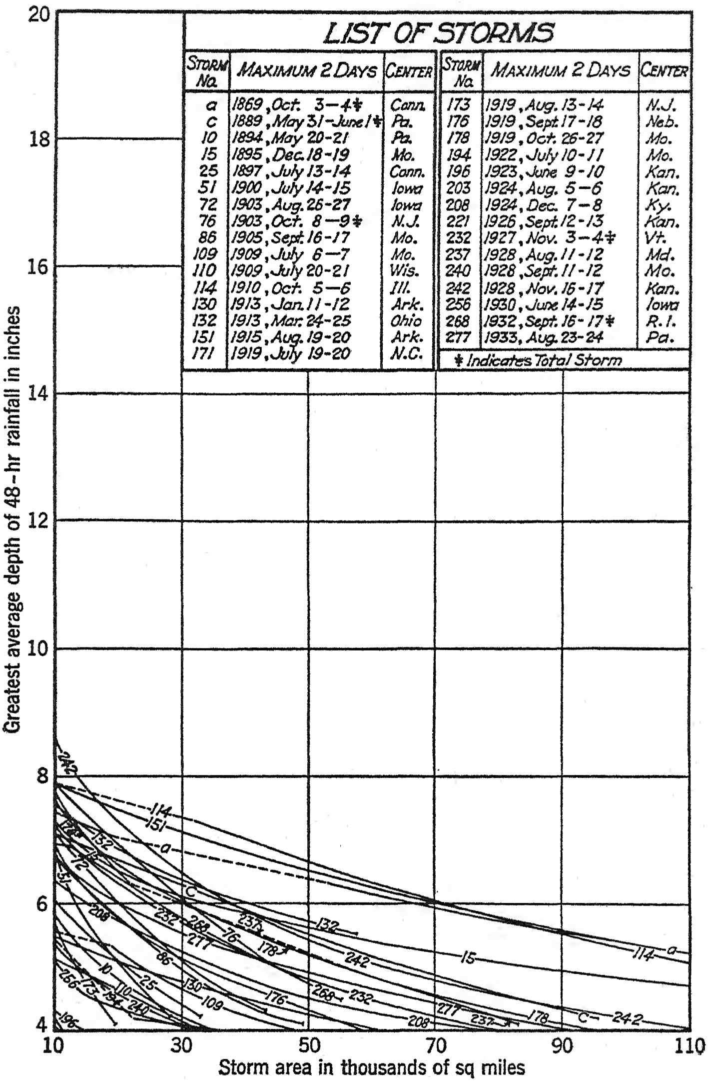


FIG. 35b. Continued. Note change in horizontal scale.

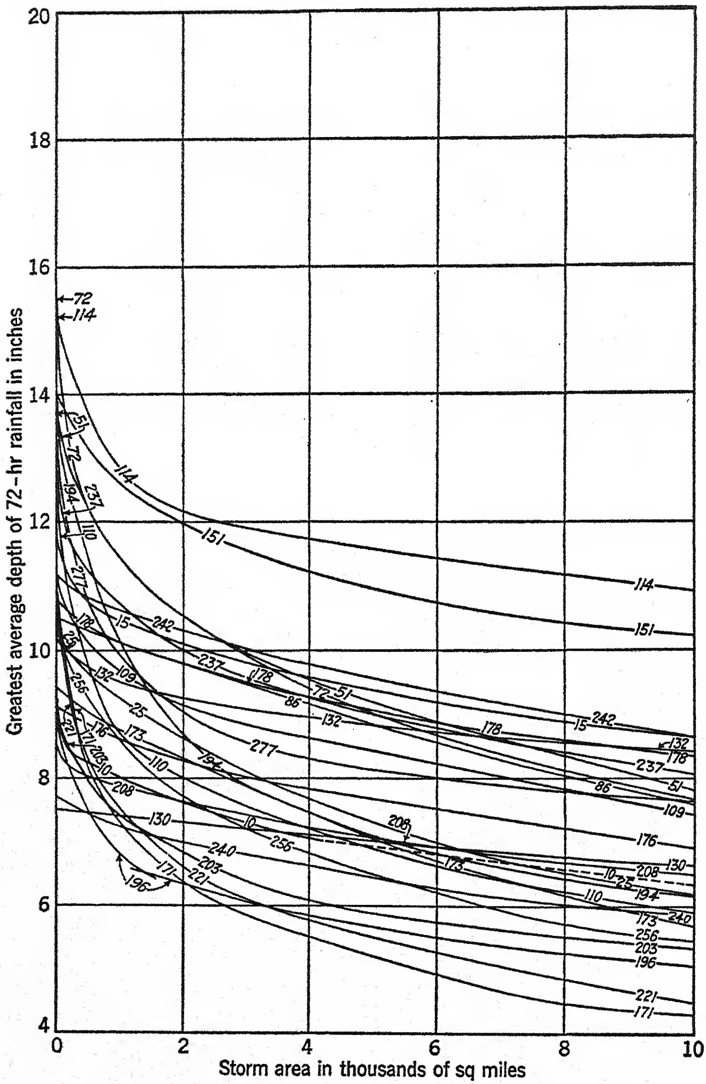


FIG. 35c. Time-area-depth curves for storms over northern states, showing greatest average depth of rainfall during 3 days.

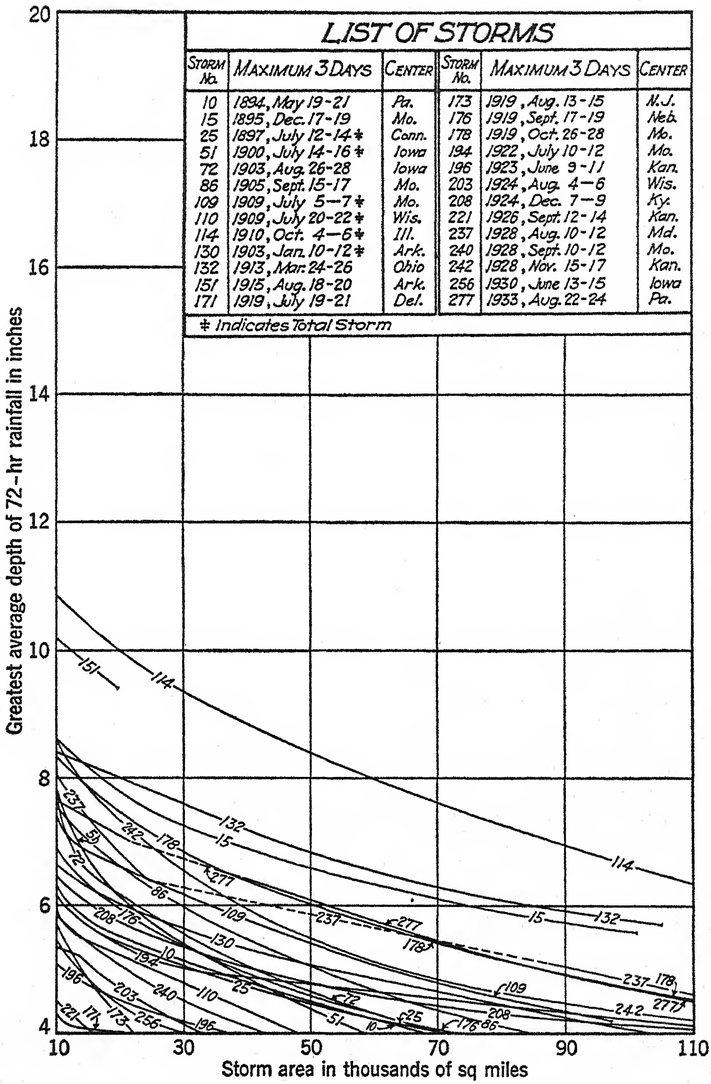


Fig. 35c. Continued. Note change in horizontal scale.

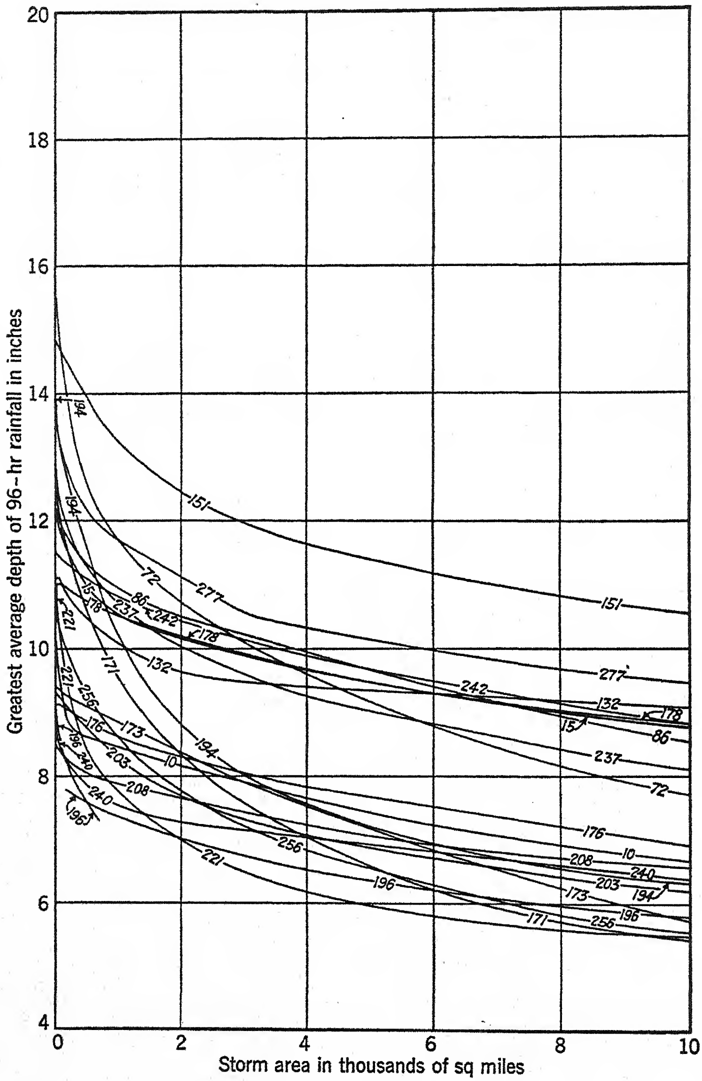


FIG. 35d. Time-area-depth curves for storms over northern states, showing greatest average depth of rainfall during 4 days.

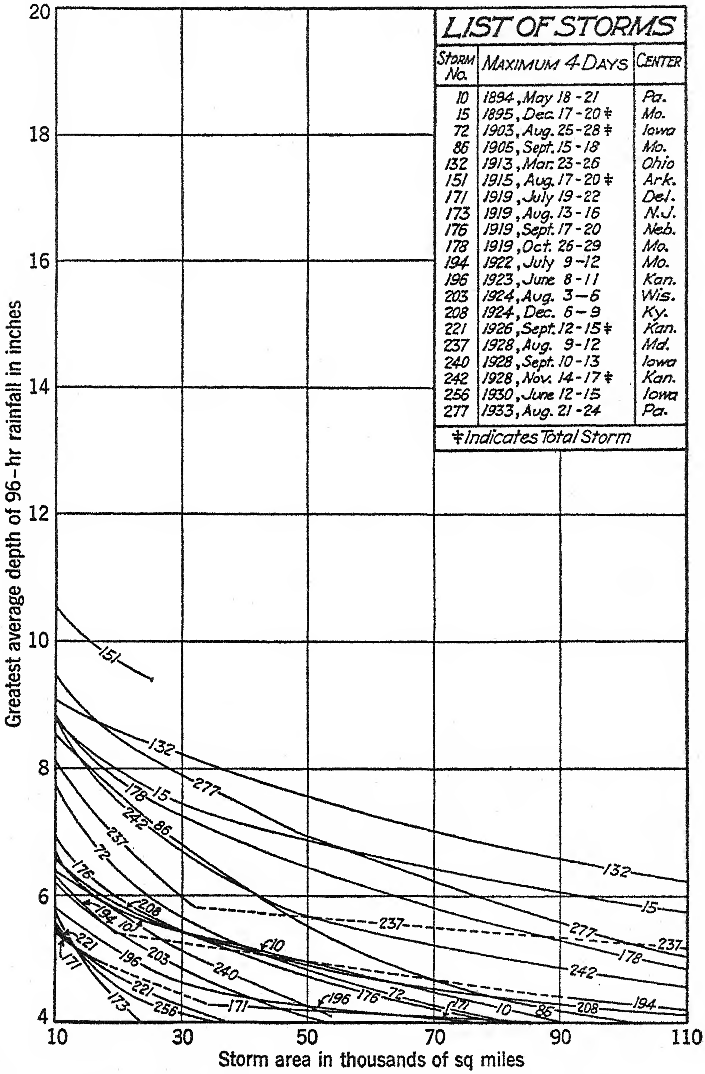


FIG. 35*d*. Continued. Note change in horizontal scale.

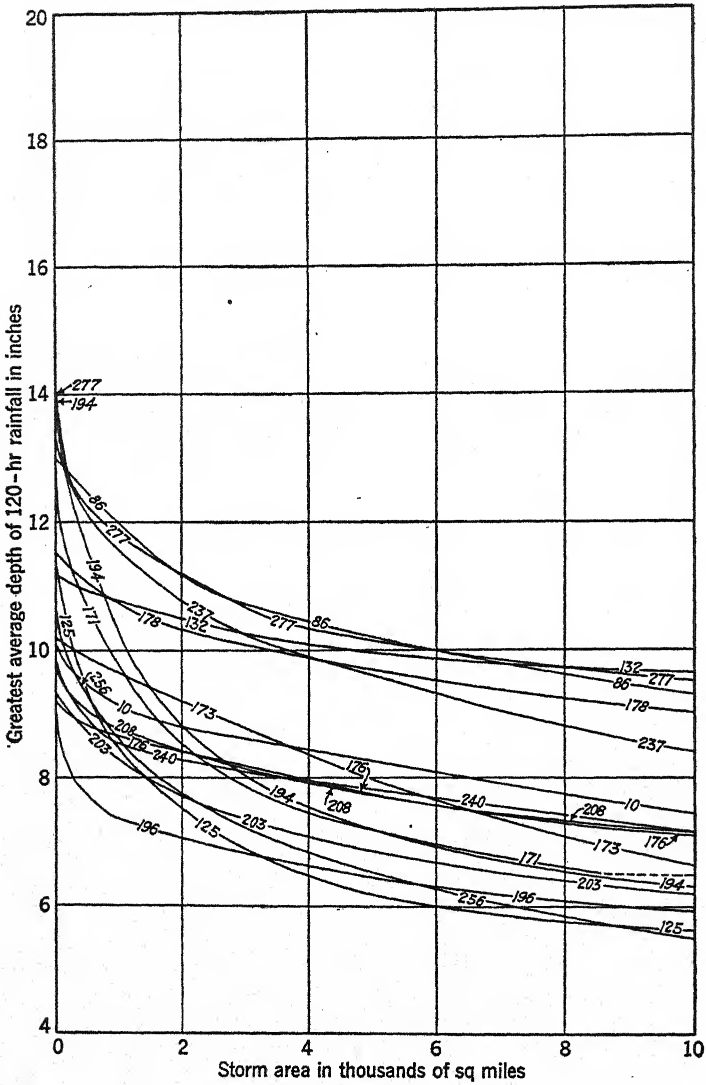


FIG. 35e. Time-area-depth curves for storms over northern states, showing greatest average depth of rainfall during 5 days.

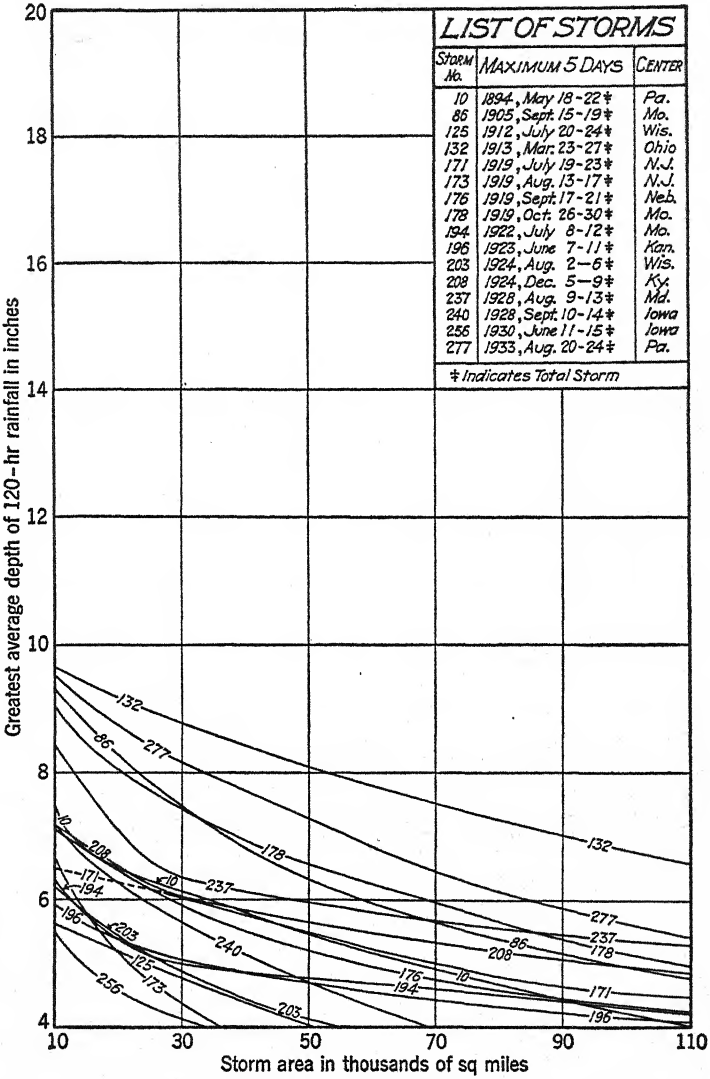


Fig. 35e. Continued. Note change in horizontal scale.

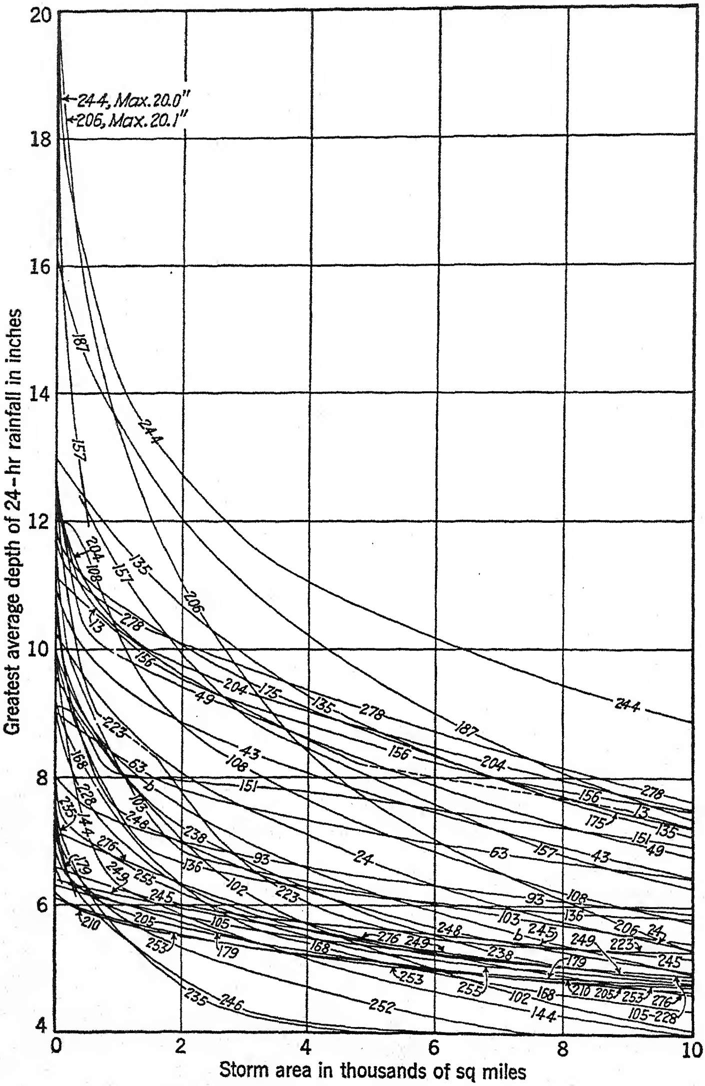
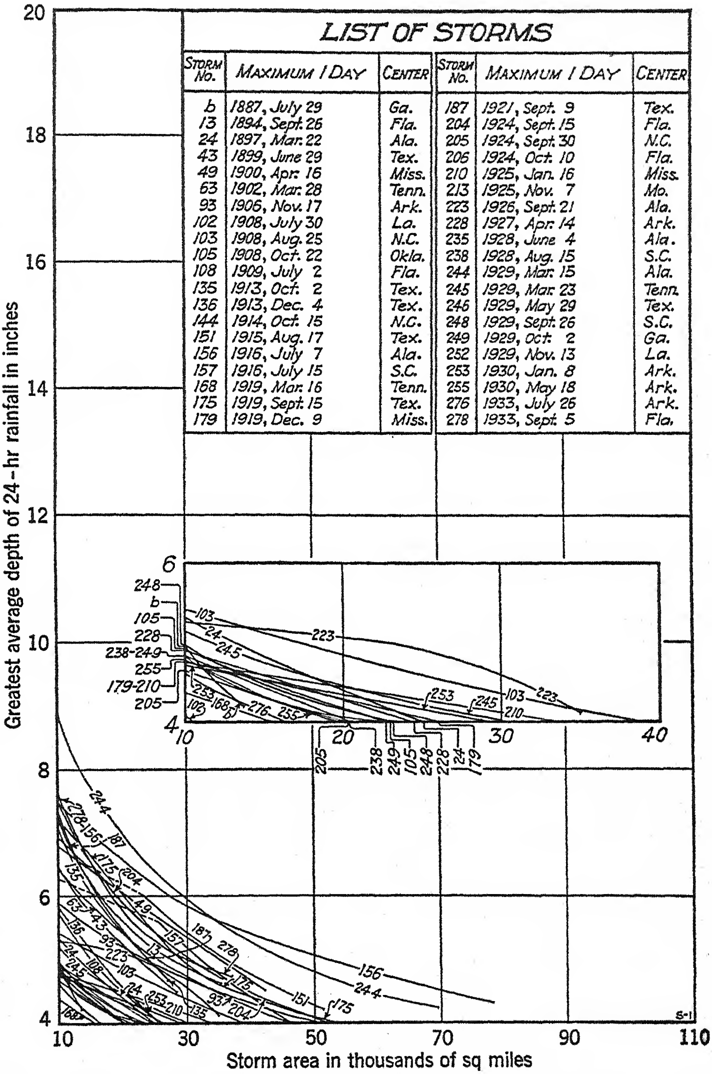


FIG. 36*a*. Time-area-depth curves for storms over southern states, showing greatest average depth of rainfall during 1 day.



LIST OF STORMS

Storm No.	Maximum 1 Day	Center	Storm No.	Maximum 1 Day	Center
b	1887, July 29	Ga.	187	1921, Sept. 9	Tex.
13	1894, Sept. 26	Fla.	204	1924, Sept. 15	Fla.
24	1897, Mar. 22	Ala.	205	1924, Sept. 30	N.C.
43	1899, June 29	Tex.	206	1924, Oct. 10	Fla.
49	1900, Apr. 16	Miss.	210	1925, Jan. 16	Miss.
63	1902, Mar. 28	Tenn.	213	1925, Nov. 7	Mo.
93	1906, Nov. 17	Ark.	223	1926, Sept. 21	Ala.
102	1908, July 30	La.	228	1927, Apr. 14	Ark.
103	1908, Aug. 25	N.C.	235	1928, June 4	Ala.
105	1908, Oct. 22	Okla.	238	1928, Aug. 15	S.C.
108	1909, July 2	Fla.	244	1929, Mar. 15	Ala.
135	1913, Oct. 2	Tex.	245	1929, Mar. 23	Tenn.
136	1913, Dec. 4	Tex.	246	1929, May 29	Tex.
144	1914, Oct. 15	N.C.	248	1929, Sept. 26	S.C.
151	1915, Aug. 17	Tex.	249	1929, Oct. 2	Ga.
156	1916, July 7	Ala.	252	1929, Nov. 13	La.
157	1916, July 15	S.C.	253	1930, Jan. 8	Ark.
168	1919, Mar. 16	Tenn.	255	1930, May 18	Ark.
175	1919, Sept. 15	Tex.	276	1933, July 26	Ark.
179	1919, Dec. 9	Miss.	278	1933, Sept. 5	Fla.

FIG. 36a. Continued. Note change in horizontal scale.

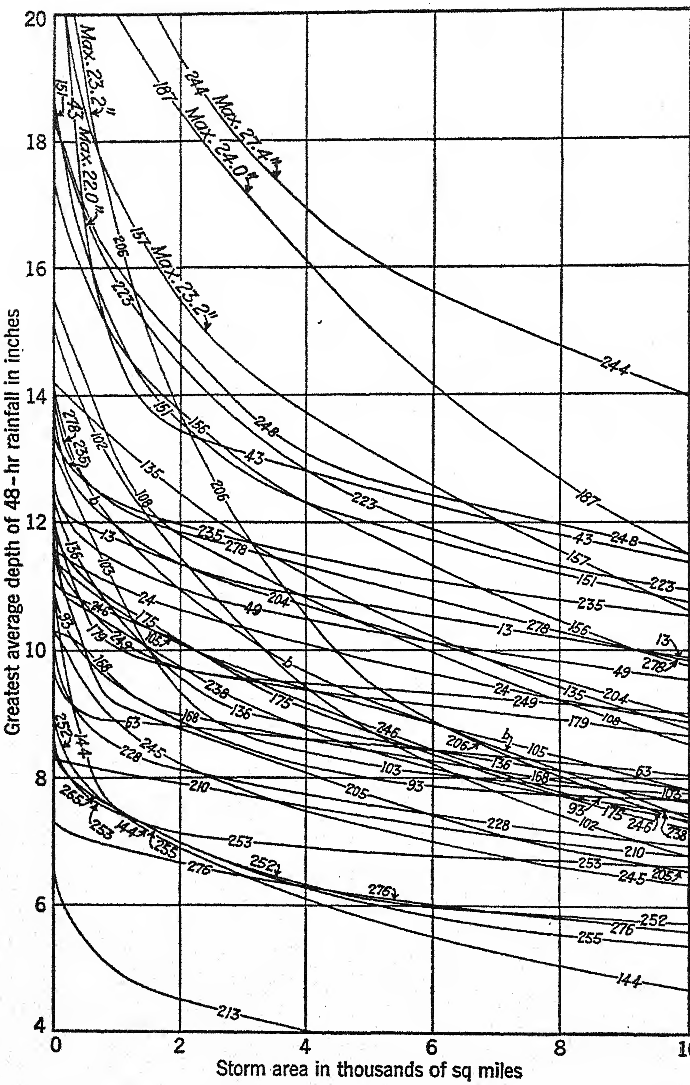
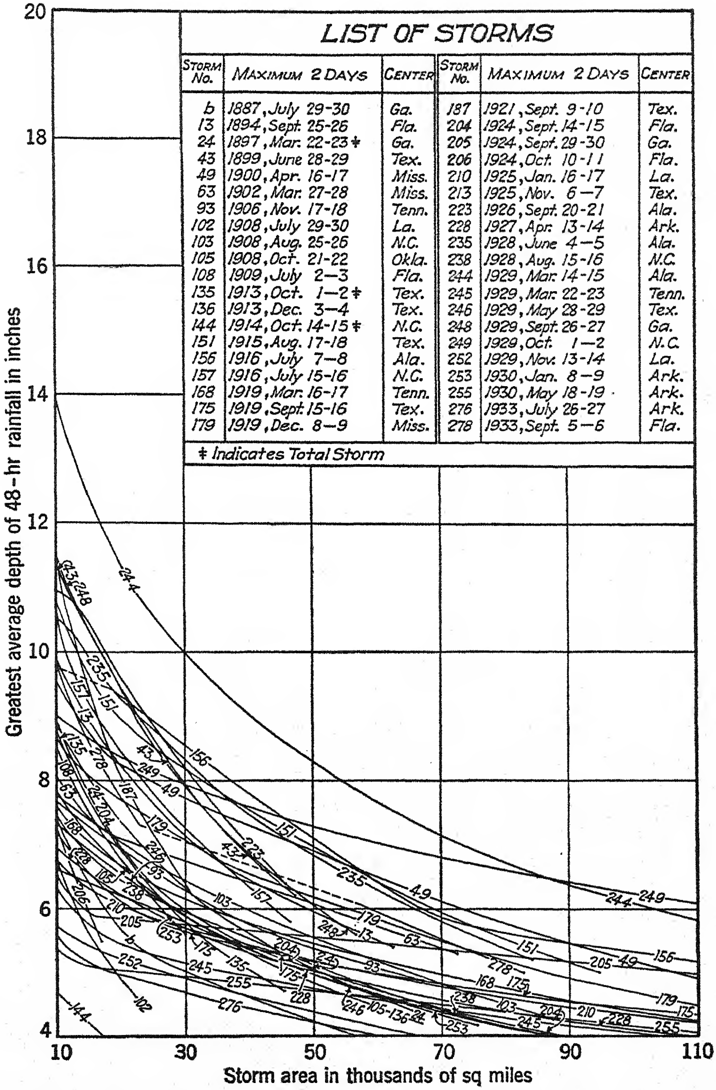


Fig. 36*b*. Time-area-depth curves for storms over southern states, showing greatest average depth of rainfall during 2 days.



LIST OF STORMS

Storm No.	Maximum 2 Days	Center	Storm No.	Maximum 2 Days	Center
b	1887, July 29-30	Ga.	187	1921, Sept. 9-10	Tex.
13	1894, Sept. 25-26	Fla.	204	1924, Sept. 14-15	Fla.
24	1897, Mar. 22-23 ‡	Ga.	205	1924, Sept. 29-30	Ga.
43	1899, June 28-29	Tex.	206	1924, Oct. 10-11	Fla.
49	1900, Apr. 16-17	Miss.	210	1925, Jan. 16-17	La.
63	1902, Mar. 27-28	Miss.	213	1925, Nov. 6-7	Tex.
93	1906, Nov. 17-18	Tenn.	223	1926, Sept. 20-21	Ala.
102	1908, July 29-30	La.	228	1927, Apr. 13-14	Ark.
103	1908, Aug. 25-26	N.C.	235	1928, June 4-5	Ala.
105	1908, Oct. 21-22	Okla.	238	1928, Aug. 15-16	N.C.
108	1909, July 2-3	Fla.	244	1929, Mar. 14-15	Ala.
135	1913, Oct. 1-2 ‡	Tex.	245	1929, Mar. 22-23	Tenn.
136	1913, Dec. 3-4	Tex.	246	1929, May 28-29	Tex.
144	1914, Oct. 14-15 ‡	N.C.	248	1929, Sept. 26-27	Ga.
151	1915, Aug. 17-18	Tex.	249	1929, Oct. 1-2	N.C.
156	1916, July 7-8	Ala.	252	1929, Nov. 13-14	La.
157	1916, July 15-16	N.C.	253	1930, Jan. 8-9	Ark.
168	1919, Mar. 16-17	Tenn.	255	1930, May 18-19	Ark.
175	1919, Sept. 15-16	Tex.	276	1933, July 26-27	Ark.
179	1919, Dec. 8-9	Miss.	278	1933, Sept. 5-6	Fla.

‡ Indicates Total Storm

FIG. 36*b*. Continued. Note change in horizontal scale.

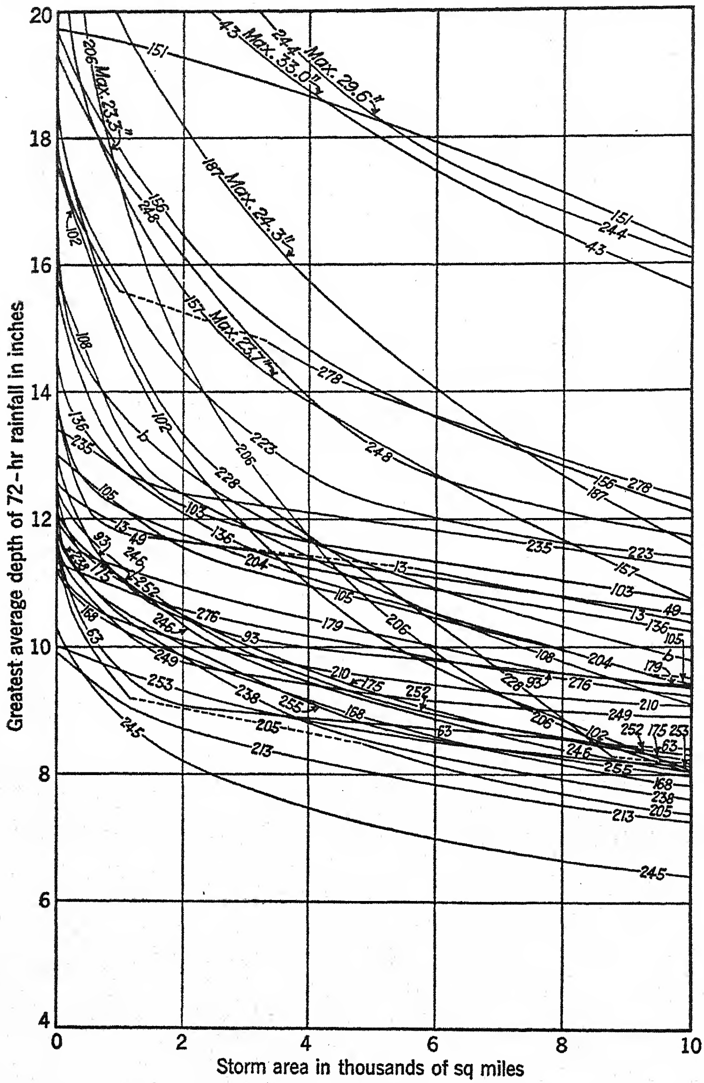
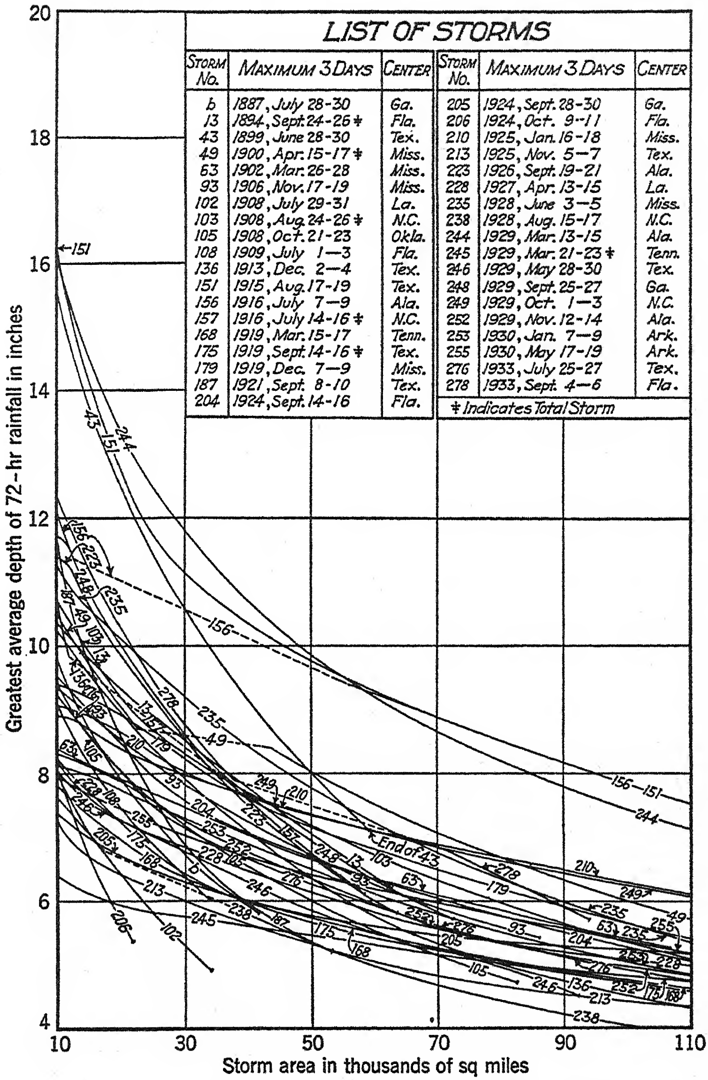


FIG. 36c. Time-area-depth curves for storms over southern states, showing greatest average depth of rainfall during 3 days.



Storm No.	Maximum 3 Days	Center	Storm No.	Maximum 3 Days	Center
b	1887, July 28-30	Ga.	205	1924, Sept. 28-30	Ga.
13	1894, Sept. 24-26 ‡	Fla.	206	1924, Oct. 9-11	Fla.
43	1899, June 28-30	Tex.	210	1925, Jan. 16-18	Miss.
49	1900, Apr. 15-17 ‡	Miss.	213	1925, Nov. 5—7	Tex.
63	1902, Mar. 26-28	Miss.	223	1926, Sept. 19-21	Ala.
93	1906, Nov. 17-19	Miss.	228	1927, Apr. 13-15	La.
102	1908, July 29-31	La.	235	1928, June 3—5	Miss.
103	1908, Aug. 24-26 ‡	N.C.	238	1928, Aug. 15-17	N.C.
105	1908, Oct. 21-23	Okla.	244	1929, Mar. 13-15	Ala.
108	1909, July 1—3	Fla.	245	1929, Mar. 21-23 ‡	Tenn.
136	1913, Dec. 2—4	Tex.	246	1929, May 28-30	Tex.
151	1915, Aug. 17-19	Tex.	248	1929, Sept. 25-27	Ga.
156	1916, July 7—9	Ala.	249	1929, Oct. 1—3	N.C.
157	1916, July 14-16 ‡	N.C.	252	1929, Nov. 12-14	Ala.
168	1919, Mar. 15-17	Tenn.	253	1930, Jan. 7—9	Ark.
175	1919, Sept. 14-16 ‡	Tex.	255	1930, May 17-19	Ark.
179	1919, Dec. 7—9	Miss.	276	1933, July 25-27	Tex.
187	1921, Sept. 8-10	Tex.	278	1933, Sept. 4—6	Fla.
204	1924, Sept. 14-16	Fla.		‡ Indicates Total Storm	

Fig. 36c. Continued. Note change in horizontal scale.

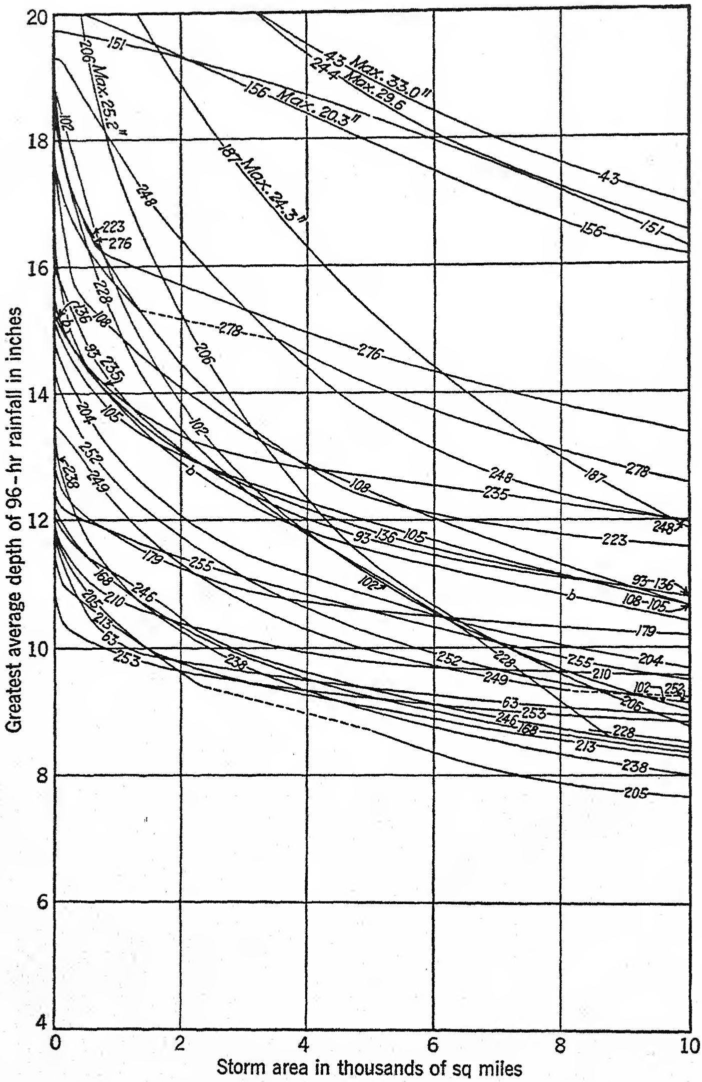
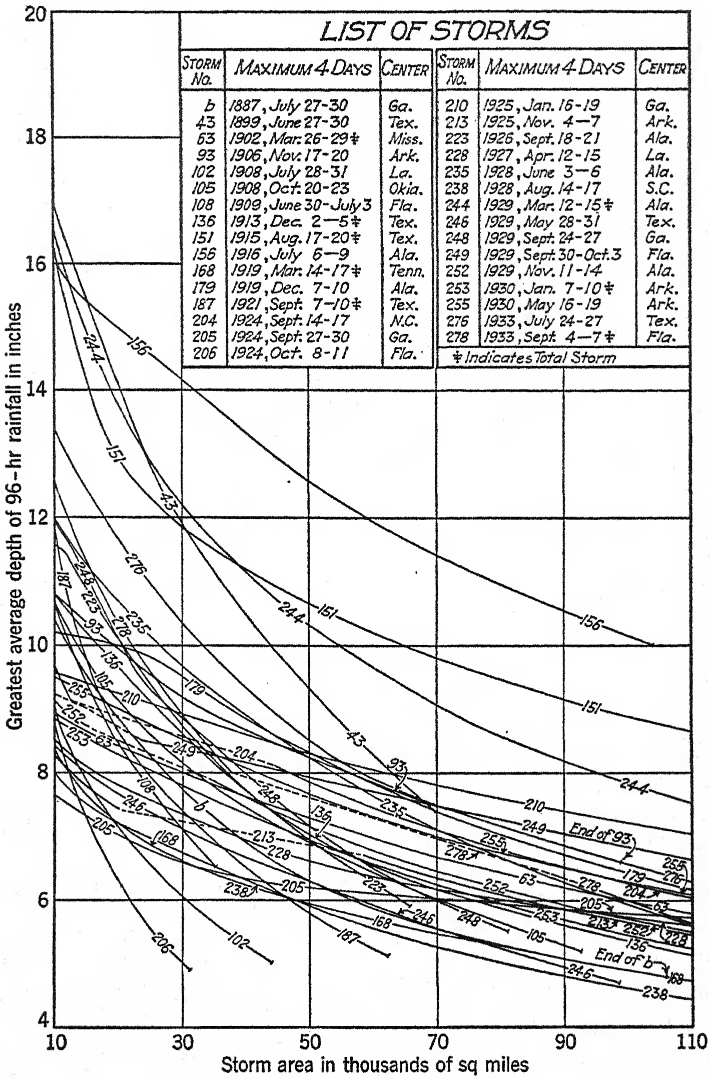


FIG. 36*d*. Time-area-depth curves for storms over southern states, showing greatest average depth of rainfall during 4 days.



Storm No.	Maximum 4 Days	Center	Storm No.	Maximum 4 Days	Center
b	1887, July 27-30	Ga.	210	1925, Jan. 16-19	Ga.
43	1899, June 27-30	Tex.	213	1925, Nov. 4—7	Ark.
63	1902, Mar. 26-29‡	Miss.	223	1926, Sept. 18-21	Ala.
93	1906, Nov. 17-20	Ark.	228	1927, Apr. 12-15	La.
102	1908, July 28-31	La.	235	1928, June 3—6	Ala.
105	1908, Oct. 20-23	Okla.	238	1928, Aug. 14-17	S.C.
108	1909, June 30-July 3	Fla.	244	1929, Mar. 12-15‡	Ala.
136	1913, Dec. 2—5‡	Tex.	246	1929, May 28-31	Tex.
151	1915, Aug. 17-20‡	Tex.	248	1929, Sept. 24-27	Ga.
156	1916, July 6—9	Ala.	249	1929, Sept. 30-Oct. 3	Fla.
168	1919, Mar. 14-17‡	Tenn.	252	1929, Nov. 11-14	Ala.
179	1919, Dec. 7-10	Ala.	253	1930, Jan. 7-10‡	Ark.
187	1921, Sept. 7—10‡	Tex.	255	1930, May 16-19	Ark.
204	1924, Sept. 14-17	N.C.	276	1933, July 24-27	Tex.
205	1924, Sept. 27-30	Ga.	278	1933, Sept. 4—7‡	Fla.
206	1924, Oct. 8-11	Fla.			

‡ Indicates Total Storm

Fig. 36*d*. Continued. Note change in horizontal scale.

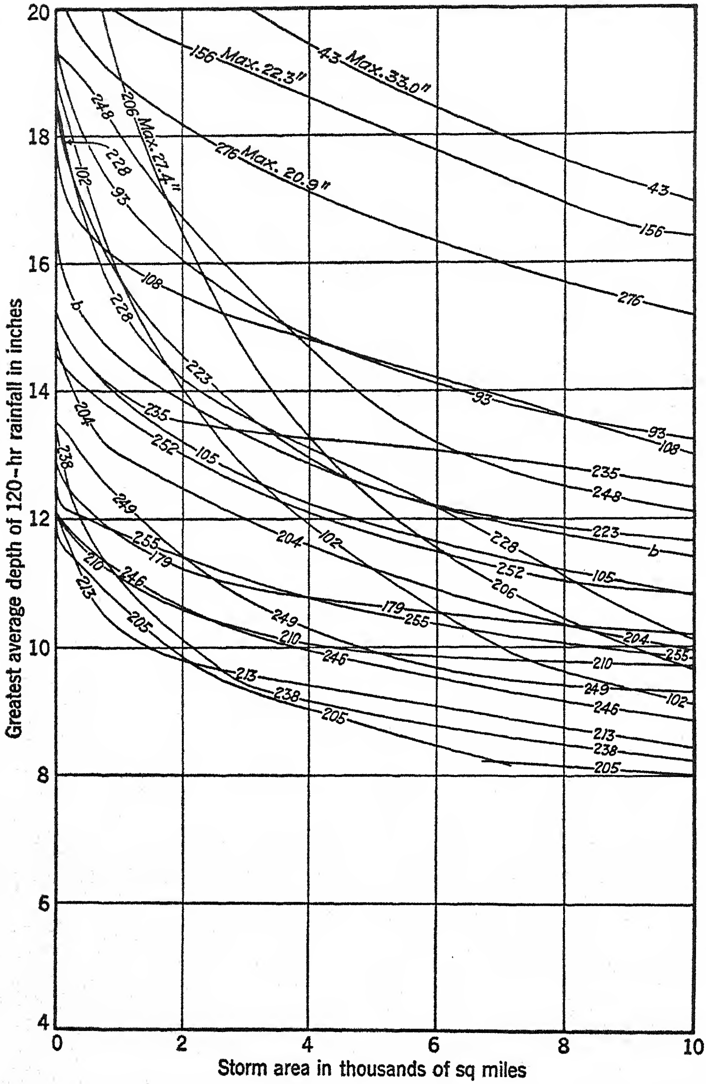


FIG. 36e. Time-area-depth curves for storms over southern states, showing greatest average depth of rainfall during 5 days.

that have occurred in the southern states. These charts are reproductions of Figs. 188 to 197 inclusive, Part V, *Technical Reports*, The Miami Conservancy District, Dayton, Ohio. These curves may be used as a basis for estimating the probable average depth

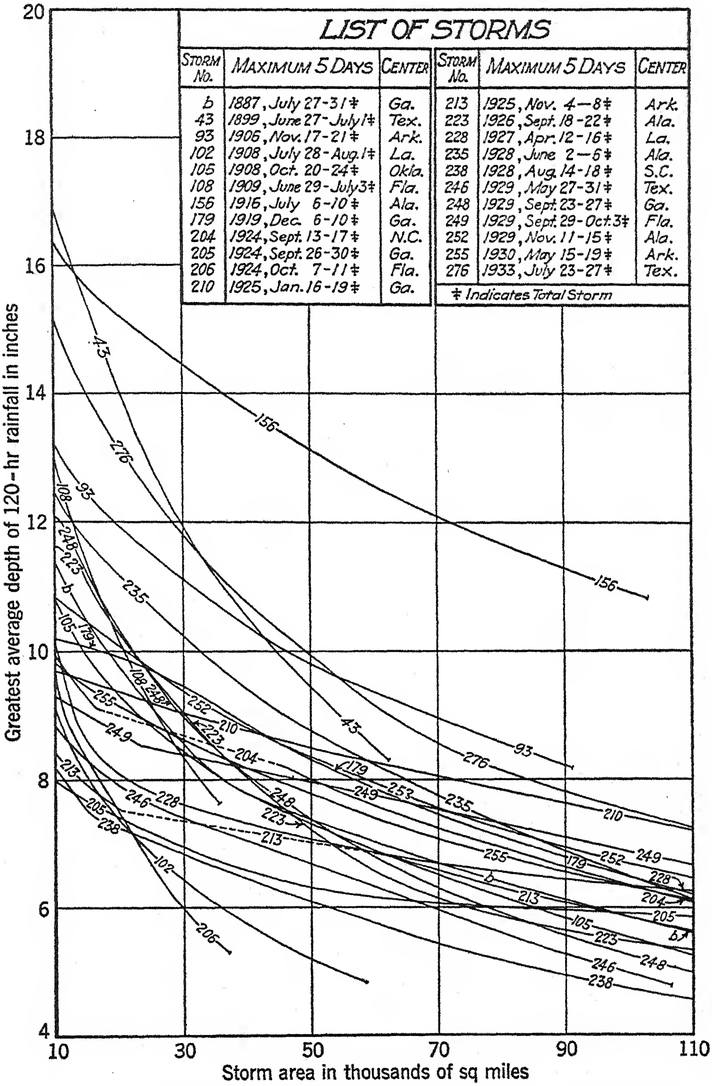


FIG. 36e. Continued. Note change in horizontal scale.

of rainfall on an area of any given size, knowing only the maximum rainfall at the center of the storm.

Suppose, for instance, that it is necessary to determine the maximum depth of rainfall that may be expected to occur in 1

day on an area of 2500 sq miles with a frequency of once in 100 yr. If the combined length of records at all the stations within the area does not provide at least ten periods of observation, which in this case means 1000 station-years of records, additional records should be obtained from adjacent stations that have the same mean annual rainfall and the same average number of rainfall days per year. It is not to be expected that both these characteristics will be exactly the same at each of the adjacent stations as the mean of those within the area, but neither should they show a deviation from that mean more than 10 per cent greater than is shown by any station within the area.

The records at the selected stations are then combined, and by use of the station-year method as above described the maximum depth of 1-day rainfall that may be expected to occur at any station with an average frequency of once in a hundred years is determined. Suppose that that depth is found to be 8 in. If this study concerns a basin located in the northern states an examination should then be made of the curves shown in Fig. 35. From this figure a number of curves are selected which preferably represent storms that occurred somewhere in the vicinity of the area being investigated and whose maximum depths are in the neighborhood of 8 in. For each of these representative storms the ratio is determined between the depth of rainfall on an area of 2500 sq miles and the maximum depth. Suppose that these various ratios are found to be about 0.75, some being as low as 0.60 and others as high as 0.90. If it is desirable to be on the safe side the higher value should be used, otherwise use the average; in other words, the depth decided upon would be 7.2 in., or 6 in., respectively.

In addition to the studies of the Miami Conservancy District referred to above, another rather extensive study of rainfall frequency was made by the U. S. Department of Agriculture.[1] These data are presented as curves such as those shown in Figs. 37 and 38, giving depths of precipitation of various durations to be expected with frequencies varying from 2 yr to 100 yr. In using generalized curves of this type it should be understood that all local conditions for the entire United States may not be fully represented. Whenever possible it is desirable to determine the rainfall frequency by the station-year method.

[1] David L. Yarnell, Rainfall Intensity-Frequency Data, *U. S. Department of Agriculture Misc. Pub.* 204, August, 1935.

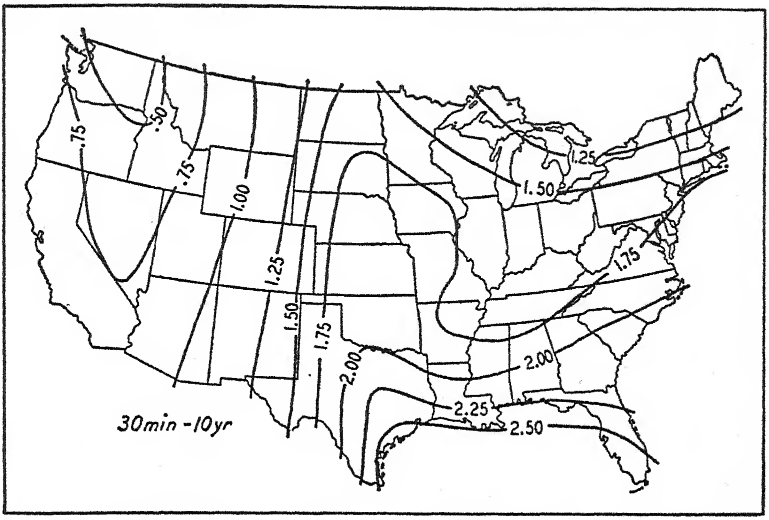


FIG. 37. Thirty-minute rainfall, in inches, to be expected once in 10 yr. Yarnell.

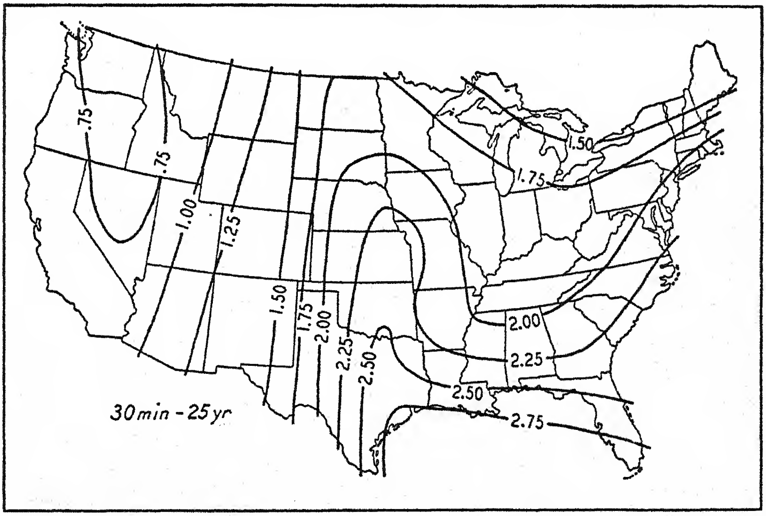


FIG. 38. Thirty-minute rainfall, in inches, to be expected once in 25 yr. Yarnell.

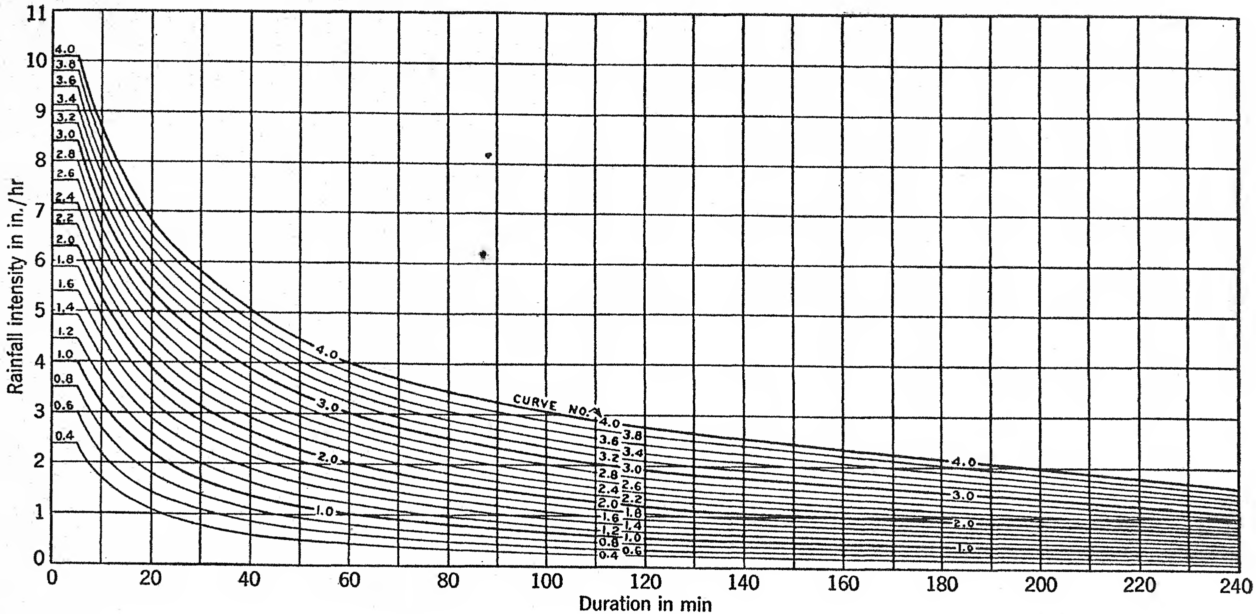


FIG. 39. Airfield drainage. Standard rainfall intensity-duration curves or standard supply curves. Notes: Curve numbers correspond to 1-hr values of rainfall or supply indicated by respective curves; all points on the same curve are assumed to have the same average frequency of occurrence. From *Engineering Manual* by Corps of Engineers, U. S. Army.

The U. S. Army Engineers have developed a set of curves,[1] Fig. 39, showing the relation between rainfall intensity and duration for any given storm frequency. As an illustration of the use of these curves, suppose that at some station a 1-hr rainfall of 2.0 in. or more occurs with a frequency of once in 10 yr. Then curve 2.0 shows that a 30-min rain of 3.1 in. per hr, or a 3-hr rain of 0.9 in. per hr, will also occur with a frequency of once in 10 yr. If, however, at this or at any other station a 1-hr rainfall of 2.0 in. occurs with a frequency of once in 2 yr, this same curve may be used for determining the intensity of a storm of any other duration that occurs with that same frequency, namely, once in 2 yr. It is possible but not necessarily true that all points on any of these curves may apply to a single storm.

Numerous other more or less extensive studies of rainfall intensity and frequency have been made.[2]

Storm Intensity Pattern

The manner in which the rate of rainfall varies throughout the storm period is called the *storm intensity pattern*. The solutions of at least two different types of problems, both of which are important in the field of hydrology, are vitally dependent upon this characteristic.

Horner and Jens[3] have shown, Fig. 40, how two storms of equal magnitude but of different intensity patterns could occur on the same drainage basin at times when the infiltration capacity at the beginning of the storm was the same in both cases and yet produce entirely different flood hydrographs. In this figure the shaded areas represent rainfall excess. Because of the difference in the volumes of rainfall excess, storm B will produce a much greater peak flow than storm A. The storm intensity pattern must therefore be taken into consideration along with the total depth of rainfall in computing the resulting runoff.

Rainfall records as provided by the daily summaries that appear

[1] *Engineer Manual,* Fig. 2, Chap. 1, Part XIII, Corps of Engineers, U. S. Army.

[2] Among them are Bailey and Schneider, The Maximum Probable Flood and Its Relation to Spillway Capacity, *Civil Eng.*, January 1939; Horner, Breihan, and Armistead, Studies of Rainfall Intensity, *Civil Eng.*, May 1940; H. F. Kennison, Sixty-year Rainfall Record Analyzed, *Civil Eng.*, November 1940.

[3] W. W. Horner and S. W. Jens, Surface Runoff Determination from Rainfall without Using Coefficients, *Trans. A.S.C.E.*, 1942, p. 1039.

in the U. S. Weather Bureau publications are frequently used as a basis for determining the frequency of storms of a given intensity at any station. The results of these frequency studies are used in the determination of the frequency of floods of different magnitudes. In such studies, daily rainfall summaries do not reveal the intensity pattern in sufficient detail to permit their direct use without adjustment. This is well illustrated in the following example.

The published records of the daily rainfall at station 15–26 in the Muskingum basin show that 2.73 in. fell on June 27, 1939, and 1.96 in. on June 28. These records are for the 24-hr periods beginning at 8 AM on each of these days. If this rain fell at a uniform

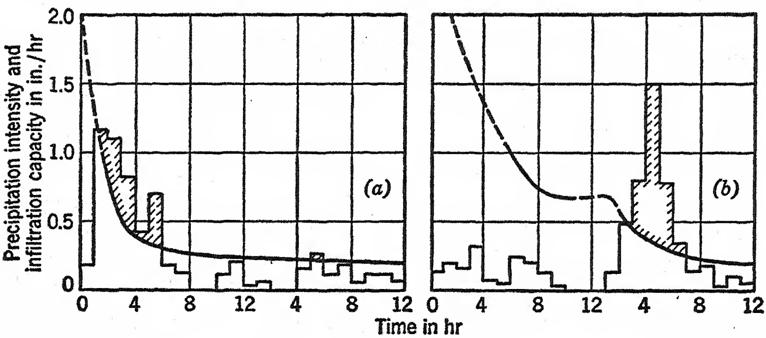


FIG. 40.

rate its intensity might have been only slightly in excess of 0.1 in. per hr. Many soils are capable of absorbing such a rain and hence would yield no surface runoff. Consider, however, the intensity pattern, Fig. 41, which shows the depth of rainfall by 30-min periods. In the figure it is seen that during this storm there were two major periods of intense rainfall. During the first, 2.66 in. of rain fell in 4.5 hr, and during the second 1.86 in. fell in 1.5 hr. The average intensities during these periods were 0.59 in. per hr and 1.24 in. per hr respectively. Few soils are capable of absorbing such high rates of rainfall, and, therefore, because of this extremely variable rainfall pattern, surface runoff would occur.

Furthermore, instead of the picture presented by the daily records which show that 4.69 in. fell in 2 days, actually no rain fell on June 27, and 4.65 in. fell in a period of 17.5 hr on June 28. Nor is this an isolated example. A study of continuous gage records as

shown either by a graph or by short-time-interval totals when compared with daily totals reveals the fact that usually the actual peak intensities are far greater than indicated by the published daily rainfall totals.

A study by Sherman[1] of the continuous rainfall records obtained at Chestnut Hill reservoir (Boston) extending over a period of 25 yr led him to the conclusion that the actual average duration of

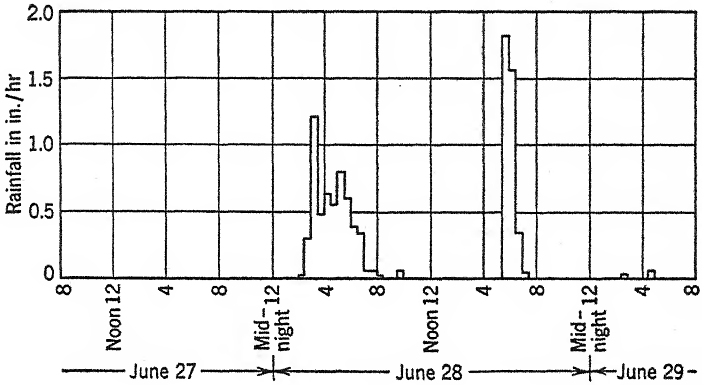


FIG. 41.

storms recorded as occurring in different numbers of days is as follows.

Recorded Duration	Actual Duration
1-day storms	13 to 14 hr
2-day storms	21 to 31 hr
3-day storms	43 to 47 hr
4-day storms	71 to 74 hr
5-day storms	83 to 84 hr

He also found that the above relationships hold true regardless of the intensities of the storms.

Because the vast majority of the rainfall records that are available in any given locality were obtained by cooperative observers and give only the total rainfall for each 24-hr period prior to the time of observation, for the reasons above stated such records must be corrected before they are suitable for frequency-intensity studies. For this purpose two general methods may be employed,

[1] C. W. Sherman, Actual Duration of One-Day and Two-Day Rain Storms, *Civil Eng.*, March 1939.

viz., the recorded duration in days may be reduced to an estimated actual duration as indicated by Sherman's studies or the recorded 1-day, 2-day, etc., storm depths may be corrected in some manner so that they will more nearly represent the actual maximum depths of rainfall occurring in any of these time periods.

If the first of these correction methods is employed, other studies similar to those made by Sherman in Boston should be carried on at various points throughout the country in order to determine whether the Boston relationships hold true generally. If it is found that they do not, such studies should determine the extent of the variations in different sections and the causes producing them.

The exact procedure to be followed in the correction of the recorded storm depth depends upon whether or not a recording gage was located within the storm area, preferably near the center. Two methods will be described. Their use will be illustrated in connection with a 2-day rainfall record.

1. If at least one continuous record is available, from this may be determined the maximum amount of rain that fell during any 24-hr period and also the percentage that this amount is of the total 2-day rainfall. This same percentage is then applied to the total 2-day rainfall as measured at each of the nonrecording stations located within the storm area. If, for instance, at the recording station 2.54 in. fell on the first day and 1.71 in. fell on the day following, and the graph shows that 3.74 in., or 88 per cent of the total, fell within a 24-hr period, and the record at a nonrecording station shows 2.45 in. on the first day and 3.70 in. on the second, then $0.88(2.45 + 3.70) = 5.41$ in., which is taken as the maximum 24-hr rainfall at this station. Although this value may not be correct the chances are that it is much more nearly correct than the 3.70 in. as indicated by the records.

2. The second method, which is used only when no continuous record of the storm is available at any station, is based upon the law of probability. If the observer reads the gage at 6 PM, a 24-hr rainfall is just as likely to start at 6 AM (or at any other hour) as at 6 PM. If it begins at 6 AM and is of uniform intensity, then this 24-hr rain having a depth of, say, 6 in. will be recorded as a 2-day rain with 3 in. falling each day. With no continuous record available, however, it would be impossible to determine from the records whether this rain fell in 48 hr or in 24 hr or even in a much shorter period. As far as may be learned from the published records it is

possible that the maximum rainfall in any 24-hr period may have been 3 in. It is equally possible, however, that it may have been 6 in. These two figures therefore represent the minimum and the maximum depths of rain that could possibly have fallen at this station in any 24-hr period during this storm. The most probable value should be the mean of these two or 4.5 in.

Therefore, in using daily rainfall records, if these records indicate that rain fell on two or more successive days, in order to determine the probable maximum depth of rain that fell in any 24-hr period during that storm, to the maximum recorded total should be added one half of the larger total that fell on either the preceding or succeeding day, and so on for longer storms. For

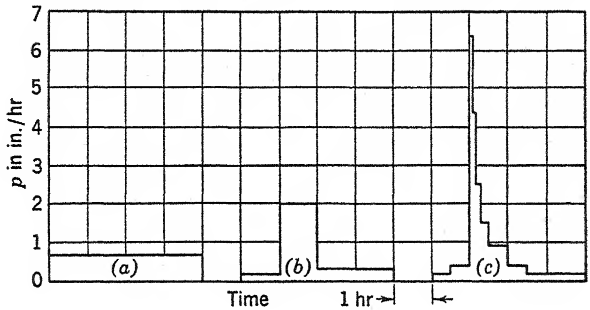


FIG. 42.

instance, suppose that the daily totals for three successive days are 1.20, 2.75, and 0.86 in. The maximum 24-hr total would be

$$2.75 + \tfrac{1}{2} \times 1.20 = 3.35 \text{ in.}$$

and the maximum 48-hr total would be

$$2.75 + 1.20 + \tfrac{1}{2} \times 0.86 = 4.38 \text{ in.}$$

Attention should be called to the fact, however, that often additional information as to the time of beginning and end of rainfall may be obtained from the observer's notes, and when this is done the above approximation is unnecessary.

From the preceding paragraph it appears that the hydrograph of runoff that may result from a rainfall of any given depth on a basin cannot be foretold without a knowledge of the intensity pattern of the storm. Inasmuch as that pattern may vary considerably, it is necessary to assume several different patterns and

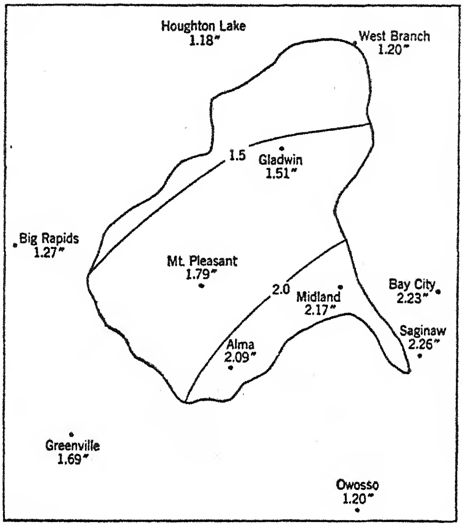
determine the flood peak that would result from each. As an illustration, suppose we assume that for a given purpose and with a specified frequency a rainfall of 2.8 in. occurs in 4 hr and we wish to determine the resulting runoff. Each of the three intensity patterns shown in Fig. 42 would produce a total rainfall of 2.8 in. All these are consistent with Curve 2 of Fig. 39. Many similar patterns could be derived. A numerical example explaining the development of such rainfall patterns is given on page 303.[1]

Storm Distribution Pattern

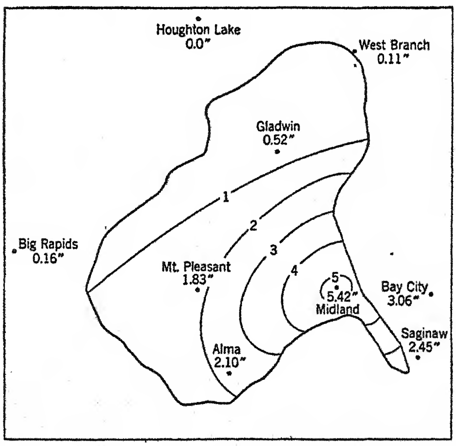
The storm distribution pattern is represented by an isohyetal map of the storm area that shows the depth and the variation in depth of rain from station to station throughout the area. This is sometimes called the storm smear. It affects the resulting hydrograph in a very pronounced manner. Consider for instance the two storms shown in Fig. 43*a* and *b*. The total average depth of rain on the basin during these two storms was practically the same. The storm intensity patterns figured as inches depth per hour on the entire basin might be very similar. Yet the hydrographs of surface runoff resulting from these two storms would be radically different. In fact, little or no surface runoff might result from the storm shown in Fig. 43*a* whereas a severe flood may have resulted from the storm shown in Fig. 43*b*. The reason for this is the fact that in the first storm the rain is fairly uniformly distributed over the basin and perhaps nowhere did the intensity at any time exceed the infiltration capacity. In the second storm, the infiltration capacity was in all probability greatly exceeded, resulting in heavy surface runoff.

During most storms the shape of the area upon which rain is falling at any instant is perhaps more nearly that of a circle than of any other geometrical figure. However, because of the translation of the storm along its path during the period of rainfall the shape of the area covered usually becomes more nearly that of an ellipse with its major axis coincident with the path of the storm. Although most storm-distribution patterns approach the ellipse in shape, many of them especially the larger ones are likely to be exceedingly irregular.

[1] For additional discussion of this subject see Stifel W. Jens, Drainage of Airport Surfaces — Some Basic Design Considerations, *Proc. A.S.C.E.*, September 1947 and June 1948.


Houghton Lake
1.18"
West Branch
1.20"
1.5
Gladwin
1.51"
Big Rapids
1.27"
Mt. Pleasant
1.79"
2.0
Midland
2.17"
Bay City
2.23"
Saginaw
2.26"
Alma
2.09"
Greenville
1.69"
Owosso
1.20"

(a)


Houghton Lake
0.0"
West Branch
0.11"
Gladwin
0.52"
1
2
3
4
5
Big Rapids
0.16"
Mt. Pleasant
1.83"
5.42"
Midland
Bay City
3.06"
Saginaw
2.45"
Alma
2.10"

(b)

Fig. 43.

The Melting of Snow

The melting of snow deposits water on the ground surface and results in runoff in much the same manner as rainfall. However, the many factors involved in causing high rates of snow melt are difficult to coordinate for the purpose of predicting quantitative values. The following discussion of this subject is by no means complete. For a more detailed study of snow and snow-melting processes the reader may refer to the literature that will be cited, which will in turn supply an additional bibliography.

Freshly fallen snow has an average specific gravity of about 10 per cent, or, in other words, a 10-in. column of new snow contains about 1 in. of water. However, the longer the snow remains on the ground, the denser it becomes. This is the result of packing, alternate thawing and freezing, and the presence of absorbed rainfall and melt water. Specific gravities of 20 or 30 per cent are common, and values as high as 50 or 60 per cent are said to occur occasionally.[1] The snow mantle acts as a sponge and is capable of absorbing and holding rather large amounts of rainfall and water derived from melting snow. The percentage by weight of a given amount of snow which is unmelted is called the "quality" or "thermal quality" of the snow. This is found by measuring the heat required to melt a quantity of snow and dividing by the heat required to melt an equal weight of ice. The quality of snow is usually no lower than 95 per cent. However, the quality of coarse grainy snow may be as low as 70 or 80 per cent, whereas new snow has been observed to have qualities of less than 50 per cent.[2] Snow containing free water, i.e., having a quality of less than 100 per cent, is said to have "ripened." It is ripened snow that is likely to contribute most effectively to rapid melting because the application of a given amount of heat not only melts a definite quantity of snow but at the same time releases the free water.

The heat that causes snow to melt may be derived from the atmosphere, from warm rainfall, from radiation, and from the soil. The amount of heat that can be transmitted to the snow by con-

[1] *Engineering Manual for Civil Works*, Part III, Chap. 5, Hydrologic and Hydraulic Analyses, War Department, Office of the Chief of Engineers, Washington, D. C.

[2] Merrill Bernard and Walter T. Wilson, A New Technique for Determining the Heat Necessary to Melt Snow, *Trans. Am. Geophys. Union*, 1941, Part I, p. 178.

duction from motionless air has been shown to be negligible.[1] However, the heat carried to the snow surface by means of the turbulent mixing process which occurs when the air is in motion is one of the principal causes of snow melting. Wilson[1] has suggested that the depth of water melted from snow in this manner may be expressed as follows:

$$D = KV(T - 32°) \tag{3}$$

where D is the depth of water in inches derived from melting in 6 hr, V is the wind velocity in miles per hour, T is the dry-bulb temperature in degrees F, and K is a constant involving the latent heat of ice, instrument exposure, air density, conversion of units, and the turbulence of the air. Tests conducted by the Weather Bureau in Yellowstone Park, Wyoming, indicated a value of K of approximately 0.001.

Heat is also derived from the atmosphere as the result of the condensation of moisture on the snow surface. Here, again, the turbulence in the atmosphere is important as a means of transporting moist air to the snow surface. Wilson[1] has suggested the following formula for computing the depth of water derived in this manner

$$D = K_1V(e - 6.11) \tag{4}$$

where D and V have the same meaning as in equation 3, K_1 is a constant similar to K in equation 3, and e is the vapor pressure in millibars. Concerning e, Wilson states: "If e is measured at the same height above the ground as T, K_1 is approximately 3.2 times K, for basins of elevation from 0 to 3200 feet, and for units of measurement used."

The quantity of heat carried to the snow by a given amount of warm rain water may be computed if the temperature of the rain is known and if it is assumed that the rain is cooled to 32° F. The heat lost by each pound of rain water would be $(T_r - 32)$ Btu, where T_r is the temperature of the rain in degrees F. Because it requires 144 Btu to melt 1 lb of ice, each pound of rain water will melt $(T_r - 32)/144$ lb of snow. It follows that any depth of rain, P, will provide the depth of water from snow melt, D, shown by equation 5, where D and P must be in the same units.

$$D = \frac{P(T_r - 32)}{144} \tag{5}$$

[1] W. T. Wilson, An Outline of the Thermodynamics of Snow-Melt, *Trans. Am. Geophys. Union*, 1941, Part I, p. 182.

It has been suggested[1] that the temperature of the rain will be about the same as the wet-bulb temperature.

Radiation is an important source of heat. However, it is difficult to determine the amount of heat energy retained by the snow because information regarding the intensity of solar radiation and the amount of radiation reflected by the snow is usually lacking. Fortunately, during flood periods there is usually a cloud cover that is likely to reduce the effect of radiation to a negligible amount.

The rate of conduction of heat to the snow from the soil depends on the texture of the soil, the moisture content of the soil, and the amount of insulation provided by forest litter, grasses, and other vegetation. The temperature gradient at the surface of the ground will be quite steep for a short time following the deposition of the initial snow on warm ground. However, the temperature gradient flattens rapidly, so that after several days the rate of heat flow is small compared with the other sources of heat.

It may be seen that the accurate predictions of snow melt during a selected period is a complex process. It would be necessary first to derive values of K and K_1 that apply to a particular region. It would be necessary to search the records for a period when the combination of wind velocity, air temperature, vapor pressure, and possibly wet-bulb temperature together with rainfall were coordinated in such a manner as to produce maximum rates of melting. Owing to the difficulties involved, estimates of snow melt are sometimes based on temperatures alone.

It is first necessary to know or estimate the maximum water equivalent of the snow that is likely to exist at any time. This information can usually be obtained from data of the U. S. Weather Bureau, the U. S. Geological Survey, interested state agencies, power companies, etc.[2] The next step is to estimate the rate at which the snow will melt. A number of studies have been reported regarding the relation of melting of snow to temperature.[3] These

[1] W. T. Wilson, An Outline of the Thermodynamics of Snow-Melt, *Trans. Am. Geophys. Union*, 1941, Part I, p. 182.

[2] See for example The Floods of 1936, *U. S. Geological Survey Water-Supply Paper* 800, Part 3, pp. 59 and 288.

[3] R. E. Horton, The Melting of Snow, *Monthly Weather Rev.*, 1915, **43**, 604.

George D. Clyde, Change in Density of Snow Cover with Melting and The Effect of Rain on Snow Cover, *Monthly Weather Rev.*, 1929, **57**, 326.

Report of The Committee on Floods, March, 1930, *J. Boston Soc. Civil Engrs.*, No. 7, **17**, 254.

studies indicate that there is a definite increase in snow melt with degree-days of temperature above 32° F. The rate of melting per degree-day has been given as low as 0.0126 in. and as high as 0.09 in. An average value of about 0.05 in. per degree-day may be used in the absence of more specific information. Whenever possible this value should be based on records from the same basin or a nearby basin. It is then only necessary to estimate the extent of a warm spell during the snow season in order to compute the rate of snow melt. Information concerning temperatures may be obtained from U. S. Weather Bureau records. If, for example, it is found that the average temperature during any day is 40° F, the amount of snow melt during that day will be approximately $(40 - 32) \times 0.05 = 0.40$ in.

In some portions of the United States, particularly in the West, a very large percentage of the stream flow may result from the melting of high-altitude snow during the spring and early summer. On such drainage basins, more detailed and systematic measurements of snow depth and water content are made. These *snow surveys* are usually carried out by cooperating federal, state, and local agencies. The data that are collected permit estimates to be made of the stream flow that will be available for irrigation and other uses during the following summer. They also provide a basis for the operation of reservoirs in a manner that will allow for the maximum storage of usable water without causing a flood hazard.

CHAPTER V

WATER LOSSES

Most of the problems that the hydrologist is called upon to answer may be divided into two general classes, viz., (1) a determination of the maximum flood that may be expected with a certain frequency and (2) a determination of the monthly, seasonal, annual, or long-term average yield, that may be anticipated from any given basin. Classification 2 includes problems pertaining to the low water flow and its probable duration and frequency of occurrence.

For the solution of these two types of problems entirely different techniques are employed. For the first, a detailed study of the characteristics of the hydrograph of surface runoff from the basin is required, together with a knowledge of infiltration capacity and its variations. Evaporation, transpiration, and other losses are ordinarily ignored because of the short period during which they are effective. For the solution of problems of the second type, the relative importance of these two kinds of data is reversed. Several different procedures have been suggested for determining long-term yield. The best use of any of them requires a knowledge of water losses and the many factors that affect and determine those losses. One of the oldest methods of determining the yield of a basin for which no runoff records were available was to estimate the losses and then deduct them from the total precipitation. Although the authors believe that in most cases a better procedure is available for determining monthly, seasonal, or annual yield, they recognize the value of a knowledge of water losses in the application of all these methods and therefore present the following discussion of this subject.

Water losses are here defined as the difference between the total precipitation and the total runoff from any given area. This may at once be recognized as being an engineer's definition of water losses for the reason that the engineer is usually interested in the utilization of water for power, navigation, water supply, and so forth, and, therefore, he considers that portion of the rainfall that

is not available for those purposes as lost. On the other hand, the agriculturist who is interested in seeing that water is available for transpiration by his crops would probably define water losses as being the difference between precipitation and transpiration.

Although nearly all water losses are, in the final analysis, evaporative, they may be subdivided as follows:

1. Interception.
2. Evaporation from free water surfaces.
3. Plant transpiration.
4. Soil or land evaporation.
5. Watershed leakage.

These losses occur in successive and overlapping stages, starting with the beginning of precipitation and continuing until all the water that fell as precipitation has left the watershed.

In the determination of the water losses from a drainage basin any part of which is irrigated, that water which is artificially applied to the land must be considered an integral part of the precipitation. If this fact is kept in mind no special discussion will be necessary to make the following principles applicable to irrigated areas. It has become customary to speak of the water that is used by interception, transpiration, and evaporation during the growth of any particular crop as being the *consumptive use* for that type of vegetation. When these losses are made to cover all such losses occurring in the valley or drainage basin they are then called the *valley consumptive use.* This quantity is equal to the total water losses except for those basins from which there is watershed leakage.

The water that is intercepted by vegetation, buildings, and other objects and does not reach the ground is evaporated and returned to the air as water vapor. In a similar manner the water that goes to depression storage and does not later infiltrate into the soil is returned to the air by evaporation. Especially in summertime when the ground surface has been heated, the rate of evaporation from that surface during the period immediately following precipitation is unusually high and continues at a diminishing rate for a long time after rainfall ceases. Even that water which infiltrates into the soil and joins the ground-water table is often subject to these losses, for, if the capillary fringe that lies just above the water table extends up to the ground surface, water will con-

tinuously be drawn up to the ground surface and evaporated therefrom; or, if the capillary water is not within reach of ground surface evaporation but is within reach of the roots of trees and other vegetation, it will be subject to the transpirational demands of that vegetation.

These different losses are interwoven to such an extent that they cannot be easily segregated so as to permit the separate and independent measurement of each. Fortunately, however, the interrelationship between these losses which makes separate measurement difficult tends to make their total more nearly constant for any particular region and climate and, therefore, reduces the importance and the necessity for their separate measurements. For instance, the presence of vegetation affects and reduces the rate of evaporation from the ground surface and, in turn, the evaporation from the ground surface reduces the moisture available for transpiration thereby making their sum more nearly uniform over any given basin regardless of the variation in vegetal cover.

The actual amount of the loss on any drainage basin depends upon the following factors:

1. The nature of the precipitation.
2. The type and development of the vegetation.
3. The area covered by buildings, pavements, and other objects.
4. The climatological factors such as temperature, humidity, and wind velocity.

INTERCEPTION

The interception rate expressed, for example, in inches per hour, is greatest at the beginning of storms and decreases with their duration. At first the surfaces of the leaves, branches, and trunks of trees and the stems of vegetation are dry and capable of retaining a considerable amount of moisture as interception storage, but after they have once become thoroughly wetted the interception rate becomes equal to the evaporation rate from those surfaces for the remainder of that particular shower.

Many storms consist of a series of showers separated by intervals of varying length. Upon the cessation of any shower, evaporation begins to deplete the interception storage and continues until all

the available moisture has been returned to the air in the form of vapor or until another shower occurs. From this it appears that the total interception during different storms is constant neither in quantity nor in percentage of total rainfall. For any given area and condition of vegetation the equation is of the form

$$x = a + bt \tag{1}$$

where x represents the total interception in inches depth on basin, a is the interception storage capacity in inches depth on basin, b is the evaporation rate in inches per hour, and t is the duration of the shower in hours. The above equation is, of course, applicable only to storms exceeding the interception storage capacity, a.

As indicated by the formula the total amount of interception increases with the duration of the storm, and since depth of precipitation increases with duration there is a general correlation between total interception and total rainfall although there is little difference between the losses resulting from a heavy downpour and from a light rain of the same duration. However, the percentage of the total precipitation that is lost through interception decreases as the amount of precipitation increases. As an illustration, for 0.25 in. rainfall in a dense forest perhaps 0.2 in. or 80 per cent is intercepted and never reaches the ground, whereas for a rainfall of 1 in. perhaps 0.3 in. or 30 per cent is intercepted, and for heavier rains the percentage is still less.

For coniferous trees the amount of interception is greater than for deciduous trees even in full leaf; the difference as given by various writers ranging all the way from zero to 100 per cent. For any given depth of precipitation, winter and summer losses appear to be about the same for conifers, but for deciduous trees summer losses are two or three times as great as winter losses. For dense grasses, shrubs, and grains approaching full growth, the interception loss is nearly as great as for deciduous trees in full leaf, but their season is short and as a result their total annual interception is considerably less than for deciduous trees.

Figure 44 shows the results of measurements of interception for two types of fairly dense forest cover made by the U. S. Forest Service, Southeastern Forest Experiment Station, Asheville, North Carolina. The values of interception plotted in the graphs were obtained by subtracting the ground rainfall plus tree-bole

drainage from the total precipitation. The tree-bole drainage amounted to from 10 to 30 per cent of the total interception.[1]

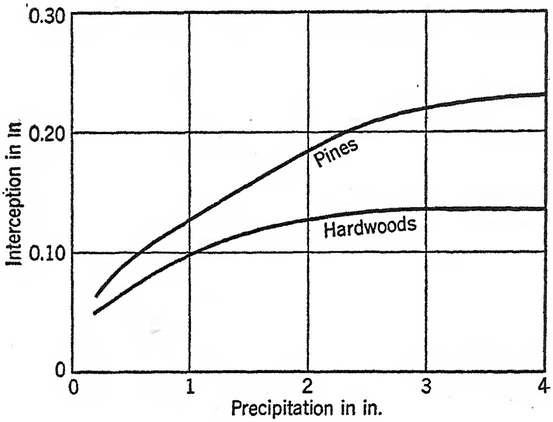


FIG. 44.

EVAPORATION FROM WATER SURFACES

Evaporation is the process by which a liquid or a solid is changed into a vapor. The exact laws governing this process are not clearly understood. To obtain the best conception of this phenomenon and of the laws governing it, it may be desirable first to review briefly the physical structure of water.

Any body regardless of its size is made up of a large number of molecules, each of which is constantly in motion at varying velocities and in different directions. The average velocity of all the molecules comprising any given mass determines its temperature. At absolute zero temperature all molecular activity is supposed to cease. In any given mass every molecule is attracted to every other one by a force that is inversely proportional to the square of the distances between them and directly proportional to the product of their masses. Those molecules that happen to be nearest to

[1] A number of other investigations of rainfall interception are:

R. E. Horton, Rainfall Interception, *Monthly Weather Rev.*, 1919, **47**, 603–623.

H. W. Beall, The Penetration of Rainfall through Hardwood and Softwood Forest Canopy, *Ecology*, 1939, **15**, 412–415.

J. Kittredge, H. S. Loughhead, and A. Niazurat, Interception and Stemflow in a Pine Plantation, *J. Forestry*, 1941, **39**, 505–522.

C. H. Niederhof and H. G. Wilson, Effect of Cutting Mature Lodgepole-Pine Stands on Rainfall Interception, *J. Forestry*, 1943, **41**, 57–61.

the free surface of a liquid are, therefore, acted upon by more of these forces from underneath than from above. Molecules that are near the surface, therefore, must overcome this additional force in order to escape into the air as water vapor. It thus appears that only the more rapidly moving molecules escape. Hence, the remaining mass will have a lower average temperature than before. Evaporation is therefore a cooling process.

Although molecules are continuously leaving the water surface, others are returning and the rate of evaporation is determined by the excess rate of those leaving over those returning. If more are returning than are leaving, condensation is said to be taking place. Immediately adjacent to the water surface is a thin layer of air whose temperature is the same as that of the water. This film quickly becomes saturated with water vapor. If, in the air above, the vapor pressure is the same as in the film, there can be no further evaporation. If, however, the vapor pressure in the air above is less than that in the film, through diffusion, convection, and wind action the vapor near the water surface will be dispersed and evaporation will continue. The rate of evaporation depends upon the difference between these two vapor pressures or, in other words, upon the pressure gradient. This principle is known as Dalton's law and is expressed by the formula

$$E = C(p_w - p_a) \tag{2}$$

in which E is the rate of evaporation in inches per day; p_w is the vapor pressure in the film of air next to the water surface or, in other words, the maximum vapor pressure corresponding to the temperature of the water; p_a is the vapor pressure in the air above; and C is a coefficient that is dependent upon barometric pressure, wind velocity, and perhaps other variables.

Influence of Depth

Depth has a very pronounced influence upon the rate of evaporation from any large body of water. The explanation of this phenomenon can be found in a comparison of the cycle of events that occurs in a shallow lake with that which occurs in a deep lake.

In late winter or early spring the temperature of an entire body of shallow water will be at or near 32° F. As the temperature of the air rises the temperature of the water on the surface also rises, but as it does so the weight of the water increases and the warmer

water sinks to the bottom as the colder but lighter water rises to the surface where it in turn is warmed to a still higher temperature and greater density, and the turnover process is repeated. This continues until the temperature of all the water becomes 39.2° F, the temperature corresponding to the maximum density of water. From then on, as the surface water becomes warmer it also becomes lighter and therefore remains on the surface. The water below the surface heats more slowly by direct radiation and conduction, as well as by mechanical mixing produced by currents and wave action.

In fall the overturning process is repeated in reverse. When the surface water cools below the temperature of the lower water it becomes heavier and sinks to the bottom, and the warmer water at the bottom rises to the surface. This continues until the entire body of water has a temperature of 39.2° F. After that, as the surface water further cools it becomes lighter and remains on the surface. This explains why, for a given reduction in air temperature, the surface-water temperature will drop much more quickly from 39° to 32° than from 46° to 39°.

Consider now a deep body of water. Either in spring or in fall, after the temperature of the entire mass has reached 39.2°, further changes resulting from radiation, conduction, and mixing are slow and extend only to a limited depth. Below 200 ft the temperature remains at or near 40° throughout the year. However, the amount of heat that is required to raise the temperature of a large mass of water from 39.2° to the temperature which it attains by autumn is enormous, and as a result there is a considerable lag between the temperature of the water and that of the air. The temperature of the water is lower than the air temperature in summer and higher in winter, whereas in shallow bodies of water the mean daily temperatures of the air and water do not differ so greatly.

This lag in the temperature of the water in deep lakes behind the temperature of the air above exerts two important influences on the rate of evaporation from such lakes, both of which combine to reduce the summer evaporation below and increase the winter evaporation above the observed rates on shallow lakes. The first of these influences is apparent from a consideration of Dalton's law (see page 145). Inasmuch as E varies with $(p_w - p_a)$, any reduction in the temperature of the water reduces p_w and also $(p_w - p_a)$ and likewise E. Also in winter an increase in the tem-

perature of the water increases p_w, $(p_w - p_a)$, and E. The other influence results from the fact that in summertime the colder the water is with respect to the air the colder and heavier and, therefore, the more stable will be that vapor-laden film of air that is in direct contact with the water. Because of this increased stability of the air adjacent to the water, the effect of wind action is partially nullified. On the other hand, in early winter the saturated layer of air next to the water is considerably warmer than the air above. Therefore, being unstable, it rises even without the aid of wind action and is replaced by other air of lower humidity. In Fig. 45 are shown monthly variations in the rate of evaporation from deep and shallow lakes. Graph A represents the rate of evaporation from Lake Superior as determined by Rohwer's formula, equation 6,

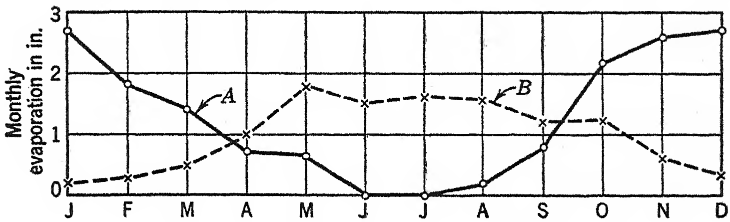


FIG. 45.

page 152, using average mean monthly air and water temperatures as given by Hickman.[1] Graph B represents the rate of evaporation from a shallow lake as determined by this same formula, using the same air temperatures as before but assuming the water temperatures to be the same as those of the air. This means that in winter the lake surface would freeze over and the evaporation from the ice would be given by equation 6. Although the latter assumption may not be exactly true, Rohwer found that it is approximately correct.

Methods of Determining Evaporation

Several methods may be used to determine the rate of evaporation from the water surface of reservoirs, natural lakes, or rivers. These methods are:

1. The storage equation.
2. Measurement in an auxiliary pan, and reduction of the

[1] H. O. Hickman, *Trans. A.S.C.E.*, 1940, **105**, 813.

pan evaporation to natural water-surface evaporation by means of a coefficient.

3. An evaporation formula.
4. The humidity and wind velocity gradients.
5. Measurement of insolation.

Storage Equation Method. The first of these methods involves the equation

$$P + I \pm U = E + O \pm \Delta S$$

in which P is the precipitation on the water surface, I the surface inflow, U the net underground inflow or outflow, E the evaporation from the water surface, O the surface outflow, and ΔS is the change in storage. This last quantity is positive for an increase and negative for a decrease in storage. The quantities are usually expressed as inches depth on the water area for some convenient time interval.

The chief disadvantage of this method is that these quantities may be determined with only varying degrees of accuracy. All the errors in measurement are combined and thrown into the resulting value of E. It is particularly difficult to make accurate measurements or estimates of the underground flow. In some cases this quantity is negligible, whereas in others it is an important factor. Large springs may occur in the lake bed or if the bed and surrounding area are highly permeable the direct underground inflow may be large. In other instances, and especially is this true of artificial reservoirs, large underground seepage losses may occur.

In his study of the hydrology of Lake Superior,[1] Colonel Pettis concluded that the underground flow into the lake is nearly equal to the surface supply. This would amount to about 16 or 17 in. per yr. In contrast to this, Meinzer[2] states, "On the basis of the great mass of information that is available in regard to the geology and ground-water conditions of the United States, it can be stated confidently that for most of the large drainage basins in the United States, the loss or gain of water by subterranean flow is relatively small and the discharge by transpiration and evapora-

[1] Col. C. R. Pettis (retired), Corps of Engineers, U.S. Army, *Trans. A.S.C.E.*, **105**, 795.

[2] O. E. Meinzer, Geologist in Charge of Division of Ground Water, Water Resources Branch, U.S. Geological Survey, Washington, D.C., *Trans. A.S.C.E.*, **105**, 834.

tion approximates the difference between precipitation and stream discharge."

Pan Measurements. Because of the ease with which accurate measurements can be made with small pans, a tremendous amount of work has been done by this method. However, it has been found that the rate of evaporation from a small pan is not the same as from a large body of water; nor is the rate of evaporation the same for all pans, it being affected by size, depth, and location. Hence for any given type of pan installation, it is necessary not only to measure the rate of evaporation under all different conditions but also to determine the coefficient or coefficients to be used in reducing the pan results to what may be anticipated from a lake or reservoir.

The following are brief descriptions of some of the more commonly used types of pans.

1. *Class A. U. S. Weather Bureau Land Pan.* This pan is 4 ft in diameter, 10 in. deep, and the bottom is raised 6 in. above the ground surface. The water surface is supposed to be at least 2 in. and never more than 3 in. below the top of the pan.

2. *U. S. Bureau of Plant Industry Sunken Pan.* This pan is 6 ft in diameter, 2 ft deep, and buried in the ground within 4 in. of the top. The water surface in the pan is supposed to be not more than ½ in. above or below ground level.

3. *Colorado Sunken Pan.* The Colorado Experiment Station has developed a pan 3 ft square, with a depth ranging from less than 18 in. to 3 ft, and buried in the ground within about 4 in. of the top. The water is supposed to be maintained within 1 in. of ground level.

4. *U. S. Geological Survey Floating Pan.* Because it was believed that the evaporation from a pan floating in a large body of water would be nearly the same as from the surrounding water, the U. S. Geological Survey has used a pan 3 ft square by 18 in. deep, supported by drum floats in the center of a raft 14 ft by 16 ft. The water level in the pan is supposed to be at the same elevation as that of the surrounding body, with the sides of the pan projecting 3 in. above.

5. On a number of occasions U. S. Weather Bureau land pans have been mounted on rafts and used as floating pans. However, they appear to have no particular advantage over the U. S. Geological Survey floating pan.

After a thorough study of the results obtained by each of these

different types of pans, the Subcommittee on Evaporation, of the Special Committee on Irrigation Hydraulics, of the American Society of Civil Engineers, recommended the U. S. Weather Bureau Class *A* land pan as their first choice. Next in order of preference was the Colorado sunken pan, and third was the U. S. Geological Survey floating pan. In their summary for and against the use of the U. S. Weather Bureau land pan they state,[1]

The advantages are:

(1) More data on this pan are available for comparison than on any other particular type of pan.
(2) The coefficient has been found to be about the same for this pan in comparison with large water surfaces in many locations and under many conditions.
(3) It is easy of access for observations.
(4) It is not subject to uncertainty due to inward or outward wash of water as is the case for a floating pan.
(5) It is raised above reasonable drift of dirt, debris, and snow.
(6) It is reasonable in cost of installation.

The disadvantages are:

(1) The coefficient is considerably less than unity; that is, the rate of evaporation is much greater from the pan than from a reservoir surface.
(2) For the data available, coefficients for certain months vary greatly from those for other months. This inconsistency, however, is true for all types of pans thus far studied.

As a coefficient to be applied to the observed evaporation from a Class *A* U. S. Weather Bureau land pan for the determination of the evaporation from a lake or reservoir, this committee recommended 0.70, with a range between 0.60 and 0.82. For the Colorado sunken pan they recommended 0.78 with a range between 0.75 and 0.86, and for the U. S. Geological Survey floating pan they recommended 0.80, with a range from 0.70 to 0.82. For a more complete discussion of this subject see the article by Rohwer and Follansbee previously referred to.[1] This same article gives the results of numerous evaporation measurements, some of which are shown in Table 6.

[1] Carl Rohwer, Robert Follansbee, and others, Symposium on Evaporation from Water Surfaces, *Trans. A.S.C.E.*, 1934, **99**, 673–747.

Summary of Evaporation Records Reduced to Reservoir Surface Evaporation*

Station	Elevation in Ft.	Years	Temperature of Air in °F.	Wind Velocity at the Pan, in Miles/Hr	Relative Humidity(%)	Reservoir Surface Evaporation in In. April–September	Annual	Maximum per Month
Gardiner, Me.	100	1915–24	45	2.2	..	18.10	24.26	4.56
Ithaca, N. Y.	800	1918–30	47	1.8	78	17.11	22.54	4.17
Pleasantville, N. J.	40	1924–30	51	2.9	74	23.02	31.81	5.40
Washington, D. C.	280	1915–17	54	2.3	69	23.52	34.53	4.87
Chapel Hill, N. C.	500	1921–30	61	1.1	69	20.03	28.56	4.71
Birmingham, Ala.	650	1910	63	...	72	32.18	42.99	6.25
Grand River Lock, Wis.	780	1905–12	46	...	..	21.64	28.57	5.75
Madison, Wis.	860	1906–11	46	...	75	12.91	19.82	3.04
Centerville, Minn.	880	1919–27	45	4.0	..	24.11	30.97	7.37
Iowa City, Iowa	610	1907–10	46	...	..	19.59	30.08	5.36
Columbus, Ohio	763	1918–30	52	2.0	74	19.21	26.81	4.94
Cincinnati, Ohio	520	1910	54	...	68	29.46	38.19	6.02
Clarksburg, W. Va.	1030	1923–30	53	2.6	79	20.65	26.60	6.05
Columbia, Mo. #1	750	1916–27	54	1.5	71	20.28	28.13	5.31
Columbia, Mo. #2	750	1916–26	54	2.9	71	26.31	35.82	6.85
Silverhill, Ala.	250	1918–30	67	1.8	78	25.35	38.27	5.73
Crowley, La.	21	1910–19	68	2.8	77	30.76	46.90	7.18
Williston, N. Dak.	1875	1909–16	39	5.5	74	32.12	38.79	8.45
Rapid City, S. Dak.	3240	1916–21	46	2.0	58	25.61	36.43	7.02
Lincoln, Nebr.	1250	1917–30	51	4.1	69	32.06	42.04	9.92
Wichita, Kans.	1300	1918–27	56	4.2	68	34.60	49.62	9.27
Lawton, Okla.	1111	1916–20	61	6.4	..	41.75	54.35	11.42
San Antonio, Tex.	700	1907–12	70	...	68	43.46	63.71	10.14
Spur, Tex.	....	1922–30	61	6.0	61	42.84	62.44	10.80
El Paso, Tex.	3700	1889–93	64	...	36	49.95	71.16	10.79
Bozeman, Mont.	4754	1918–30	41	2.7	..	24.72	33.77	7.58
Shoshone Reservoir, Wyo.	5390	1916–29	46	...	..	31.70	43.27	9.46
Fort Collins, Colo.	4998	1887–1927	46	1.7	68	30.41	42.19	6.12
Wagonwheel Gap, Colo.	9610	1920–24	34	2.0	64	16.12	22.32	3.22
Santa Fe, N. Mex.	7010	1917–30	48	2.6	54	33.66	44.82	8.46
Elephant Butte, N. Mex.	4265	1916–30	60	3.9	..	47.24	66.99	11.75
Arrowrock, Idaho	3220	1916–30	49	0.8	..	29.32	40.09	8.07
Salt Lake City, U.	4250	1928–30	52	3.6	50	40.67	50.94	10.10
Fallon, Nev.	3970	1908–30	51	3.0	52	44.50	56.74	11.10
Tucson, Ariz.	2400	1929–30	67	1.3	40	41.82	60.26	9.39
Walla Walla, Wash.	1000	1917–30	53	2.0	58	29.68	36.85	8.64
Yakima, Wash.	1060	1910	50	...	..	35.07	47.25	7.41
Warm Springs Reservoir, Oreg.	....	1927–30	47	2.5	..	38.39	54.04	10.42
Klamath Falls, Oreg.	4100	1924–29	48	...	..	24.08	36.71	6.33
Corvallis, Oreg.	235	1922–30	52	1.5	..	21.88	30.68	5.92
Tahoe, Calif.	6230	1916–30	42	3.0	..	22.34	32.09	6.67
E. Park Reservoir, Calif.	1200	1911–29	59	4.0	..	39.48	50.84	9.51
Oakdale, Calif.	215	1918–30	60	5.2	..	45.75	56.89	11.76
Chula Vista, Calif.	9	1918–30	59	3.8	..	26.84	42.26	5.45
Keewatin, Ont.	....	1913–28	36	...	72	11.24	14.29	3.84
Saskatoon, Sask.	....	1918–30	..	...	..	23.51	28.89	7.14
Edmonton, Alta.	....	1918, '21–22	..	...	..	18.65	24.00	5.46
San Juan, Puerto Rico	82	1919–30	78	...	78	31.34	55.29	6.54
Gatun, Canal Zone	85	1912–30	80	7.1	84	22.47	48.38	7.03
Maunawili, Oahu	250	1921–30	72	1.9	..	17.23	30.92	3.80
Alexandria, Egypt	....	1920–29	68	...	72	.....	28.76	....
Atbara, Sudan	....	1905–29	83	...	29	.....	123.67	....

* Data taken from "Evaporation from Reservoir Surfaces" by Robert Follansbee, *Trans. A.S.C.E.*, 1934, **99**.

Evaporation Formulas. Most of the evaporation formulas that have been proposed are based upon Dalton's law and therefore are of the general form of equation 2 in which a value of C has been derived from observed evaporation from small pans. Therefore, to determine the evaporation from large bodies of water, a carefully selected coefficient (see preceding article and reference page 150) must be applied to the results obtained by means of any of these formulas. Two of the earlier formulas are those of Fitzgerald and Russell. Fitzgerald's formula which includes a wind correction factor is

$$E = (0.40 + 0.199w)(p_w - p_a) \tag{3}$$

in which E is the evaporation, in inches per day; w the velocity of the wind at the ground surface, in miles per hour; p_w the maximum vapor pressure corresponding to the temperature of the water, in inches of mercury; p_a the vapor pressure in the air above, in inches of mercury.

Russell's formula ignores the influence of wind but corrects for barometric pressure; it is

$$E = \frac{(1.96p_w + 43.88)}{B}(p_w - p_a) \tag{4}$$

where B is the mean barometric pressure in inches of mercury at 32° F.

It is generally recognized, however, that wind has a greater influence upon evaporation than has barometric pressure.

Horton's formula[1] which corrects for wind effect is

$$E = 0.4(xp_w - p_a) \tag{5}$$

where $x = 2 - e^{-0.2w}$, in which e is the base in the Napierian system of logarithms.

As a result of studies made by the Bureau of Agricultural Engineering of the U. S. Department of Agriculture,[2] Rohwer derived the formula

$$E = 0.771(1.465 - 0.0186B)(0.44 + 0.118w)(P_w - P_a) \tag{6}$$

The same nomenclature as in the preceding formulas applies here. This formula provides corrections both for wind velocity and for barometric pressure.

[1] *Eng. News-Record,* April 26, 1917, pp. 196–199.

[2] C. Rohwer, Evaporation from Free Water Surfaces, *U. S. Department of Agriculture Tech. Bul.* 271, December 1931.

Meyer's formula for evaporation from small bodies of water is

$$E = 15(V - v)\left(1 + \frac{w}{10}\right) \tag{7}$$

in which E is evaporation in inches per 30 days; V the maximum vapor pressure in inches of mercury corresponding to the monthly mean air temperature as observed at nearby Weather Bureau stations; v the actual vapor pressure in the air, based upon Weather Bureau records of monthly mean air temperature and relative humidity at nearby stations; w the monthly mean wind velocity in miles per hour, as observed about 30 ft above the ground at nearby Weather Bureau stations.

Many other formulas have been proposed by different experimenters for the determination of the rate of evaporation from free water surfaces. For a more complete discussion of this subject reference should be made to the articles above cited.

Inasmuch as wind velocity in most of these formulas refers to the velocity at a point about 6 in. above the ground, it is usually necessary to reduce measured velocities to ground velocities. Figure 46 is a curve presented by Rohwer for this purpose. Rohwer points out, however, that this curve is based on limited data. Recent work done by the Department of Agriculture provides more extensive information concerning the variation in wind velocity near the ground. A year of records was obtained at Arlington, Virginia,[1] of the wind velocity at elevations of 1, 7, 13, and 25 ft, together with temperatures at elevations of 2 ft and 24.5 ft. A number of careful observations were also made at New Philadelphia, Ohio.[2] In the latter case the wind velocity was measured at twelve points, the lowest being 0.5 ft above the ground and the highest 28 ft above the ground.

Various sets of data from the latter publication are plotted in Fig. 47. The ordinates are elevations above the ground, plotted logarithmically, and the abscissas are the ratios of the velocities at the various elevations to the velocity at 1 ft. It may be noted that curves A, B, and C occur at various times during the same day. This is a typical variation. Curve D shows the most uniform

[1] C. W. Thornthwaite and Benjamin Holzman, Measurement of Evaporation from the Land and Water Surfaces, *U. S. Department of Agriculture Tech. Bul.* 817, 1942.

[2] C. W. Thornthwaite and Paul Kaser, Wind Gradient Observations, *Trans. Am. Geophys. Union*, 1943, p. 166.

velocity gradient, and curve *E* the steepest gradient of the 97 complete sets given in the paper. The difference in the slopes of the various curves is due in part to the effect of the temperature gradient.

When the upper air is warmer than the lower air, during a temperature inversion, the air is relatively stable since there is

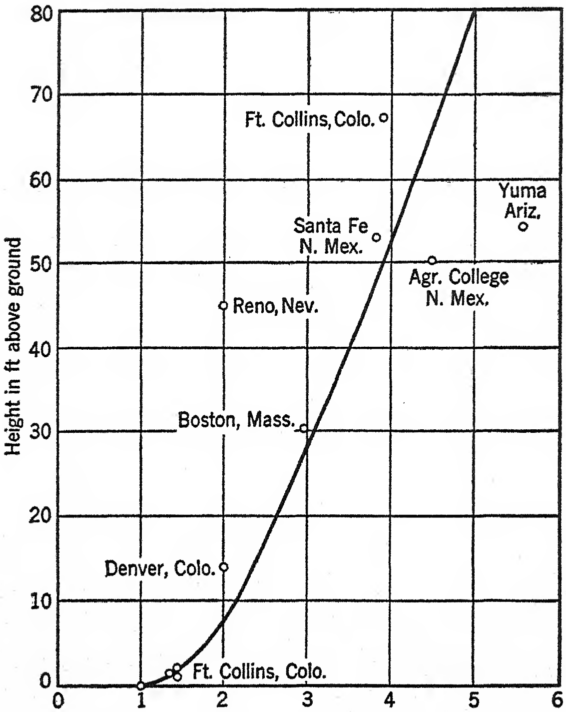


FIG. 46. Ratio of observed velocity to ground velocity. From *U. S. Department of Agriculture Tech. Bul.* 271 by C. Rohwer.

little tendency for the upper air to mix with lower layers. As a result the velocity gradient tends to remain steep. Such a condition as shown by Curves *C* and *E* of Fig. 47 quite regularly prevails at night. During the daytime the lower air is warmed and a temperature lapse rate greater than the normal adiabatic rate is often established. At such times the heavier upper air tends to fall and mix with the slower-moving lower air. The result is a tendency for a more uniform velocity at all elevations. Curves *A* and *D* of Fig. 47

illustrate these conditions. Curve *B* is typical of those occurring in the morning or evening when normal adiabatic lapse rates usually exist.

When attempts were made to correlate the ratio of the velocity at 25 ft to the velocity at 1 ft (V_{25}/V_1) with temperature gradient,

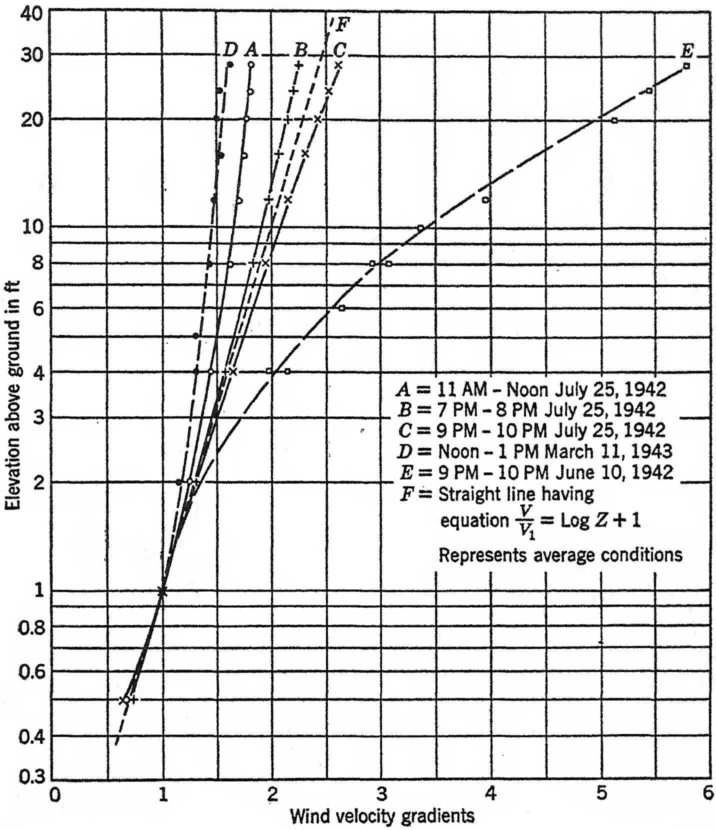


FIG. 47.

it was found that another important factor apparently overshadowed temperature gradient in importance. This factor is the magnitude of the velocity at any point near the ground. In this case the velocity at 1 ft was used. The effect of this factor is shown in Fig. 48 where values V_{25}/V_1 are plotted against V_1 for 220 observations selected at random from *Bulletin* 817, together with the

97 values of V_{24}/V_1 taken from *Wind Gradient Observations.* The two groups of ratios, i.e., V_{25}/V_1 and V_{24}/V_1, were plotted together since the difference between the two is small compared with experimental errors and the influence of other factors. It may be

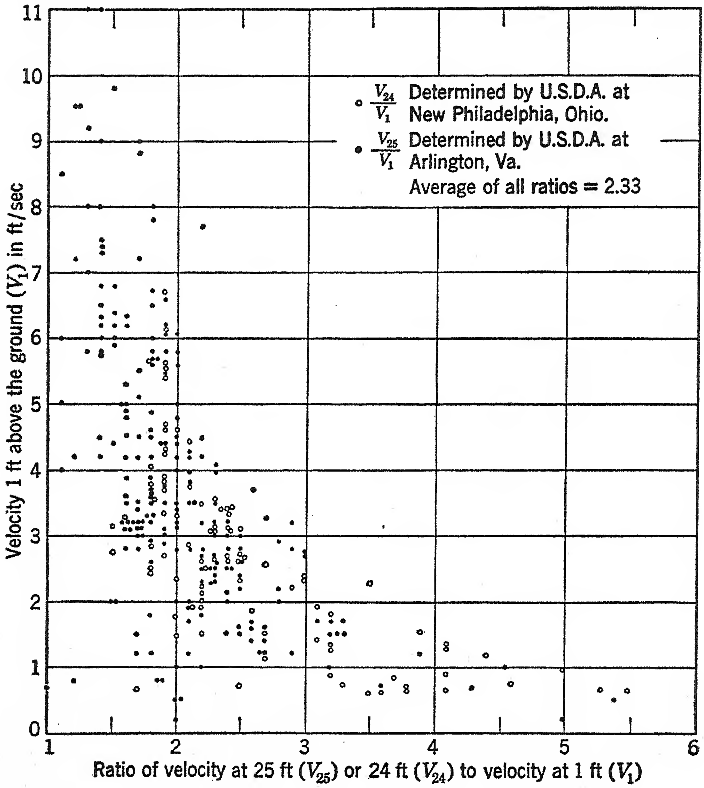


FIG. 48.

seen that there is a tendency for the velocity gradient between the 1-ft and the 25-ft levels to be steep for low values of V_1 and flat for high values.

The average value of the ratio for all the points plotted in Fig. 48 was found to be 2.33. If this value is plotted at an elevation of 24.5 ft on Fig. 47, it will be seen that Curves B and F very nearly represent average conditions. Curve F is a straight line having the simple equation

$$\frac{V}{V_1} = \log Z + 1 \tag{7}$$

where Z is the elevation above the ground at which V is measured. As a practical method of reducing the wind velocity at any elevation to that at 6 in. above the ground, it is suggested that the ratios be computed from equation 7 or read from Curve F. For example, if V is measured at a point 10 ft above the ground (V_{10}) and found to be 5 ft per sec, equation 7 becomes $V_{10}/V_1 = (\log 10) + 1$ or $V_{10}/V_1 = 2$. Also, $V_{0.5}/V_1 = (\log 0.5) + 1$ or $V_{0.5}/V_1 = 0.70$. The ratio of the wind velocity at 10 ft (V_{10}) to the velocity at the ground ($V_{0.5}$) may then be found by combining the above values as follows: $V_{10}/V_{0.5} = V_{10}/V_1 \times V_1/V_{0.5} = 2 \times 1/0.70 = 2.86$. Using the value of 5 ft per sec for V_{10}, $V_{0.5} = 5/2.86 = 1.75$ ft per sec. All the ratios determined in the above example from equation 7 could also have been read from Curve F, Fig. 47.

A greater degree of refinement may be obtained by first solving for the velocity at 1 ft from Curve F, then from Fig. 48, checking for the corresponding V_{25}/V_1. If this is far different from 2.4, the ratios may be redetermined from another curve of the family of curves in Fig. 47. If in the above example, V_{10} had been 10 ft per sec, the ratios should have been determined from Curve A rather than Curve F.

It should be understood that many other factors also influence the wind velocity pattern. Such factors as the roughness and slope of the ground and the presence of buildings or trees in the vicinity may cause great fluctuations in the velocity. It would, for example, be difficult to decide whether an anemometer placed 4 ft above the roof of a 20-ft building should be corrected for an elevation of 4 ft or 24 ft. Neither is likely to give the proper result owing to the presence of eddies and cross currents which exist under such conditions.

Use of Humidity and Wind Velocity Gradients. The fourth method of measuring evaporation is based on the premises (1) that, if a moisture gradient exists in the air, water vapor will move toward points of lower moisture content, and (2) that the rate of movement of the water vapor is accentuated by the intensity of turbulence in the air. This method has the great advantage of being applicable to both natural land and water surfaces. The difficulty involved in its use is the fact that sensitive and relatively expensive equipment is required. The humidity gradient may be determined from simultaneous measurements of the moisture content of the air at two elevations above the ground with some type of hygrometer. The intensity of turbulence is determined from corresponding

measurements of wind velocity gradients. The development of a relationship between the two quantities and the rate of evaporation is based upon the assumption that water vapor will be transported in a manner similar to the interchange of momentum by the turbulent mixing process. The formula for evaporation also includes a turbulence constant that must be evaluated experimentally. The U. S. Department of Agriculture has been conducting an extensive series of tests[1] involving this method. Hourly evaporation-transpiration records covering nearly a year have been obtained for a meadow in Arlington, Virginia. The reference cited above gives a review of the theoretical background for this method, including a derivation of the formula as well as a description of the instruments and methods used in its application.

Insolation Method. The fifth method of determining evaporation is based on the idea of conservation of heat energy within a body of water. For any given body of water a balance must exist between (1) insolation; (2) heat transferred from the water surface by radiation, conduction, or convection; (3) heat energy acquired or lost in raising or lowering the temperature of the water; and (4) heat dissipated or acquired by evaporation or condensation. If items (1), (2), and (3) are measured, the evaporation or condensation may be determined. This relation between evaporation and insolation was suggested by A. Angstrom[2] in 1920. A method of utilizing this principle to determine evaporation from natural water surface has been presented by Richardson.[3] However, its application requires records of air temperatures above lakes, temperature gradients within lakes, and clearness of the atmosphere, that are rarely available. As a result, this method is of limited usefulness for the present.

TRANSPIRATION

Transpiration is the process whereby the moisture that has circulated through the plant structure is returned to the atmosphere principally in the form of water vapor. For any given plant the rate of transpiration varies throughout the 24 hr of the day; it

[1] C. W. Thornthwaite and Benjamin Holzman, Measurement of Evaporation from Land and Water Surfaces, *U. S. Department of Agriculture Bul.* 817, May 1942.

[2] *Geografiska Annala*, 1920, H. 3, p. 237.

[3] Burt Richardson, Evaporation as a Function of Insolation, *Trans. A.S.C.E.*, 1931, **95**, 996.

varies from one day to another depending upon the temperature, sunlight, moisture available, and other atmospheric conditions; for annual plants it varies throughout their period of growth, and for perennial plants it varies from year to year depending upon their stage of development.

Some of these factors influence transpiration and evaporation in a very similar manner; others do not. For instance, temperature seems to exert the same influence on both; in other words, the rate of transpiration is doubled for approximately every 18° rise in temperature. Also relative humidity, wind velocity, and perhaps barometric pressure appear to exert practically the same effect upon transpiration as they do upon evaporation from free water surfaces. On the other hand, transpiration is to a considerable extent influenced and controlled by the amount of sunlight, moisture available, and stage of plant development.

Sunlight

Although the rate of evaporation from a free water surface is usually lower at night than in daytime, the reduction is principally due to the lower night temperature of the air and to the resulting increase in relative humidity. Transpiration rates however experience a much greater variation between day and night, principally because transpiration varies directly with plant growth and the latter is almost wholly dependent upon sunlight. As a result, transpiration is virtually restricted to daylight hours.

A striking illustration of the effect of transpiration and of its variations between day and night is provided in Figs. 49 and 50. In Fig. 49 is shown the variation in flow July 7 to 14, 1938, for Stream 14, which drains an area of 152 acres in Coweeta Experimental Forest of the Southeastern Forest Experiment Station near Asheville, North Carolina. In Fig. 50 are shown the fluctuations in ground-water level September 19 to 26, as recorded in Well G–2 which is located in the valley bottom of a dry cove where the water table is 10.5 ft below the ground surface. The water level in Well G–1 is also shown. This well is located about 30 ft from G–2, where the water table is 16.5 ft from the ground surface. Dr. C. R. Hursh, Senior Forest Ecologist at the station, explains these fluctuations in the following manner.

A much greater density of trees and shrubs occurs near the banks of the streams and in the valley bottoms than is to be found

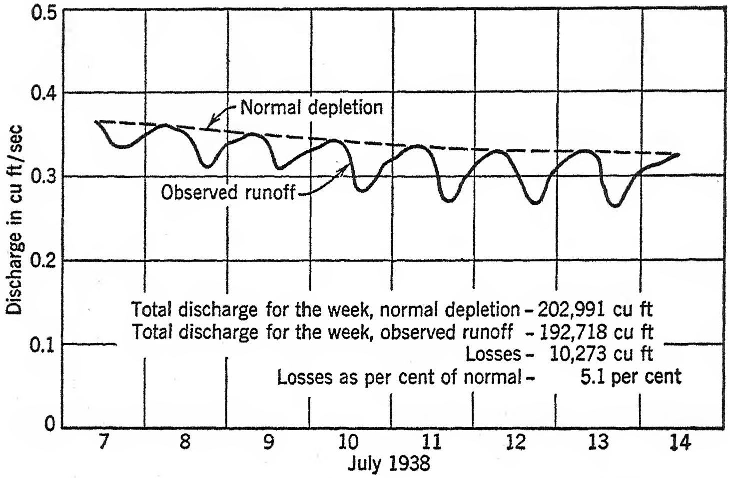


FIG. 49. Stream hydrograph reflecting transpiration draft. Stream 14, drainage area, 152 Acres, Coweeta Experimental Forest, Southeastern Forest Experiment Station, Asheville, N. C.

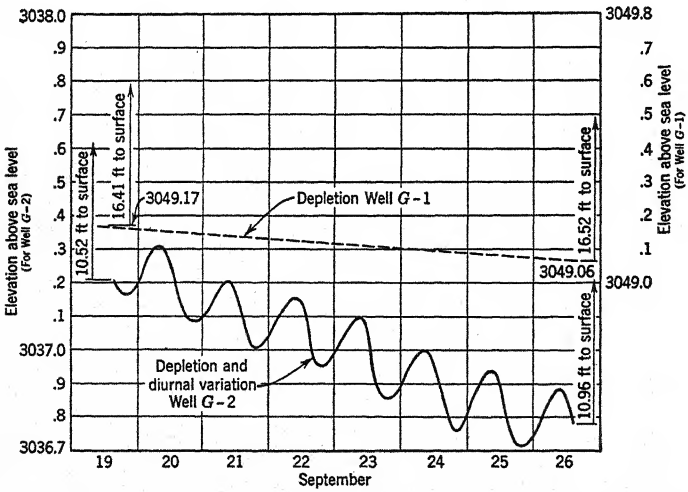


FIG. 50. Ground-water elevations, Coweeta Experimental Forest, Southeastern Forest Experiment Station, Asheville, N. C.

farther back and nearer to the divide. The roots of this vegetation penetrate into the capillary fringe, thus lowering the water table in the immediate vicinity during the daylight hours when transpiration is most active. Back further from the stream channel the water table is so far below the ground surface that the plant roots cannot reach the capillary fringe and, as a result, in this region there is no transpirational draft during the daytime followed by a night recovery, but instead there is only a continuous decline in the water table until it is replenished from precipitation. These conditions are illustrated in Fig. 51, in which *abc* represents the water level in the stream and adjacent ground water in the morning

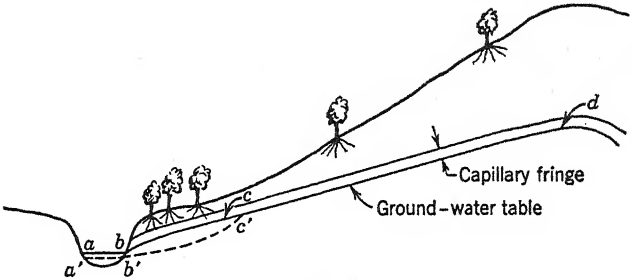


FIG. 51.

before the effects of transpiration are felt. To a greatly exaggerated scale *a′b′c′* shows the water surface in the stream and in that portion of the water table that is directly affected by the day's transpiration, as they appear in the evening. During the following night, water from *cd* flows in and restores the lower portion of the water table almost but not quite to its initial level *abc*.

The effect of transpiration draft as shown in Fig. 49 is most frequently observed in small headwater streams and even then only when the water table or capillary water in the adjacent banks is within reach of the roots of vegetation. It is less distinctly seen in hydrographs of large streams because of the ironing-out effect produced by the large number of inflowing tributaries, each having a different timing.

Moisture Available

Quite naturally the amount of moisture that is used by vegetation is limited to the amount that is available. Except for aquatic and aerial vegetation, plants have the following possible sources of

moisture supply, viz., gravity water, pellicular water, and capillary water. Gravity water is available only for short periods during and immediately following rains, and under the action of gravity it is always moving downward toward the water table. Pellicular water exists above the capillary fringe as a thin film covering the soil grains, being held there against the action of gravity. Vegetation whose roots do not penetrate into the capillary fringe is, therefore, principally dependent upon pellicular water for its moisture supply. Vegetal roots are incapable of drawing all the pellicular water from the soil, and in this respect various types of plants seem to differ but little. Assuming an average depth of penetration of plant roots of 5 ft, Tolman[1] estimates a moisture content of a little less than 1 ft of water in this depth for a soil of average texture. Throughout a growing season most vegetation requires from 6 in. to 24 in. of water or more. Therefore those plants whose roots do not penetrate to the capillary fringe are dependent upon frequent replenishment of soil moisture by rains. When the available moisture has been exhausted to the point where permanent wilting occurs, the ratio of the weight of the remaining moisture to the weight of the dry soil is called the *wilting coefficient* for that soil. Its value varies inversely with the sizes of the soil grains, ranging from less than 1 per cent for coarse sand to 25 or 30 per cent for heavy clay.

Plants whose roots penetrate into the capillary fringe are not so directly dependent upon rains for the replenishment of their supply, for as this moisture is drawn off it is continuously replenished from the water table. The rate of replenishment, however, depends upon the sizes of the soil grains and upon the height to which the water must be raised. If, for instance, the capillary fringe is 4 ft in thickness and the roots penetrate halfway through or to within 2 ft of the water table, the rate of replenishment will be very much greater than if the depth of penetration were only a few inches. In Fig. 52 are shown the rates of capillary rise in a number of California soils as determined by W. A. Packard in an unpublished research reported by Tolman.[2] In this figure it will be noted that the average height of rise in the first 4 days is about 75 per cent of the total height observed in 25 days.

[1] C. F. Tolman, *Ground Water*, McGraw-Hill, 1937, p. 31.

[2] By permission from *Ground Water*, by C. F. Tolman, copyrighted by McGraw-Hill, 1937.

Raising or lowering the water table may result either beneficially or detrimentally to vegetation. If, for instance, the water table is at such an elevation that the natural penetration of the roots is well into the capillary fringe but not into the water table, the conditions for plant growth are at their best. If the water table is lowered the plants will at times suffer from lack of moisture, and if it is raised for any great length of time the submerged roots will be killed by drowning. On the other hand, if the water table is

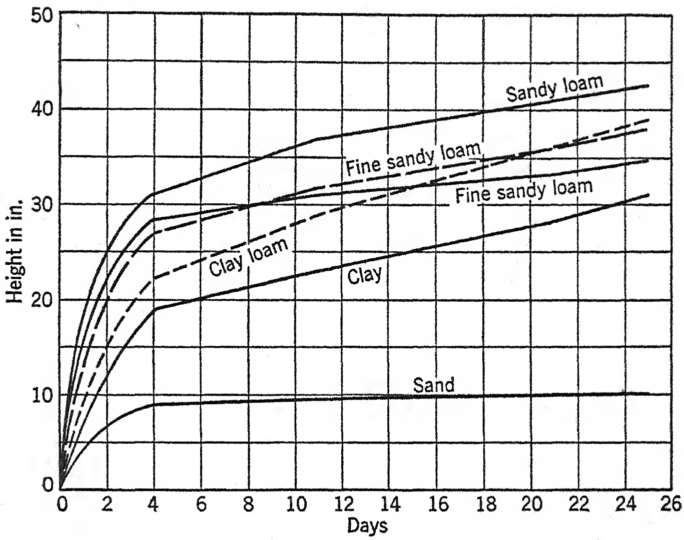


FIG. 52.

naturally so low that the capillary water is beyond the reach of the roots, a proper amount of raising will benefit vegetation; and, if it is naturally so high that the roots can penetrate only to an insufficient depth without being drowned, a certain amount of lowering will prove beneficial.

Stage of Plant Development

In addition to the diurnal variation in transpiration that has already been discussed, there are also seasonal and annual variations depending upon the stage of development and rate of growth. All plants grow more rapidly at certain seasons of the year than at others. The fact is quite well established that for any particular

type of vegetation the rate of transpiration varies with the weight of dry matter produced. Inasmuch as the production of dry matter is the result of growth it varies directly with vegetative activity. In other words, transpiration is restricted to the growing season and is most rapid during the period when vegetation is growing the fastest.

Transpiration Ratio

Because from month to month there are large variations in the rate of growth for different types of vegetation, the monthly changes in transpiration are correspondingly large. Because of the

TABLE 7

Plant	Transpiration Ratios: Usual Range	Transpiration Ratios: Average
Alfalfa	700–1000	850
Wheat	300– 550	450
Oats	400– 650	500
Corn	250– 350	300
Rye	400– 600	500
Barley	300– 600	450
Rice	600– 800	700
Cotton		600
Red clover	300–1000	800
Buffalo grass	250– 350	300
Weeds	200–1000	600
Potatoes	300– 600	500
Sugar beets	300– 500	400

difficulties encountered in measuring monthly changes in ground-water and soil-moisture storage, but few data are available on monthly transpiration. Nor is much known about the changes from year to year resulting from the growth and development of annual plants. For any particular kind of vegetation the ratio between the weight of water consumed and the weight of dry matter produced is called its transpiration ratio. In determining this factor the weight of the dry matter of the entire plant, exclusive of the roots, should be used. Sometimes, however, to make the results more generally useful, only the marketed crop is used, such as the grain of wheat, corn, beans, and rice, the tubes of potatoes and sugar beets, and so on.

For any type of plant the transpiration ratio varies between

wide limits depending upon the kind of soil, available moisture, relative humidity, and other climatic factors. In Table 7 are given the usual range and approximate average values for some of the more common types of vegetation.[1] These transpiration ratios are based upon the weight of dry matter produced rather than upon the weight of the marketed product.

Measurement of Transpiration

Although a number of laboratory methods of measuring transpiration have been used, especially by botanists, the one that has met with most favor by engineers, foresters, and agriculturists has been the closed-phytometer method. This instrument consists of a watertight tank containing sufficient earth to nourish the plant. A cover is sealed on to prevent any rain from entering or any water from escaping from the tank except through plant transpiration. Means are provided for adding water as desired. The transpirational losses for any period are equal to the original weight plus the weight of water added minus the final weight. This method is, of course, restricted to plants having relatively small root systems.

In Column 5, Table 8, are shown some transpiration ratios determined from experiments by Briggs and Shantz. These values are based upon the weight of the dry matter contained in the marketed product. The weights and depths shown in Columns 6 and 7 were computed by applying these ratios to the average yields shown in Column 2. These experiments were conducted at Akron in eastern Colorado and do not necessarily represent the transpiration requirements of crops grown under different soil and climatic conditions elsewhere. In fact, they do not truly represent the water uses of crops grown in eastern Colorado under actual field conditions. All these plants were grown in closed phytometers and in screened shelters. The experimenters concluded that the shelters caused a reduction in the water requirement of approxi-

[1] For detailed results of experimental investigations see the following references:

L. J. Briggs and H. L. Shantz, *U. S. Department of Agriculture Bureau of Plant Industry Bul.* 285, 1913.

L. J. Briggs and H. L. Shantz, *J. Agr. Research,* Vol. 3, No. 1, pp. 1–63, 1914.

H. L. Shantz and L. N. Piemeisel, *J. Agr. Research,* Vol. 34, No. 12, pp. 1093–1190, 1927.

Physics of the Earth IX, *Hydrology,* McGraw-Hill, 1942, pp. 283–288.

TABLE 8

Crop (1)	Bu per acre (2)	Lb per bu (3)	Lb per acre (4)	Transpiration Ratio§ (5)	Lb of water per acre (6)	Depth in in. (7)
Wheat	30	60	1800	1000–1600	1,800,000–2,880,000	8–12.5
Oats	50	32	1600	1100–1900	1,760,000–3,040,000	8–13.5
Barley	35	48	1680	950–1250	1,600,000–2,050,000	7–9
Rye	25	56	1400	1800	2,520,000	11
Corn	80	35	2800	850–1240	2,240,000–3,470,000	10–15
Potatoes	100	60	1320†	1380–2200	1,820,000–2,900,000	8–13
Sugar Beets	10*		2400‡	570	1,360,000	6
Alfalfa	1.5*		3000	420–1060	1,260,000–3,180,000	5.5–14
Red Clover	1.5*		3000	720–1020	3,060,000	9.5–13.5

* Tons per acre.
† Dry matter taken as 22% of total weight.
‡ Dry matter taken as 12% of total weight.
§ Based upon the results of experiments by Briggs and Shantz, *J. Agr. Research*, Vol. 3, No. L, pp. 1–63, 1914.

mately 20 per cent. The extent of the influence of the phytometers upon the water consumption is unknown.

As far as is known to the authors, the only large-scale experimental work that has ever been done to determine the amount of transpiration of any given type of vegetation under perfectly natural conditions is that which was conducted by the Southeastern Forest Experiment Station, Asheville, North Carolina.[1]

In this work which was carried on under the direction of Dr. C. R. Hursh, Senior Forest Ecologist for the station, very careful measurements were made of the rainfall, ground-water levels, and runoff on several small drainage basins in the Coweeta Experimental Forest covering a period of 4½ yr. At the end of that period in 1941, one of the basins having an area of 32.8 acres was completely deforested. Care was taken to prevent as completely as possible any change in the ground-cover conditions so that any observed

[1] M. D. Hoover, Effect of Removal of Forest Vegetation upon Water-Yields, *Trans. Am. Geophys. Union*, 1944, Part VI, p. 969.

change in the rainfall-runoff relationship would be the direct result of forest removal. Unfortunately because of labor shortages resulting from the war, except for the first year after deforestation, second growth was permitted to develop and, to an extent, obscure the effects of forest removal. However, during 4 yr before cutting, the average water losses from the basin were 42.5 in. and the average runoff corrected for storage was 27.9 in. During this period the average annual precipitation was 70.4 in. and the mean temperature was 53.8°. During the first year after cutting, when no second growth was permitted, the water losses were 21.9 in., only slightly more than half of what they were before, and the runoff corrected for storage was 40.5 in., an increase of nearly 50 per cent. During this year the precipitation was only 62.4 in. or 8 in. below the average for the preceding period, and the temperature was 55° or 1.2° above the average for the preceding period. During the second year after cutting but with some second growth developing, the water losses were 31.6 in. and the runoff corrected for storage was 46.8 in. During this year the rainfall was 78.4 in. and the average temperature was 54.6°. Although 2 yr is a rather short period upon which to base conclusions, the indications are that the annual runoff has been increased about 20 in. as a result of deforestation.

SOIL EVAPORATION

Soil evaporation or land evaporation is the loss of moisture through direct evaporation from the individual soil grains located at and near the ground surface. Its rate is governed by the same factors that affect the rate of evaporation from a free water surface plus a factor which Horton has called "evaporation opportunity." This factor, as its name implies, is a measure of the opportunity or of the possibility of the occurrence of evaporation from the ground surface. It may be expressed as the percentage that the actual evaporation from the ground surface is of the rate of evaporation from a free water surface. It is principally controlled by the amount of moisture present in the surface strata of the soil and ranges from zero for a thoroughly dry ground surface to 100 per cent or more for a bare soil immediately after a rain. Under a hot sun, with a relatively low humidity and a high wind velocity it will continue at a high rate for some time after the cessation of rainfall until the ground surface starts drying off and will then

gradually decrease as the surface layers of the ground become more and more dried out.

Influence of High or Low Water Table

The water table exerts an influence upon soil evaporation that is only to a limited extent similar to its influence upon transpiration. In transpiration there is always a certain elevation of the water table for which the vegetation will use the maximum amount of moisture. If the water table rises too high the roots will drown, and the plant will die. For soil evaporation, however, the rate increases until the water table rises to the ground surface when the soil-evaporation opportunity becomes 100 per cent.

In most drainage basins there are certain areas in which the ground surface is never within the capillary fringe and throughout which the rate of soil evaporation is extremely variable, ranging from a maximum right after rains to zero during dry periods. In other areas the ground surface is always being supplied with capillary water and the soil-evaporation rate is continuous and much more nearly uniform. When these areas are artificially drained, the water table is lowered, and the soil evaporation is greatly reduced. The effect of drainage upon flood and low water flows was discussed in Chapter III. (See page 61.) Intermediate between these two areas is a third area throughout which during a portion of the year the ground surface is within the capillary fringe and the soil evaporation is continuous and fairly uniform, and during the remainder of the year it is above the capillary water and the evaporation rate is discontinuous. Depending on the size of this intermediate area and upon the length of time each year when the ground surface is within the capillary fringe, soil evaporation will vary one year from another even though all other factors such as temperature, humidity, and precipitation remain the same.

Measurement of Soil Evaporation

From the above it appears that experiments to determine soil evaporation must be conducted under two different sets of conditions, namely, (1) for a freely draining soil and (2) for a soil whose surface is constantly being supplied with capillary water and which, therefore, has a higher evaporation opportunity.

For measuring soil evaporation under the first of these conditions, lysimeters are commonly used. A lysimeter is a tank, usually

from 4 to 6 ft square and from 3 to 6 ft deep, filled with earth, and imbedded with the surface practically flush with the ground. The bottom is funnel shaped and drains into a closed receptacle located in an underground gallery or passageway. The soil evaporation is the difference between the rainfall and the drainage.

To measure the evaporation from a soil whose surface is within the capillary fringe, tanks may be used that are equipped to maintain the water table at any desired elevation. The soil evaporation is determined by weighing the tanks at stated intervals and knowing the amount of water that was added in the interim.

Many experiments have been conducted in this country and in Europe to determine the evaporation from different kinds of soils and for various depths to the water tables.

In general, these experiments[1] indicate that for soils whose surface is always above the capillary fringe the annual soil evaporation (1) varies with the annual precipitation; (2) varies with the soil texture, being greater for fine compact soils than for loose coarse soils; (3) is negligible below a depth of 1 ft; (4) is approximately equal to one half of the annual precipitation.

For soils whose surface is supplied with capillary water, the evaporation depends upon the distance down to the water table and upon the soil texture. With the water table at or within a fraction of an inch of the ground surface, the evaporation approaches that from a free water surface. When the water table is 4 ft below the ground surface the evaporation rate is only slightly more than for greater depths. For intermediate depths to the water table the evaporation rate is influenced in the same manner as for freely draining soils.

In general, experiments that are designed to isolate and determine separately plant transpiration and soil evaporation must necessarily be conducted under conditions that rarely exist in nature. These two quantities are so interdependent that they cannot easily be separated. Vegetation reduces soil evaporation in

[1] For more detailed results of experiments on soil evaporation see the following references:

R. B. Sleight, Evaporation from the Surface of Water and Riverbed Materials, *J. Agr. Research*, Vol. X, pp. 209–261, Gov. Printing Office.

R. L. Parshall, Experiments to Determine Rate of Evaporation from Saturated Soils and River-Bed Sands, *Trans. A.S.C.E.*, **94**, 961–999.

S. Fortier, *Use of Water in Irrigation*, McGraw-Hill, 1915.

C. H. Lee, Transpiration and Total Evaporation, Physics of the Earth IX, *Hydrology*, McGraw-Hill, 1942.

two ways: first, by using some of the moisture that would otherwise be available for soil evaporation and, second, by reducing the wind velocity and thereby increasing the relative humidity at the ground surface. On the other hand, soil evaporation removes moisture that would otherwise be available for transpiration. Consequently any figures on soil evaporation and transpiration that have been determined separately are of little practical value, for they cannot be added in determining the total water losses from any basin.

The humidity gradient method of determining evaporation from water surfaces described on page 157 is ideally suited to measuring combined evaporation and transpiration rates. It may be hoped that carefully controlled applications of this method under various conditions will provide valuable information in regard to both soil evaporation and combined evaporation-transpiration rates.

WATERSHED LEAKAGE

The geological formation under many drainage basins is such that precipitation falling on one basin finds its way underground through fissures and water-bearing strata to an outlet either in a nearby or a remote drainage basin, or directly to the sea. This is called *watershed leakage.* Although it is believed to occur quite commonly, in many cases it is relatively unimportant, and the losses from one basin are frequently balanced by accretions from another.

Occasionally, however, these losses are large and may constitute a major loss from a basin as in the north branch of the Thunder Bay River and in the Rainy River in northern Michigan. Both these basins are underlain with limestone. Sinkholes are numerous, and much of the runoff finds its way into them and thence in all probability to Lake Huron. Similar conditions are known to occur in Wisconsin and in many other states.

A thorough knowledge of the geology of the basin usually provides the best evidence as to the probability of such leakage. Without such information, runoff records from the basin in question should be compared with records from similar basins nearby. The results of this comparison together with a study of the rainfall records and a knowledge of the soil, topography, and vegetal cover should give a good indication of the probability of any appreciable amount of watershed leakage.

TOTAL WATER LOSSES

As stated early in this chapter the total water losses from any drainage basin consist of interception, evaporation from land and water surfaces, transpiration, and ground-water outflow. Ordinarily these losses for any given drainage basin can be either measured or estimated more accurately and more easily collectively than separately. The relationship between these collective losses, rainfall, runoff, and storage, is expressed by the equation

$$\boxed{L = P - Q \pm \Delta S} \qquad (8)$$

in which L is the total losses for the period, P the total precipitation for the period, Q the total runoff for the period, and ΔS the increment in surface and subsurface storage during the period, the sign being plus for a decrease in storage and minus for an increase. All these quantities are expressed in inches depth on the drainage basin. The quantity ΔS is made up of three separate factors, viz., (1) the increment in surface storage in lakes, ponds, swamps, and streams; (2) the change in ground-water storage below the water table, which for any given area and period is equal to the product of the average change in level and the average porosity of the soil; and (3) the change in field moisture above the water table.

For any region, a study of the rainfall and ground-water records reveals the fact that well-defined cycles are maintained for each of these phenomena throughout the year. As an illustration, in southern Michigan about 18 per cent of the annual precipitation occurs in winter, 27 per cent in spring, 29 per cent in summer, and 26 per cent in fall. It is not to be inferred that these percentages hold rigidly each year but rather that they represent the average values and indicate the general cyclic variation in precipitation throughout the year.

Also the ground-water level usually reaches its maximum stage about the first of May and then is subjected to a more or less general decline until the latter part of September or the first of October. Furthermore, although during the year there may be fluctuations amounting to several feet or even more, the observed level on October 1 of any year does not ordinarily differ greatly from the long-term mean elevation for that date except as a result of unnatural changes or developments made within the basin. Also at this time of year the amount of surface storage and the soil-

moisture deficiency do not usually vary much one year from another. Because of these facts, the average annual water loss for any period of 5 yr or longer, beginning and ending with October 1, may be determined from equation 8, ignoring ΔS. If the values of P and Q have been accurately determined for the period, the average annual water loss should be correct within an inch or two at the most.

If the water loss for a single year is determined in this same manner the error will be considerably greater, depending upon the value of ΔS for that year. For instance, if the water table on October 1 of any year is 18 in. higher or lower than it was on the same date for the preceding year and the average soil porosity is $33\frac{1}{3}$ per cent, the change in ground-water storage alone amounts to 6 in. which added to the changes in soil-moisture and surface storage would probably give a total value for ΔS of 8 or 10 in. This large error should, however, be detected, and at least partial correction should be made. This is possible because, except during a period of surface runoff immediately following a storm, all the stream flow is derived from ground water and from surface storage. Inasmuch as the surface storage and the soil-moisture content also vary more or less in parallel with the ground-water storage, it is seen that, except when surface runoff is occurring at the end of the water year, the stream flow at that time provides a good index of ΔS for that year. If, for example, on October 1 of two successive years the entire flow is apparently coming from ground water and if that flow is approximately the same on those dates, the value of ΔS is negligible for that year. If, however, the flow is considerably greater or less at the end of the year than it was at the beginning, ΔS must be taken into account and correction must be made for it, or the annual loss for such a year will be correspondingly in error. The best procedure to be followed in making this correction depends upon the character of the data at hand in each case. If, for instance, sufficient ground-water levels are available, this change in ground-water storage can be approximated fairly well, depending upon knowledge of the average soil porosity. If this change represents an increase in storage, in all probability there will also be increases in the amounts of soil moisture and surface storage. On the basis of a knowledge of surface-storage conditions, depth and character of soil, and also the antecedent rainfall, these quantities may be

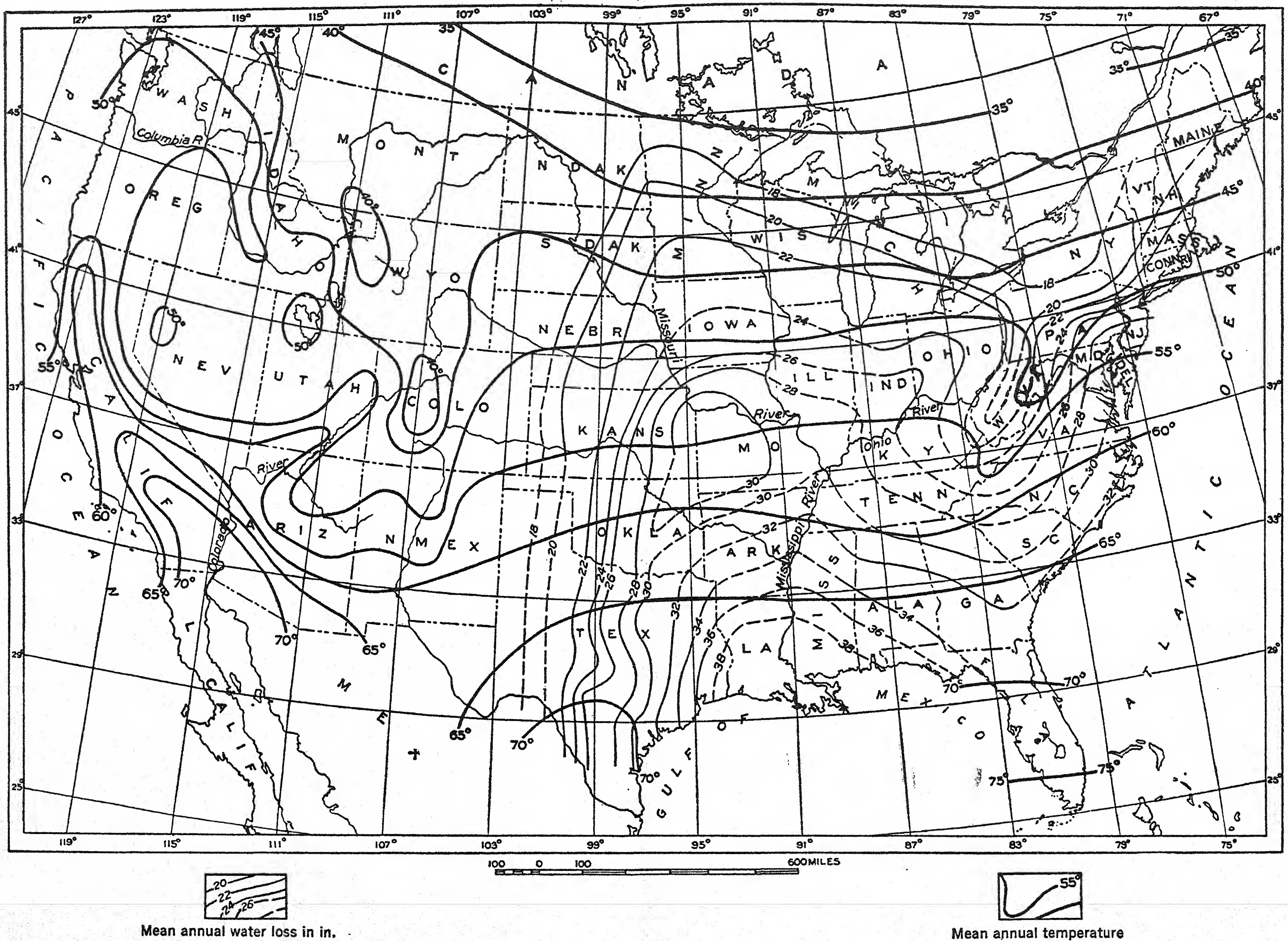


FIG. 53. Map of the United States showing generalized lines of mean annual water loss and lines of mean annual temperature. From *U. S. Geological Survey Water-Supply Paper* 846.

estimated and added to the change in ground-water storage to determine the total value of ΔS.

The value of ΔS in equation 8 may also be determined from a normal depletion curve of a stream (see page 24). Because the depletion curve represents the rate of depletion of ground-water storage, ΔS is equal to the area beneath the portion of the curve bounded by the discharge rates occurring at the beginning and the end of the period for which ΔS is being determined.

It thus appears that for any drainage basin the average annual water losses can be determined quite accurately for a long period of time from only the precipitation and runoff records. On the other hand, for the accurate determination of annual water losses, ΔS must be taken into account and its value may be at least closely approximated for each successive year from ground-water levels or from the hydrograph. For seasonal or monthly losses, the determinations will be less accurate.

Factors Affecting Total Water Losses

The total water losses for any year are the combined result of the influence of many factors, the principal of which are the total annual precipitation, its intensity and distribution throughout the year, temperature, humidity, and wind velocity. Of the last three factors, temperature is perhaps the most important.

In *U. S. Geological Survey Water-Supply Paper* 846 are given the results of a study made to determine the annual water losses on a large number of drainage basins in many of the eastern and central states. In this paper an attempt is also made to correlate water loss and temperature. Figure 53 has been reproduced from this paper. In this figure the isothermals are shown heavy, and the lighter lines are generalized lines of mean annual water loss. It will be observed that in a general way these two sets of lines are nearly parallel east of the Mississippi River, but, because of reduced precipitation and the resultant reduction in evaporation opportunity at points not far west of the Mississippi, the water-loss lines dip to the south and cross the isothermal lines nearly at right angles.

In Fig. 54 are shown the annual water losses from the drainage basins of the south branch of the Nashua River and of the Sudbury River, as given in this paper, each plotted against the mean tem-

perature for the corresponding years. This figure shows a general lack of correlation between annual water loss and annual temperature. There are two reasons for this. In the first place the quantities that are plotted as annual water loss are in fact annual

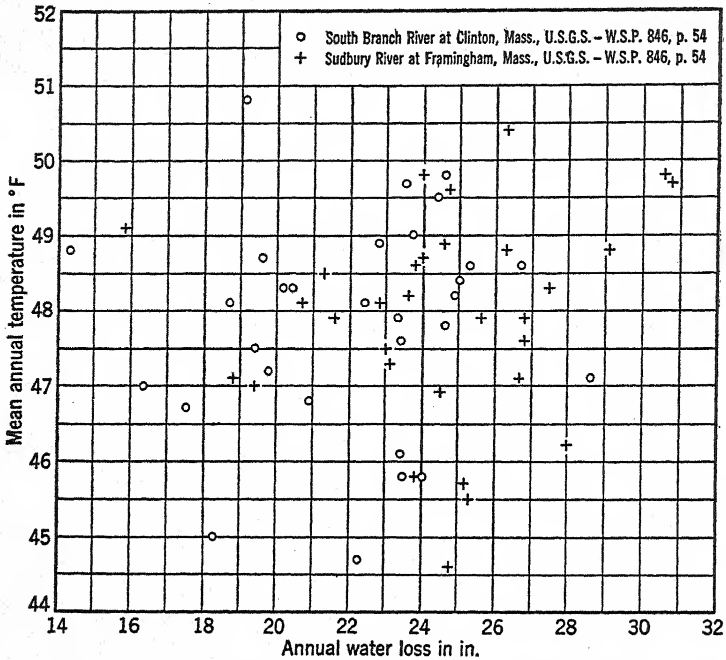


Fig. 54.

water loss plus or minus ΔS. It has been shown that for yearly periods ΔS may be as much as 8 or 10 in. Furthermore, in addition to temperature, water losses are influenced by amount and distribution of rainfall, depth to water table, humidity, wind velocity, and other factors.

The part that water losses play in the determination of yield will be discussed in Chapter VIII.

CHAPTER VI

INFILTRATION

Attention has already been called to the great diversity in the characteristics of the hydrographs of different streams. It has been shown that the flow of some is relatively steady throughout the year, whereas others are extremely fluctuating and some are intermittent. (See Figs. 9 and 10.) The most influential factor in determining the steadiness or variability of flow is the source of supply. If the principal source is from surface runoff, the stream is almost certain to have large floods and small rates of low-water flow. On the other hand, if the drainage basin is very permeable, as one having a coarse sandy soil, and if there is no relatively impervious stratum above the water table, there may be no surface runoff, and the flow will be well sustained and fairly uniform throughout the year.

The ability of a drainage basin to absorb and detain the water that falls upon it as rain or that comes to it from the melting of snows therefore provides a key to the character of the resulting hydrograph and becomes a matter of fundamental importance to the hydrologist. It appears that Horton was the first to recognize this fact and to suggest the theory of *infiltration capacity*,[1] which together with the theory of the unit hydrograph as proposed by Sherman provides the most useful tools that we now have in the field of hydrology.

Definition

Infiltration is the process whereby water enters the surface strata of the soil and moves downward toward the water table. This water first replenishes the soil moisture deficiency, and thereafter any excess moves on downward and becomes ground water. The maximum rate at which a soil in any given condition is capable of absorbing water in this manner is called its *infiltration*

[1] R. E. Horton, *Surface Runoff Phenomena*, Publication 101, Edwards Bros., Ann Arbor, Mich.

capacity. For drainage basins from which there is subsurface storm flow (see p. 20), the true infiltration capacity is not measured by the ability of the surface strata to absorb water but rather by the ability of the relatively impervious substratum to absorb and transmit water through it and on down to the water table.

Oftentimes rain falls at a rate that is less than the infiltration capacity of the soil. Then no surface runoff results. The prevailing infiltration rate, f', is equal to the infiltration capacity, f, only during and immediately following periods of excess rainfall. In determining infiltration capacity it is therefore necessary to consider only that time interval during which infiltration is occurring at capacity rate. This rate depends upon many factors which may be divided into two general classes: (1) areal factors or those which cause the infiltration capacity of one area to differ from that of another and (2) time factors or those which cause the infiltration capacity of any area to change from time to time. Not all the factors that affect infiltration capacity can be definitely assigned to one or the other of these classifications, for some belong in both. For instance, although soil type belongs in the first group, and soil moisture falls in the second, vegetal cover belongs in both because the character of the vegetation is never uniform over any large area nor is it the same throughout the year or from one year to another.

In the following pages will be presented a brief discussion of these various factors and some of the different methods of determining the infiltration capacity of any area. First, however, it is desirable to present a general picture of the manner in which infiltration occurs and describe some of its characteristics.[1]

Variability of Infiltration Capacity

One of the most striking characteristics of infiltration capacity is its extreme variability, with respect to both area and time. At the beginning of any storm the infiltration capacity, f_0, is likely to be high; it then decreases rapidly during the first half hour or so and finally levels off and approaches a constant value, f_c, after the next hour or two. The ratio of f_0 to f_c depends greatly upon permeability. For a well-compacted clay it is high, and for a

[1] For an excellent discussion of this subject see Howard L. Cook, The Infiltration Approach to the Calculation of Surface Runoff, *Trans. Am. Geophys. Union*, October 1946, pp. 726–747; and discussions.

coarse sandy soil it is relatively low. This characteristic may be more readily understood when it is realized that the infiltration capacity of any particular area is determined at the ground surface. Ordinarily it is the nature of the openings that exist in the top ¼ in. of the soil that determines infiltration capacity. However, where subsurface storm flow occurs it is the condition of the surface layers of the relatively impervious strata which determines the infiltration capacity. Inasmuch as in all cases f is dependent upon the character and condition of this layer it is readily understandable that any disturbance of that layer may completely change the infiltration capacity.

For rock, shale, and well-compacted clay it may approach zero. On the other hand, for clean, coarse sand or for muck or peat, the infiltration capacity may exceed the most intense rate of rainfall to which these soils are ever subjected. Between these two extremes are to be found soils having every possible capacity. Especially in glaciated areas, soils possessing widely varying infiltration capacities occur within short distances of one another.

Furthermore, within an area having a single homogeneous soil, the infiltration capacity may vary greatly because of different land use and different types of cover. Also it varies from time to time throughout the year, more or less seasonally, and also from year to year as a result of the activities of boring animals, decaying roots of vegetation, varying moisture content of the soil, and many other factors. From all this it appears that infiltration capacity is not a permanent characteristic of a drainage basin that is comparable with size, length, and similar properties. Instead, when the objective is the determination of flood flows, it is necessary to determine the range within which the average infiltration capacity of a basin varies from time to time; then, knowing the factors that cause those variations and knowing the influence of each, one can make a reasonable estimate of the value that should prevail at any given time. Or, with the minimum, maximum, or average infiltration capacity existing during any design storm, it is possible to determine the greatest, smallest, or average flood that will result from such a storm. (See Chapter IX.)

Forces Affecting Infiltration

When rain falls on a pervious soil, gravity tends to pull the water down through the myriads of tiny interstitial channels that

lead from the ground surface to the water table. If there is no soil-moisture deficiency, gravity is the only active force in this process. When, however, the soil surface has become dry as a result of evaporation and transpiration, the molecular attraction of the soil grains is added to the force of gravity, thereby increasing the infiltration capacity. This increase is only temporary, however. Each soil grain is capable of attracting and holding against gravity only a certain limiting thickness of film. After any soil grain has been thus supplied, its molecular force is thereafter used only in holding onto that film and is totally inactive in producing further infiltration until its surface film has been removed or reduced by evaporation and transpiration.

It therefore appears that molecular attraction increases the infiltration capacity of a soil only during the early stages of a rain. The extent of this influence depends upon the character of the soil. For a compact, finely textured clay, it is much greater than for a coarse sand. The reason for this is that the thickness of the film of water retained by molecular attraction is practically independent of the size of soil grain, and the volume of water retained in unit volume of soil is therefore directly proportional to the aggregate surface area. If all soil grains are considered spherical, the number contained in any given volume varies inversely as the cube of the diameter. On the other hand, the surface area of an individual grain varies as the square of the diameter. Therefore, the aggregate surface area of all the soil particles contained in unit volume, and consequently the volume of water retained by molecular attraction, varies inversely as the diameter of the soil grains. Meinzer[1] calls attention to the fact that, in a cubic foot of sand composed of grains 1 mm in diameter, the total surface area of the soil particles is about 1000 sq ft; whereas, in a cubic foot of sand composed of grains 0.02 mm in diameter, the total surface area is about 50,000 sq ft or more than an acre; and, if the diameter is only 0.001 mm, the surficial area is about 1,000,000 sq ft or more than 20 acres. These figures give an idea of the tremendous influence of texture upon the ability of a soil through molecular attraction to retain water against the action of gravity. This fact explains to a considerable extent why the infiltration capacity of a dry finely textured soil decreases so much more rapidly during the

[1] Oscar Edward Meinzer, The Occurrence of Ground Water in the United States, *U. S. Geological Survey Water-Supply Paper* 489, 1923.

early stages of a storm than does that of a soil of coarse texture. It also explains why a fine clay retains so much more water against gravity and can therefore keep vegetation supplied with its moisture requirements throughout a much longer period without wilting than can a coarse sand.

Other Factors Affecting Infiltration Capacity

In addition to the difference in the forces that are active at different times in causing infiltration, a number of other factors affect and determine the infiltration capacity of any given area, all of which vary from time to time. These factors include (1) the moisture content of the soil, (2) the shrinking or swelling of the colloidal material in the soil, (3) the effect produced by rain upon the soil surface, (4) changes in macrostructures resulting from animal borings, the decay of vegetal roots, sun-checking, and the dissolution of minerals, (5) condition of the vegetal cover, (6) cultivation, (7) compression of entrapped air, and (8) temperature changes. Most of these changes occur more or less seasonally although some of them are much more rapid and their effects are felt from day to day or even from hour to hour. Nearly all these factors affect infiltration capacity as a result of the changes produced in the effective sizes of the openings through which water enters the surface strata of the soil.

Moisture Content. The soil is capable of holding a considerable amount of water against the pull of gravity. Harding[1] states that the results of field observations made in many different places show that the maximum depth of water that can be retained against gravity is about 1.25 in. per ft depth of soil. On the other hand, Harris and Jones[2] report that as much as 18 in. of precipitation may be stored in the upper 6 ft of soil or 3 in. per ft of depth. On the basis of these two figures, the maximum weight of the pellicular water retained per cubic foot of soil is between 6.5 and 15.6 lb, and, for a soil having a porosity of 25 per cent, the corresponding reductions in channel area are 42 and 100 per cent respectively. For a soil having a porosity of 30 per cent the reductions range between 35 and 83 per cent, and for a porosity of 40 per cent the reductions in channel area are between 26 and 62 per cent. It is clear from these figures that many of the smallest

[1] S. T. Harding, *Soil Sciences*, October 1919, pp. 303–312.

[2] F. S. Harris and J. W. Jones, *Utah Agri. Expt. Sta. Bul.* 158, July 1917.

interstitial spaces are not available for the percolation of water after they receive their initial supply of moisture, thus causing the infiltration capacity of moist soil to be less than that of dry soil. This phenomenon also provides one of the reasons why the capacity of coarse, sandy soils is high and more nearly constant throughout a storm, whereas that of finely textured and well-compacted soils is much more variable and, although high at the beginning, diminishes rapidly during the first hour or two and thereafter approaches a nearly constant value.

Shrinking and Swelling of Colloids. Most finely textured soils contain more or less colloidal material. This is especially true of clay, silt, and loam. Upon becoming wet these colloids swell and tend to choke up the tiny pores through which the infiltering water enters the soil. This swelling of colloids provides another reason for the much greater reduction in infiltration capacity occurring in colloidal clays as compared with sandy soils, especially during the early stages of a storm. When such colloidal soils have been thoroughly wetted and then subjected to the hot rays of the sun, the reverse process occurs; the colloids shrink, and cracks or sun-checks develop. Vegetation and humus protect the soil surface from this action, but, where these protective coverings are absent and the bare soil is exposed to the direct rays of the sun, many deep and wide sun-checks form in the surface of such colloidal soils and greatly increase their infiltration capacity during the early stages of the storm.

The Effect of Rain. Where the soil is not protected by vegetal cover, rain beats down upon it, compacts it, and reduces its permeability. Dust particles, soot, and tiny bits of decayed vegetation that the rain has filtered from the air and has collected from the surfaces of vegetation, buildings, and pavements are washed into the tiny pore spaces of the already compacted surface strata and thus further reduce the infiltration capacity. The surface of a cultivated soil that before a rain was loosely textured will afterwards be found coated with a densely compacted crust of earth $\frac{1}{8}$ in. or more in thickness. A heavy vegetal cover will, however, tend to protect the soil surface and prevent this compaction. More effective though in providing this protection and in maintaining a high infiltration capacity is the thick mulch of leaves, grass, and decayed vegetation that usually covers the ground surface throughout a forest.

Other Factors Affecting Structure. Macro openings in the soil result principally from (1) the activities of boring animals, (2) the decay of vegetal roots, (3) cultivation, (4) compaction and (5) chemical action. Each of these causes has its own peculiar type of variation with no uniformity in their periods of occurrence or in the extent of their influence.

Boring animals include earthworms, crayfish, ants, bugs, beetles, ground moles, gophers, woodchucks, and a great many others including the countless millions of tiny insects that are found throughout the surface humus and especially beneath forest litter. Each of these types of animals has its own habits, and the effects upon infiltration are not always parallel. For instance, earthworms and crayfish bore deeply into the soil during drought in search of water and come to the surface only after rains. On the other hand, the activities of ants and certain other animals are more in evidence during periods of drought than at any other time.

The decaying of vegetal roots extends over long periods of time. The period of influence depends upon the character of the vegetation, but the greatest increase in infiltration capacity resulting from this cause occurs most commonly in the fall.

In a study of the factors affecting the infiltration capacities of a dozen different soil types, Lewis and Powers[1] found that the effect of cover may be even more important than soil type. That this is true may be easily seen by a comparison of the condition of a clay or loamy soil that is barren with one that is thickly forested. In the first case the surface of the ground is hard and closely compacted from the successive beatings of rains and by the tread of animals. When protected by a thick covering of forest litter, however, the same kind of soil will be loose and pervious and will have a high infiltration capacity. A protected soil may have many times the infiltration capacity that it has when it is barren.

In a study quite similar to that discussed in the preceding paragraph, Duley and Kelly[2] found that there may be more variation in infiltration capacity resulting from surface conditions on a single soil, depending upon whether or not that soil is under cultivation, than would be found on different soils having the same surface conditions.

[1] M. R. Lewis and W. L. Powers, *Soil Sci. Soc. Am. Proc.*, 1938, pp. 334–339.
[2] F. I. Duley and L. L. Kelly, *Nebr. Agri. Expt. Sta. Res. Bul.* 112, May 1939.

Dean C. Muckel[1] reports a series of experiments that were conducted on three plots of the same type of soil, one of which was covered with native vegetation consisting chiefly of wild oats, wild clover, and Bermuda grass; a second was denuded of all vegetation and cultivated so as to form shallow furrows about 8 in. apart; and a third was denuded of all vegetation and leveled but not cultivated. Throughout the experiments the infiltration capacity of the vegetated plot was about four times as great as that of the cultivated plot and about two and a half times that of the denuded plot.

Cultivation has the effect of temporarily loosening the structure of the surface strata. However, after a cultivated soil has been subjected to the action of winds and water for a period of a week or two, its surface becomes consolidated and its infiltration capacity is reduced. Lewis[2] tells of experiments on an irrigated tract in which the soil became so nearly impervious that the moisture content below a depth of 1 ft not only did not increase but sometimes actually decreased while water was still flowing over the surface.

The mechanical action resulting from the tread of animals or from the wheels of vehicles tends to compact the surface of a soil and render it less pervious. A frequently occurring example is an overstocked pasture. Here, the vegetation becomes sparse and the soil is poorly protected against the grazing animals.

In a highly mineralized soil it is possible that dissolution will cause a slight increase in infiltration capacity. On the other hand, some cementation occasionally takes place, resulting in a decrease in infiltration capacity.

Effect of Entrapped Air. First, let us assume that the ground surface and the water table are parallel and that the soil is of uniform porosity throughout a given area so that the entering water advances as a sheet of uniform thickness completely filling the voids. If the water table is at D ft below the ground surface, and the infiltering sheet of water has penetrated to a depth of d ft, the pressure in the entrapped air will be, from the laws of hydrostatics, neglecting capillarity,

$$p = 14.7 + 0.433d$$

[1] D. C. Muckel, *Trans. Am. Geophys. Union*, 1936, pp. 471–474.
[2] M. R. Lewis, *Ore. State Hort. Soc. Proc.*, 1936, pp. 164–172.

Also, in accordance with Boyle's law,

$$14.7D = p(D - d)$$

Solving these two equations simultaneously

$$d = D - 34$$

From this equation it is seen that, under the assumed conditions, infiltration cannot occur unless the water table is more than 34 ft below the ground surface. Actually, however, the ground surface is seldom parallel with the water table. On the contrary it is irregular and contains countless ridges and mounds, and furthermore, instead of the infiltering water entering as a sheet of uniform thickness, it penetrates very unevenly, as shown in Fig. 55. The

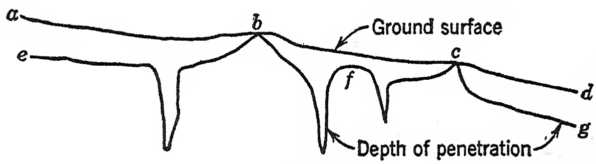


FIG. 55.

tops of the ridges and mounds, as at b and c, act as vents through which some of the compressed air escapes at the same time that water is penetrating throughout the adjacent area. There can be no question, however, but that compression of entrapped air does tend to retard infiltration and is one of the factors that cause a reduction in infiltration capacity as the storm progresses, although often it is of relatively minor importance.

Effect of Temperature Changes. Infiltration capacity increases with temperature because of the decrease in the viscosity of water. Inasmuch as the flow is usually laminar, at least for compact, finely textured soils, the rate of infiltration should vary inversely as the kinematic viscosity. On this basis, for an increase in water temperature of 50° F, the infiltration capacity should approximately double. Experimental verification of this conclusion is, however, lacking at present.

Annual and Seasonal Changes

The average infiltration capacity of a drainage basin changes both annually and seasonally. The annual change results principally

from changes in land use, from changes in the character of the annual vegetation, and from the advancing stage in the development of the perennial vegetation. Except for major changes in land use, these variations occur slowly and their effects become discernible only after a period of years. However, when an entire

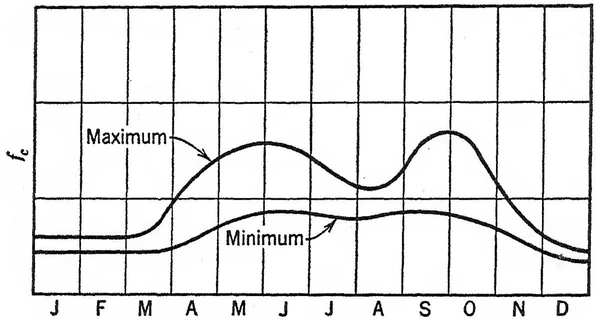


Fig. 56.

basin or any large portion of it is subjected to a sudden change in land use, such, for instance, as being deforested and put into agriculture, the resulting change in infiltration capacity may be marked.

In the preceding discussion it may have been observed that the various factors that affect infiltration capacity are not constant throughout the year. For instance, the moisture content of the soil reaches a maximum in spring and a minimum in fall. Also the changes in the macrostructures resulting from animal borings, from the decay of vegetal roots, from the condition of the vegetal cover, from cultivation, and from temperature affect infiltration capacity more or less seasonally. They do not, however, exert their influence in parallel. In Fig. 56 is shown, schematically (i.e., not to scale), Horton's idea of the combined, net result of all these influences. The upper graph represents the maximum, and the lower represents the minimum values of f as they may be expected to vary throughout the year. For coarse, sandy soils these two graphs would perhaps be closer together and would show considerably less seasonal variation, whereas for a compact clay they might be even more divergent and variable than here shown. Justification for the dip in these graphs during the summer months is questionable.

In Fig. 57 is shown a curve representing the seasonal variation of f during 1938–1939 for a small watershed at Edwardsville, Illinois, as derived by Horner and Lloyd.[1] A series of graphs of this type, drawn to scale and showing the maximum and minimum values of f for a variety of drainage basins of different physical characteristics and climatic conditions would be of great practical

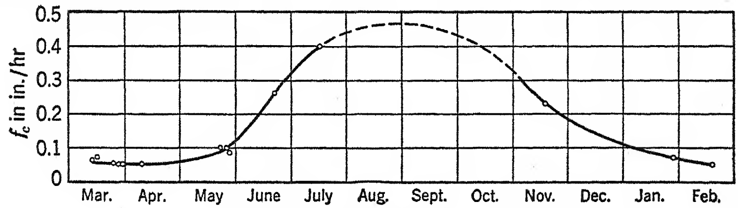


FIG. 57.

use. It is hoped that, on the basis of the data now being collected by the various government agencies, this need will soon be filled.

Methods of Determining f

There are two general methods of determining infiltration capacity. The first is with an infiltrometer of which there are many different kinds, but in all of which water is artificially applied to a small area or sample plot, and the rate of infiltration is determined more or less directly. The second is by analysis of the hydrograph of runoff resulting from a natural rainfall on a drainage basin.

The first of these methods is helpful in determining the effects of land use, slope, vegetal cover, and other variable factors over which it is desirable to have control in making studies on the prevention of soil erosion, flood reduction, underground storage, and similar problems in the solution of which hydrograph analysis often cannot be used. For reasons that will be explained presently, records obtained with infiltrometers are of value qualitatively rather than quantitatively. At least with our present limited knowledge they cannot be safely used for computing runoff from rainfall. On the other hand, infiltration capacities determined by hydrograph analysis are extremely useful for this purpose.

[1] W. W. Horner and C. Leonard Lloyd, Infiltration-Capacity Values as Determined from a Study of an Eighteen-Month Record at Edwardsville, Ill., *Trans. Am. Geophys. Union*, 1940, Part II, pp. 522–541.

Infiltrometers

Infiltrometers are of two general classes: (1) those in which the rate of intake is determined directly as the rate at which water must be added to maintain a constant depth, usually about ¼ in., within the infiltrometer, and (2) rain simulators.

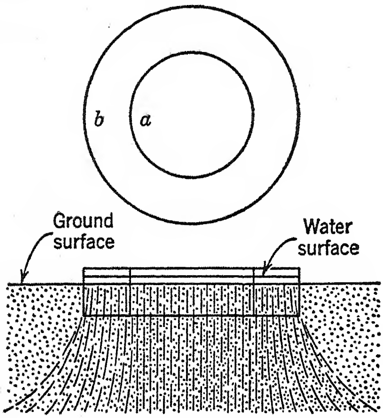


FIG. 58.

Under the first general class, the most common types consist either of two concentric rings or of a single tube. In the first type, two shallow concentric rings of sheet metal, usually ranging from 9 in. to 36 in. in diameter, are placed with their lower edges a few inches below the ground surface and with the upper portion projecting above, as shown in Fig. 58. Water is then applied in both compartments, *a* and *b*, and is always kept at the same level in both. The function of the outer ring is to prevent the water within the inner space from spreading over a larger area after penetrating below the bottom of the ring. From the rate at which water must be added to the inner ring in order to maintain a constant level, the infiltration capacity and its manner of variation is determined.

In the second type a single tube is placed in the ground at a depth at least equal to that to which the water penetrates during the experiment, and therefore no spreading can occur. The rate at which water must be added to maintain a constant depth within the tube is then observed.

A number of other types of infiltrometers have been used, in which water is applied to a small enclosed area in the form of a sheet of water of definite thickness. Although these devices provide a simple and direct method of determining the capacity at which the ground is capable of absorbing water under those conditions, the results obtained are of value only in the determination of the influence of land use, vegetation, slope, and other physical variables. This results principally because (1) the effect of the beating of raindrops, with the resulting compaction and inwash

of fine materials, is absent; (2) the effect of compression of the entrapped air is absent because of lateral escape; (3) it is impossible to place a ring or tube in the ground without disturbing the soil structure near the boundary. Because of the small area contained within this type of infiltrometer, the disturbed area may be an appreciable percentage of the total and the results are correspondingly affected.

Rain Simulators

In order to eliminate the above objectionable features to as great an extent as possible, various types of infiltrometers have been devised in which water is applied by sprinkling at a uniform rate that, except perhaps for a brief initial period, is in excess of infiltration capacity. The area covered usually ranges from 1 sq ft to 0.01 acre. The varying rate of the resulting surface runoff is measured, and from these data the f curve is derived.

Horton was one of the first to use this type of infiltrometer. Even before World War I he determined infiltration capacities of small circular tracts about 10 ft in diameter to which water was supplied by a sprinkling system consisting of a number of radial horizontal pipes about 6 ft above the ground, rotating about a vertical axis, and driven by the reaction of a series of horizontal jets. In the more recent investigations that have been conducted by the Soil Conservation Service and other agencies, the water has been supplied through a system of stationary pipes designed to secure a practically uniform distribution of water over the plot, the proper size of drops, and the desired height of fall. Following are brief descriptions of some of the more commonly used types of infiltrometers of this general class.

Pearse. In this type the water is applied through a perforated pipe at the upper edge of a plot 1 ft square, and the runoff is collected at the lower edge. The rate of application is controlled by maintaining a constant elevation of water in the supply tank.

Modified North Fork. Water is applied through sprinklers to a rectangular area of 2.5 sq ft. The rate of application is measured by means of six rain gages, each 1 in. in diameter, and the runoff is collected at the lower end of the plot and measured.

Rocky Mountain. This instrument is quite similar to the modified North Fork infiltrometer except that the water is applied to a plot 2 ft by 4 ft and is measured by twelve 1-in. gages.

Modified Type F. This type of infiltrometer, also developed by the Soil Conservation Service, has perhaps been more commonly used than any other, and probably it measures the true infiltration capacity more accurately. Simulated rainfall is applied to an area approximately 6 ft by 12 ft and is measured in two trough gages, each 12 ft long and 1 in. wide, properly centered over the area. The runoff is automatically recorded.

Many other types of infiltrometers have been used, and unquestionably still more will be developed. Their use has not been standardized, and so it cannot be said that any particular type is best. The objections to the use of infiltrometers of the ring and tube type apply also to the sprinkling type although to a lesser degree. It has been quite definitely established[1] that the results obtained by infiltrometers are qualitative and not quantitative. In other words, with infiltrometers it is possible to determine the relative effect of any change in land use or of any other controllable physical characteristic. It is not possible, however, to determine satisfactorily the runoff from a drainage basin by the direct use of infiltration capacities as determined by infiltrometers. This is because of the factors stated above and also because it is difficult to obtain sufficient samples to determine the average value of f for a large basin. For basins in which there is subsurface storm flow, values of f as determined by infiltrometers would in all probability give grossly misleading results.

Hydrograph Analysis

Accurate data on the varying intensities of rainfall during any given storm together with a continuous record of the resulting runoff provide an excellent basis for the determination of infiltration capacity. Furthermore, infiltration capacities thus determined may, with confidence, be used to determine the hydrograph of runoff resulting from any given storm occurring on the same basin under similar conditions. Before explaining this method of determining infiltration capacity by hydrograph analysis, it may be well to review briefly the runoff process.

When rain starts falling on any particular basin, it does so at varying intensities over the area. In the beginning a portion is intercepted by trees, grasses, other vegetation, buildings, and so

[1] H. G. Wilm, Methods for the Measurement of Infiltration, *Trans. Am. Geophys. Union*, 1941, pp. 678–686.

forth, and never reaches the ground surface. An additional portion flows into depressions, both small and large, and later either evaporates or soaks into the ground. The remainder starts flowing overland toward the stream channels, but in so doing a part of it infiltrates into the soil.

The rain that is not used up by either interception, depression storage, or infiltration eventually finds its way overland to the stream channels but only after some delay. In other words, there is a lag between the time when excess rainfall occurs and the time when that water appears as surface runoff at the outlet of the drainage basin. Furthermore this delay varies throughout the basin and throughout the storm period. For instance, excess rainfall may have started at noon and continued for 2 hr, and, although surface runoff may have started at 1 PM and continued for 2 days thereafter, the period during which infiltration was occurring at capacity rate over the entire basin may have started at noon and ended, let us say, at 3 PM. Then during the period, perhaps from 3 PM to 9 PM, while surface detention was flowing overland toward the stream channels, infiltration was occurring at capacity rate over an ever-decreasing area.

Because of this varying period of delay or lag between the time when rain falls on a basin and the time when the excess appears at the outlet as surface runoff, it is often difficult, and in large drainage basins it is usually impossible, to determine the exact manner in which infiltration capacity varies throughout a storm. For smaller basins in which the hydrograph is quick to respond to the varying intensities of rainfall, the actual manner in which f varies throughout the storm can often be quite accurately determined, but for larger basins it is possible to determine only the average infiltration capacity, f_a.

Determination of the f-Curve for Small Basins

A method of determining the infiltration-capacity curve for small drainage basins as suggested by Horner and Lloyd[1] will be explained and, with minor modifications, illustrated by the following example. In Fig. 59 is a graph showing the variation in

[1] W. W. Horner and C. Leonard Lloyd, Infiltration-Capacity Values as Determined from a Study of an Eighteen Month Record at Edwardsville, Illinois, *Trans. Am. Geophys. Union*, 1940, pp. 522–541.

rain intensity as it occurred on the Controlled Watershed,[1] at La Crosse, Wisconsin, on April 27, 1938, and the resulting hydrograph of surface runoff. The area of this watershed is only about 2.7 acres. Because of its small size, each period of intense rainfall produces a separate peak in the hydrograph. It will be observed, however, that the first two periods of intense rain occurred in such

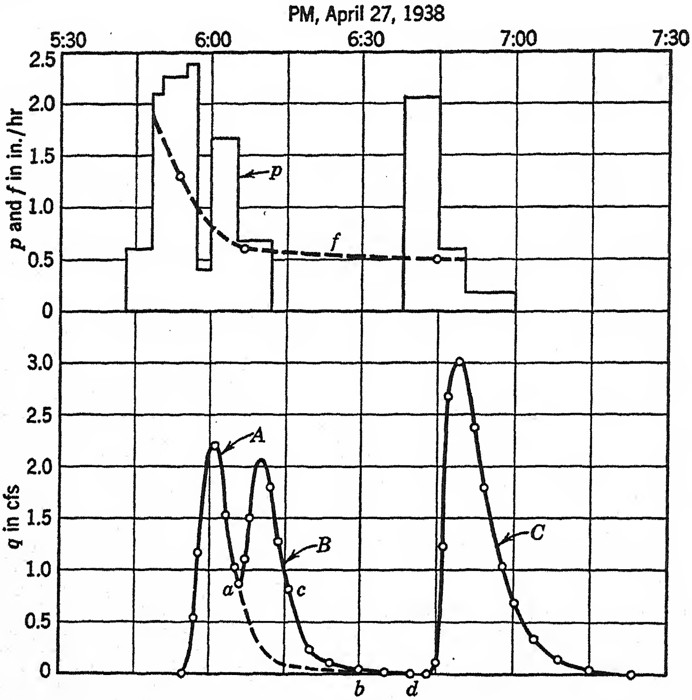


FIG. 59.

rapid succession that the resulting hydrographs overlap. The recession curve of *A* may be completed by drawing *ab* parallel with *cd* as shown in the figure. The areas beneath graphs *A*, *B*, and *C* when reduced to the proper units show depths of runoff on the basin of 0.10 in., 0.09 in., and 0.18 in. The depths of rain that produced these runoff volumes are, respectively, 0.34 in., 0.22 in., and 0.29 in. After deducting the respective depths of runoff from

[1] Hydrologic Studies, Upper Mississippi Valley Conservation Experiment Station, La Crosse, Wis., *U. S. Department of Agriculture Soil Conservation Service Tech. Paper* 29, November 1939.

each of these values, the total infiltration, F, corresponding to each of these periods of intense rain is found to be 0.24 in., 0.13 in., and 0.11 in. Strictly speaking, these are not true values of infiltration, for they include interception and depression storage. Nevertheless, inasmuch as it is impossible to determine the magnitude of interception and depression storage and since they never become surface runoff anyway, they may therefore conveniently be included with and considered as infiltration. Furthermore, by so doing, the values of f thus determined may be applied directly to any design storm to find the resulting runoff without the necessity of deducting values of interception and depression storage, which at any time would have to be estimated.

In order to reduce these values of F to capacity rates, f, it is necessary to divide each by the average length of time during which infiltration was occurring at capacity rate over the entire basin. These periods of infiltration start at the beginning of excess rainfall and continue until some time after it ends. At the moment that excess rainfall ends, if the storm covers the whole area, infiltration is occurring at capacity rate over the entire basin, but soon thereafter this area starts shrinking from the outer boundaries toward the stream channels. Inasmuch as any segment of this area approximates a triangle with its vertex at the stream channel and its base on the divide, Horton assumed that the equivalent period during which the same volume of residual infiltration would occur on the entire basin is equal to one third of the period that elapses from the end of excess rainfall until the end of overland flow. Horton[1] has called attention to the fact that overland flow ends about at the point of inflection on the recession side of the hydrograph. The reason that he selected this point is that, after the cessation of overland flow, the hydrograph represents only outflow from channel storage and is therefore an exhaustion curve and as such must be concave upward. By this method of determination, the duration of infiltration, t_1, t_2, and t_3, corresponding to each of these three periods of excess rainfall, were found to be 11, 13, and 13 min respectively. Therefore,

$$(0.34 - 0.10) \times \frac{60}{11} = 1.31 \text{ in. per hr}$$

[1] R. E. Horton, *Surface Runoff Phenomena*, Publication 101, Edwards Bros., Ann Arbor, Mich., p. 34.

$$(0.22 - 0.09) \times \frac{60}{13} = 0.60 \text{ in. per hr}$$

$$(0.29 - 0.18) \times \frac{60}{13} = 0.51 \text{ in. per hr}$$

Plotting each of these values at a time $t/2$ after the beginning of excess rainfall for each of these periods, the f curve, shown in Fig. 59, is obtained.

This is a typical infiltration-capacity curve, starting with a high value, f_0, because of the initial soil conditions and also because of the heavy initial demands of interception and depression storage, which are here included with infiltration, then dropping rapidly during the early stages of the storm and finally leveling off and approaching a constant value, f_c. In this particular case, f_0 is nearly four times as great as f_c. For different basins this ratio has, however, a wide range, depending upon the amount of interception and depression storage and upon the type, texture, and condition of the soil. Horton[1] has called attention to the fact that this curve is of the exhaustion type, that it approaches a constant value, usually after a period of 1 to 3 hr, and that it may be represented by an equation of the form

$$f = f_c + (f_0 - f_c)e^{-kt}$$

in which e is the Napierian base, k is a constant for a given curve, and f is the infiltration capacity in inches per hour at any time, t, in hours. (For an explanation of methods of determining f_0 and k in this equation see page 280.)

Attention should be called to the fact that the infiltration-capacity curve shown in Fig. 59 is, in reality, not an f curve but, rather, it is an f_a curve. In other words, it does not represent instantaneous values of infiltration capacity as it existed throughout the storm period, but instead it represents the average infiltration capacity for each of the several periods of high storm intensity. It is possible that, during the period between showers, from 6:12 to 6:38 PM, some recovery in infiltration capacity may have occurred, and, if such was the case, this curve should be made to dip and then rise again. In no case is it possible to determine the exact behavior

[1] R. E. Horton, Analysis of Runoff Plat Experiments with Varying Infiltration Capacity, *Trans. Am. Geophys. Union*, 1939, Part IV, p. 693.

because of the variable period of delay between the time when water falls as rain and the time when it appears as surface runoff at the outlet of the drainage basin. However, the divergence of the f_a curve shown in Fig. 59 and the true f curve is probably of minor significance.

Infiltration Capacity of Large Basins

For drainage basins that are larger than those for which the rain intensity may be considered as being uniform over the entire area, Horton[1] proposed the following method for determining the average infiltration capacity, f_a, that existed during any given storm. The procedure presumes that enough rainfall records are available on the basin to represent the rainfall variation satisfactorily and also that at least one of these records was obtained by an automatic recording gage. The fact should be brought out at this point and emphasized that, for drainage basins in which there is subsurface storm flow (see page 20), any value of f_a obtained by this method is not the infiltration capacity of the ground surface, but for all practical purposes it is the average infiltration capacity of the relatively impervious substratum. Therefore, values so derived may be directly applied to any design storm to determine surface runoff from that same basin but should not be used for runoff studies on any other basin on which subsurface conditions are probably quite different.

This method is based upon two assumptions: first, the fact that, in great general storms producing major floods on large basins, the rain intensity patterns at adjacent stations are very similar; second, the fact that surface runoff approximately equals the difference between the rainfall and the infiltration that occurs during the period of rainfall excess. In other words, the rain that falls during and immediately following the period of rainfall excess, but infiltrates during the subsequent period of overland flow, is ignored.

Even after the above assumptions are made, and even though the infiltration capacity were uniform over the entire basin, it still would not be permissible to divide the difference between the total rainfall and the total runoff by the duration of rainfall excess as shown by the recording gage because the period of rainfall excess

[1] R. E. Horton, Determination of Infiltration Capacity for Large Drainage Basins, *Trans. Am. Geophys. Union*, 1937, Part II, pp. 371–385.

varies throughout the basin and is different at the various stations. This fact must be taken into consideration or the result would not represent the true infiltration capacity at all. The method herein described consists of finding a value of f that, when multiplied by the period of rainfall excess and subtracted from the total rainfall for the same period, will leave a remainder equal to the total surface runoff.

It is further assumed that the periods of rainfall, but not of rainfall excess, are approximately the same at all stations, although not necessarily simultaneous. For a method of correcting for the varying duration of rainfall that is likely to occur at the different stations during a convectional rain, or during a storm accompanying a rapidly moving cold front, reference should be made to the original paper.

The station at which the recording gage is located will be called the base station, and those at which only 24-hr total rainfall records are available will be called substations. It is first necessary to determine which rainfall stations should be considered. This may be done by the Thiessen method as described on page 86. The daily rainfall records at the various stations are then adjusted to a common 24-hr basis as explained on page 80.

The average infiltration capacity may first be approximated by subtracting the total surface runoff from the total rainfall as recorded at the base station and dividing the difference by the period of rainfall at the base station. In doing this, any period of light rainfall, either at the beginning or at the end, should be ignored. The percentage of the total rainfall that fell during each hour at the base station is then computed. Next, a total rainfall of some even number of inches that is about equal to the average depth for the given storm is assumed, and the depth of rain falling each hour at the base station is determined by multiplying this assumed depth by the various percentages as above determined. The results of a set of these computations as given by Horton in the original paper are shown in Table 9. In Column 2 are shown the hourly depths of rainfall, and in Column 3 are shown the percentages of the total. In Column 4 are shown the corresponding depths of rain that would fall during each hour of a 4-in. storm having an intensity pattern similar to that of the recorded storm. In Columns 5, 6, 7, 8, and 9 are shown the depths of rainfall excess that would occur each hour as a result of this same storm, but for

infiltration capacities of 0.1, 0.2, 0.3, 0.4, and 0.5 in. per hr, respectively.

In Fig. 60, the total rainfall excesses as shown in Columns 5, 6, 7, 8, and 9 are plotted against a storm rainfall of 4 in. Similar tables are prepared for storms having depths of 2 in., 3 in., 5 in., and 6 in., and the results are plotted, producing the curves shown in the figure. It should be noted that Table 9 shows that for storms

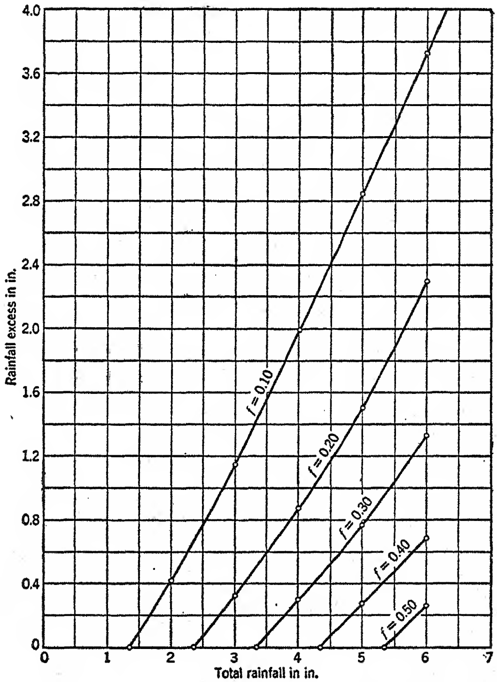


FIG. 60.

of this pattern having a total depth of 4 in. there would be no surface runoff if the infiltration capacity is 0.4 in. per hr or more.

From the curves shown in Fig. 60, the rainfall excess for the various infiltration capacities can be determined for any total depth of rain. In Table 10, Column 2, are shown the total depths of rainfall at each of the substations within the basin during this storm. In Columns 3, 4, and 5 are shown values of rainfall excess as obtained from the curves in Fig. 60 for each of these depths of total rainfall. The average of the values in any column represents

TABLE 9

Recorded Rainfall at Base Station				p = 4.0 in. Infiltration-Capacity f, in./hr				
Hour (1)	Amount, in. (2)	Portion of Total (3)	Total Amount, in. (4)	0.10 (5)	0.20 (6)	0.30 (7)	0.40 (8)	0.50 (9)
				Rainfall Excess				
5 AM	0.05	0.013						
6	.05	.013						
7	.03	.008						
8	.02	.005						
9	.05	.013						
10	.05	.013						
11	.07	.019						
12 M	.08	.022	0.088	0				
1 PM	.20	.054	.216	0.116	0.016	0	0	0
2	.20	.054	.216	.116	0.016	0	0	0
3	.13	.035	.140	.040	0	0	0	0
4	.12	.032	.128	.028	0	0	0	0
5	.03	.008						
6	.02	.005						
7	.15	.040	.160	.060	0	0	0	0
8	.15	.040	.160	.060	0	0	0	0
9	.35	.094	.376	.276	0.176	0.076	0	0
10	.35	.094	.376	.276	0.176	0.076	0	0
11	.35	.094	.376	.276	0.176	0.076	0	0
12 PM	.35	.094	.376	.276	0.176	0.076	0	0
1 AM	.25	.068	.272	.172	0.072	0	0	0
2	.25	.068	.272	.172	0.072	0	0	0
3	.15	.040	.160	.060	0	0	0	0
4	.15	.040	.160	.060	0	0	0	0
5	.05	.013						
6	.05	.013						
7	.02	.005						
8	.01	.003						
Totals	3.73	1.000		1.988	0.880	0.304	0	0

the average excess rainfall on the entire basin. In Fig. 61 these averages are plotted against infiltration capacity. From this curve, if the rainfall excess is known, the infiltration capacity is readily found.

Greater accuracy could be obtained by weighting the rainfall excess at each of the various stations by the Thiessen method, but this is not considered necessary unless there are only a few records available or if the rainfall is very nonuniformly distributed over the basin.

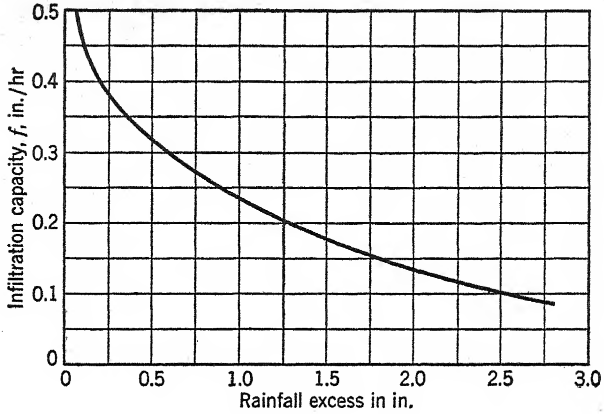


FIG. 61.

TABLE 10

Station	Total Rainfall, in.	Rainfall Excess for Values of f in in./hr of		
		0.10	0.20	0.30
(1)	(2)	(3)	(4)	(5)
A	5.91	3.65	2.22	1.27
B	3.81	1.82	0.77	0.20
C	3.76	1.76	0.74	0.18
D	4.13	2.08	0.95	0.35
E	4.01	2.00	0.90	0.30
F	4.09	2.05	0.93	0.33
G	4.92	2.76	1.48	0.74
H	6.05	3.77	2.32	1.35
Averages	4.58	2.49	1.26	0.59

CHAPTER VII

GROUND WATER[1]

by John G. Ferris[2]

This chapter presents certain phases of the ground-water hydrology with special reference to the physics of movement of ground water. It is not intended to present a complete discussion of either the general or the quantitative phases of ground-water hydrology, which is beyond the scope of this book. For more complete discussions of the occurrence and movements of ground water, the reader is referred to the papers listed at the end of the chapter.

Our study of the waters of the earth progresses from the more familiar fields of atmospheric water and surface water to the third province of hydrology, which deals with the study of subsurface or ground water. Among the many prerequisites necessary to the study of ground-water hydrology, probably the one most neglected in the training of engineers is the subject of geology. Inasmuch as it would be impossible to correct this deficiency in any single chapter, it is necessarily assumed that the student has sufficient background training in this field to recognize the degree to which geology controls the occurrence and movement of ground water.

Although man has long been familiar with the development of small water supplies from wells, it is only in recent years that much thought has been directed to the hydraulics of ground-water flow. The great advances made since the turn of the century in the improvement of well-drilling methods and pumping equipment, particularly in the development of the deep-well turbine pump, have resulted in a marked upward trend in the use of ground water for domestic, rural, municipal, and industrial water supply. It is of interest to note that in 1939 it was estimated[3] that, in the United

[1] Published by permission of the Director of the U. S. Geological Survey in the public interest.

[2] District Engineer, Ground Water Division, U. S. Geological Survey.

[3] Anonymous, Inventory of Water Supply Facilities, *Eng. News-Record,* 1939, **123**, 414.

States, about 9100 public water supplies were derived from ground water and about 3300 from surface sources. Superimpose on this established upward trend the demands of industry awakened to the economic advantages of ground water for air temperature and humidity control and as a relatively constant-quality source lending itself to almost fixed treatment. Notwithstanding the magnitude of the total withdrawal of ground water for all the above uses, this total is exceeded by the present demand for ground water in irrigation.

An ever-increasing number of problems has attended the rapid growth in the use of ground water. Those engaged in the search for answers to these problems are handicapped by the deficiencies in hydrologic research and the lack of trained technicians in this field. Our ground-water reserves have, too frequently, been called inexhaustible. Advances in hydrology show the fallacy of such terminology. Equally unfavorable, however, is the dissemination of discouraging opinions by those who have experienced water shortages that result from overdevelopment. It becomes increasingly evident that a wiser and fuller use of this great national resource can be achieved only by the sound and rational methods of the trained hydrogeologist and hydrogeological engineer.

The earth's crust, composed of its myriad and varied hard rocks and the unconsolidated overburden, serves as a vast underground reservoir for the storage and transmission of percolating ground waters. The rocks comprising the earth's crust are seldom if ever solid throughout. They contain numerous openings called interstices that vary through a wide range of sizes and shapes. Although these interstices may reach cavernous size in some rocks, it should be noted that most of them are very small. Generally, they are interconnected, permitting movement of the percolating waters, but in some rocks they are isolated, preventing the transmission of water between interstices. Accordingly, then, the mode of occurrence of ground water in the rocks of a given area is largely determined by the geology of that area.

Porosity

The physical property of a rock that defines the degree to which it contains interstices is termed its porosity and is expressed quantitatively as the percentage that the interstitial volume is of the total. The porosity of a material is dependent on the inter-

relation of size, shape, and manner of sorting of its component parts in the case of unconsolidated or pervious sedimentary material; or on the size, shape, and pattern of channeling in the case of relatively soluble rock such as limestone; or on the size, shape, and pattern of fracturing in the dense sedimentary, igneous, and metamorphic rocks. Some idea of the relation of porosity to rock texture and particle sorting may be gained by reference to Fig. 62.

The porosity of rock or unconsolidated material may range from considerably less than 1 per cent to more than 50 per cent. How-

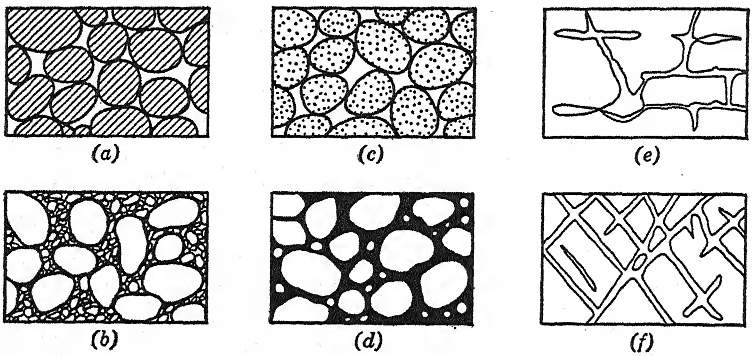


FIG. 62. Diagram showing several types of rock interstices and the relation of rock texture to porosity. *a*, Well-sorted sedimentary deposit having high porosity; *b*, poorly sorted sedimentary deposit having low porosity; *c*, well-sorted sedimentary deposit consisting of pebbles that are themselves porous, so that the deposit as a whole has a very high porosity; *d*, well-sorted sedimentary deposit whose porosity has been diminished by the deposition of mineral matter in the interstices; *e*, rock rendered porous by solution; *f*, rock rendered porous by fracturing. After Meinzer, *U. S. Geological Survey Water-Supply Paper* 489, 1923, Fig. 1, p. 3.

ever, a porosity in excess of 40 per cent is rare except in soils or poorly compacted materials. In general, we may consider a porosity greater than 20 per cent as large, between 5 and 20 per cent as medium, and less than 5 per cent as small.

There are a number of methods in use for determining the porosity of rocks or soils which are based on either volumetric or specific-gravity measurements of dry versus saturated samples. The relation of the factors most commonly required for porosity

tests is summarized by the following equation,[1]

$$P = 100\left(\frac{W}{V}\right) = 100\left(\frac{V - v}{V}\right) = 100\left(\frac{S - a}{S}\right) = 100(b - a) \quad (1)$$

where P is porosity, W volume of water required to saturate dry sample of rock or soil, V volume of sample, v aggregate volume of solid particles comprising the sample, S weighted average of specific gravities of minerals composing the soil or rock, a specific gravity of dry samples, and b specific gravity of saturated sample.

Rather elaborate core-sampling apparatus have been devised to obtain samples in an undisturbed condition. However, the removal of any sample from its original environment is certain to disturb the sample to some extent. There is no positive assurance that any laboratory procedure reproduces the original regimen of pressure, temperature, or volume. Further, in the final analysis the sample represents only an infinitesimal section of the soil or rock formation.

Mechanics of Interstitial Flow

As stated in the preceding chapter there are two principal forces that control the movement of water in rocks and unconsolidated material, namely, gravity and capillarity. As to the first, we are all familiar with its action, and it is this force of gravity that is responsible for the entrance and percolation of ground waters from the time they start from the surface, percolate downward, and move laterally in the saturated zone to emerge as springs and seeps making up the base flow of surface streams and at flowing wells, ponds, and lakes. Normally ground-water flow is laminar, but turbulent flow may occur where conditions are favorable, as for example near a stream channel or discharging well.

The second force, capillarity, is generally not accorded proper attention in proportion to its importance. Molecules possess definite size, shape, and fields of force in all states of matter. It is their degree of mobility relative to one another which distinguishes the gas, liquid, or solid state. In a gas they are separated by distances which minimize the attraction between adjacent molecules, and thus unrestrained motion occurs. In a liquid the

[1] O. E. Meinzer, The Occurrence of Ground Water in the United States, *U. S. Geological Survey Water-Supply Paper* 489, 1923, p. 12.

molecules are free to move relative to one another but are held sufficiently close to each other by the cohesive forces to prevent all but a small proportion from escaping as vapor. In a solid the molecules, though not absolutely immobile, are essentially fixed in position.

The spontaneous contraction of a liquid surface toward the shape with minimum surface area is typified in the formation of spherical droplets. A diagrammatic representation of the attractive forces which might obtain for molecules within a liquid is shown by Fig. 63. Each interior molecule is surrounded by others on every side and is in equilibrium with the attractive forces from all directions. The surface molecules, though attracted inward and to each side, are not subjected to a comparable attraction outward because there are very few molecules outside the liquid. Thus, all surface molecules are subject to a resultant inward attraction, perpendicular to the surface. Consequently, the surface molecules move inward until the concentration of interior molecules reaches a maximum and the surface contracts to the minimum area for the given volume.

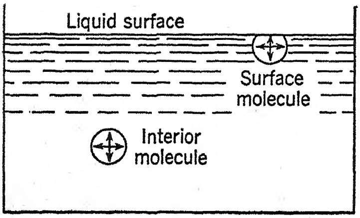


FIG. 63.

Inasmuch as work must be done to extend a surface by moving an interior molecule to the surface against the inward attractive forces, the spontaneous contraction of a liquid surface indicates that there is energy resident in the surface. It follows that, to form a free surface within a given liquid, work must be done against the mutual attraction of the molecules on each side of the interface. If T indicates the energy per unit of surface area, the work W which is required to separate a liquid column will be $2AT$ because two free surfaces of area A are formed. The quantity $W/A = 2T$ is generally termed the work of cohesion.

When a curved surface is extended by moving each element in a radial direction there is an increase in area. To increase the surface area, work must be done by the pressure differential that moves the surface. As shown by Adam,[1] this work may be evaluated by

[1] N. K. Adam, *The Physics and Chemistry of Surfaces*, Oxford Univ. Press, London, third edition, 1941, pp. 1–16.

considering the energy changes which occur in the displacement of the surface. In Fig. 64, $ABCD$ is an infinitesimal part of a surface with sides at right angles. Let the elemental area be extended in a radial direction a distance Δn, with the normals to the boundaries in the displaced position $A'B'C'D'$ the same as the normals in the original position. The normals from A and B meet at O_1; those from B and C meet at O_2. Denote the radius of curvature for arc AB as R_1 and for arc BC as R_2. Angle AO_1B measures AB/R_1 radians, and angle BO_2C is BC/R_2. Then the area of the displaced surface may be computed as

$$(A'B')(B'C') = \left(AB + \frac{AB\,\Delta n}{R_1}\right) \cdot \left(BC + \frac{BC\,\Delta n}{R_2}\right) \qquad (2)$$

Expanding equation 2 and neglecting second-order quantities there follows

$$ABCD\left(1 + \frac{\Delta n}{R_1} + \frac{\Delta n}{R_2}\right)$$

The net increase in area is

$$ABCD\,\Delta n\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

The net increase in surface energy is

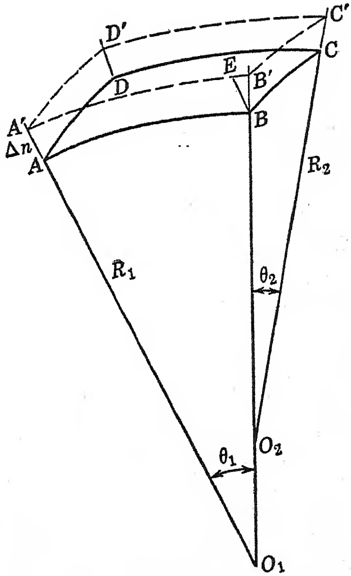


FIG. 64.

$$\Delta W = T\Delta A = TABCD\,\Delta n\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad (3)$$

Denoting the pressure on the concave side as p_1 and on the convex as p_2, the work done by this pressure differential is

$$(p_1 - p_2)ABCD\,\Delta n \qquad (4)$$

Since no work is done by other forces, the above quantities are equal, and there follows the fundamental equation of capillarity.

$$p_1 - p_2 = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad (5)$$

One of the more important and better-known applications of the forces of adhesion is the capillary rise of liquids in tubes or interstices. When water is touching glass, the glass becomes "wetted" and the angle of contact is very small. Figure 65 shows a section through a circular glass capillary tube having a diameter d. The contact angle between the water and the glass is represented by α. If it is assumed that α is zero and that the meniscus is a perfect hemisphere, $R_1 = R_2 = d/2$. Equation 5 may then be written

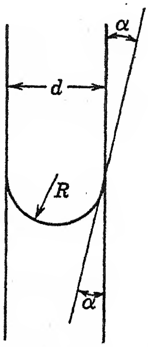


FIG. 65.

$$p_1 - p_2 = \frac{4T}{d} \tag{6}$$

This pressure difference forces the liquid up the tube to a height h, above the plane surface outside the tube, such that the weight of this liquid column just balances the pressure deficiency under the meniscus. Then

$$p_1 - p_2 = wh = \frac{4T}{d}$$

and

$$h = \frac{4T}{dw} \tag{7}$$

where h is height of capillary column, in feet; T surface tension, in pounds per foot; d tube diameter, in feet; and w unit weight, in pounds per cubic foot.

The value of T for water may be taken as 0.005 lb per ft for the range of temperatures encountered in ground water. The capillary rise of water in a glass tube 0.1 in. in diameter may be found from equation 7 to be approximately 0.46 in. As shown by this equation, the rise of water level in a capillary tube is inversely proportional to the diameter of the tube and, consequently, the capillary rise in a tube of 0.01-in. diameter under the above conditions is 10 times as great or 4.6 in. The values of h computed from equation 11 agree quite well with observed values for glass tubes having diameters less than 0.2 in., but for larger tubes observed values are less than computed values.

Inasmuch as the magnitude of the capillary rise depends on the energy resident in the liquid surface, the shape of the tube below

the meniscus does not affect the height of water level in the tube. Reference is made to Fig. 66 which shows the type of interstitial opening that may occur in consolidated or unconsolidated material. It is assumed that the effective diameter of cross section at the meniscus is equal in each case to the bore of the capillary tube. In each of the three examples the water level will be held at the same height if each interstice is filled and if the height of section A above the reservoir level is equal to h as given by equation 7. If the water level declines in either of the irregular-shaped tubes, there will be a large lowering of water level when the water surface reaches a section of enlarged area. A new equilibrium level occurs

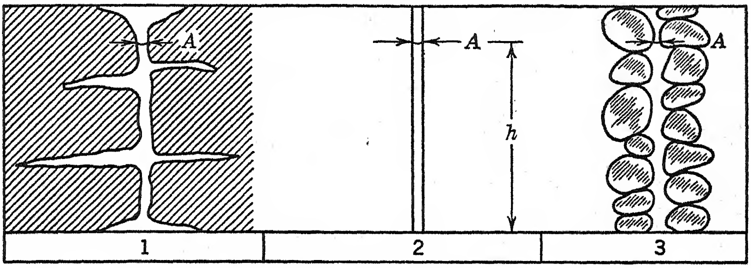


FIG. 66. Diagram showing that during a declining water level, for any given area A at the water surface, the height at which water can be held by capillarity is independent of the size and shape of the tube below the free-water surface. A rise in water level, however, may result in a much lower level in tubes 1 and 3.

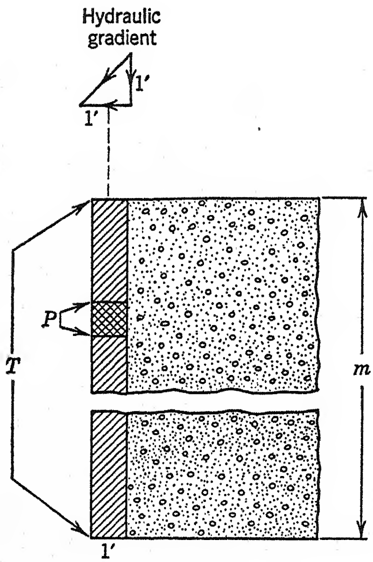
when the declining head reaches a section with sufficient surface tension to balance the weight of the residual water column. With a declining water table, the capillary fringe tends to reach a maximum though quite irregular thickness. A restricted section of capillary size, at or below the limit of capillary lift, retains water in the irregular-shaped tubes at a level above the receding water table, which is dependent on the surface energy at the critical cross section. When the water table is rising, the fringe advance is limited to the interstitial openings of continuous capillary size. It has been observed by laboratory experimentation that the height of capillary lift is somewhat greater if the water level in a glass tube recedes from a higher stage to the equilibrium level than if the water level advances toward this level from a lower stage. This phenomenon, which is termed hysteresis, is generally ascribed

to frictional forces or differences in wall adhesion under the two conditions.

Permeability and Transmissibility

The vertical percolation of ground water through capillary interstices results in the build-up of a hydraulic gradient with consequent lateral percolation of water through interconnecting interstices. The capacity of a formation for transmitting water is measured by its coefficient of permeability, which is defined by Meinzer[1] as the rate of flow of water in gallons per day through a cross-sectional area of 1 sq ft under a hydraulic gradient of 1 ft per ft at a temperature of 60° F.

The term *coefficient of transmissibility* introduced by Theis[2] is coming into popular usage in ground-water hydrology. The coefficient of transmissibility is defined as the rate of flow of water in gallons per day through a vertical strip of the aquifer 1 ft wide and extending the full saturated height under a hydraulic gradient of 100 per cent at a temperature of 60° F. The difference between the coefficients of permeability and transmissibility is shown in diagrammatic form by Fig. 67.



P = coefficient of permeability in gpd/sq ft
T = coefficient of transmissibility in gpd/ft
$T = P \cdot m$

FIG. 67.

The permeability of granular material varies with the diameter and degree of assortment of the individual particles. A well-sorted gravel has a much higher permeability than a well-sorted coarse sand. However, gravel with a

[1] N. D. Stearns, Laboratory Tests on Physical Properties of Water-Bearing Materials, *U. S. Geological Survey Water-Supply Paper* 596, 1928, p. 148.

[2] C. V. Theis, The Relation between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground Water Storage, *Trans. Am. Geophys. Union*, 1935, p. 520.

moderate percentage of medium- and fine-grained material may be considerably less permeable than a uniformly sized coarse sand. In graded material, the particles of moderate size fill the pore spaces between the larger particles, and in turn the resultant pore spaces are filled by the fine materials, thus forming a compactly knit and impervious mass such as is obtained in good concrete.

Measurements of the permeability of rocks and unconsolidated materials may be made by either field or laboratory methods as described by Muskat[1] and by Wenzel.[2] Laboratory determinations of the coefficient of permeability are made by measuring the discharge or the time rate of change in head, for the percolation of measured quantities of water through a known area and volume of soil sample. Devices used for this purpose are termed permeameters and include a supply reservoir or tank from which water is discharged through a percolation cylinder under either a constant or a variable head. The percolation cylinder is accurately machined to a fixed diameter and is equipped with a base screen which supports the soil sample and permits free inflow of water. Manometer tubes in the supply and receiving reservoirs are used to determine the loss of head that occurs for the vertical percolation of known quantities of water through the soil cylinder at measured rates. A schematic representation of the more common types of permeameter is shown by Fig. 68.

The use of permeameters to determine the permeability of unconsolidated material is invalidated to a large degree because of the great errors introduced in repacking a disturbed sample. Inasmuch as the packing arrangement is a critical factor in determining the permeability of an incoherent material it would seem advisable to apply laboratory methods only to consolidated materials or cores of unconsolidated material. Further caution should be exercised because the volume of material used in permeameter tests represents only an infinitesimal sample of a formation that is generally quite heterogeneous. Accordingly, to be of value permeameter programs should include intensive sampling and testing methods on many samples collected at frequent depth intervals and at numerous locations within the area.

[1] Morris Muskat, *Flow of Homogeneous Fluids through Porous Media*, McGraw-Hill, 1937.

[2] L. K. Wenzel and V. C. Fishel, Methods for Determining Permeability of Water-Bearing Materials, *U. S. Geological Survey Water-Supply Paper* 887, 1942.

Field determinations of permeability are made by either the velocity or the potential method. In the velocity method one well is used for the injection of salt, dye, or an electrolyte. Two or more

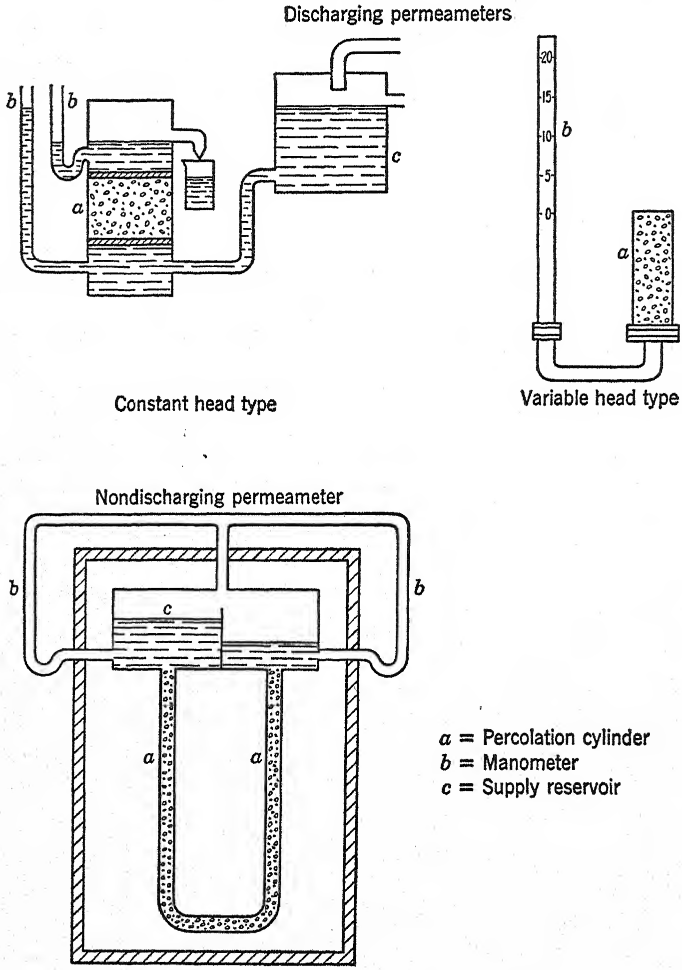


Fig. 68.

wells are used as observation stations to determine the time rate of travel of the injected substance through the water-bearing material. Fluorescein is generally used for the dye method and can be detected by eye or in more dilute form by colorimeter. The chemical or salt method requires periodic sampling and analysis

of water from each observation well to determine the time of arrival of the salted solution. The electrolyte method requires periodic readings of the electric conductivity of the water in each observation well. Measurements of the water-table gradient, the distance between observation wells, and the time of travel of the injected material provide the basis for determining the permeability of the material over the path of travel. A sketch of the equipment setup for one form of the velocity method is shown in Fig. 69.

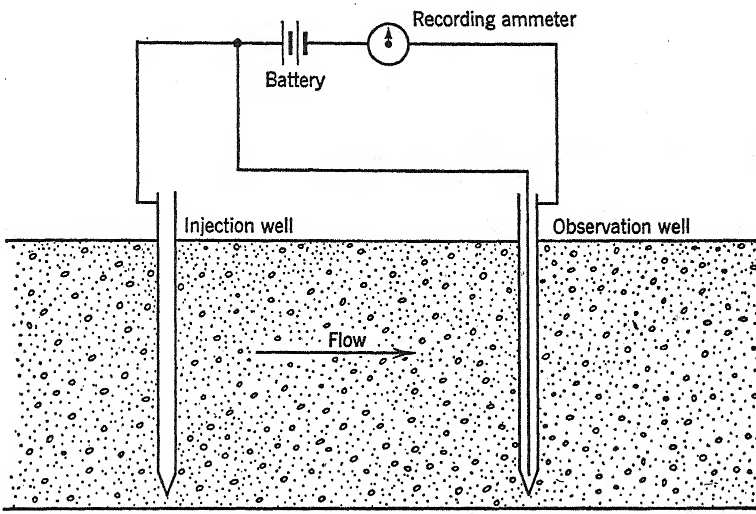


FIG. 69.

Inasmuch as the velocity of flow through most ground-water aquifers is measured in terms of a few feet per day, it is necessary that velocity observations be confined to small areas in order to secure results within a reasonable time. This method measures the velocity of the fastest thread of water that happens to intersect the two wells and not necessarily the average velocity between the wells. This method would be impractical for sampling adequately any heterogeneous aquifer that has large variations in vertical and horizontal permeability.

Potential methods of determining permeability are based on measurements of the amount and rate of drawdown or recovery of water level in observation wells at different distances from a well that is either pumping or recovering from pumping, respectively.

The principles and application of this method are covered in detail under the discussion of ground-water hydraulics. A distinct advantage of the potential method is its ability to sample large areas of the undisturbed aquifer within a limited time and at a minimum of expense.

Specific Yield

The remaining physical characteristic in the hydrology of ground water to be discussed is the specific yield. When saturated rocks or soils are drained under the action of gravity, it is found that the volume of water yielded by draining is less than the volume of void space indicated by the total porosity of the material because of the pellicular water that is retained by molecular attraction. The quantity of water yielded by gravity drainage from saturated water-bearing material is termed the specific yield and is expressed as a percentage of the total volume of the material drained. The quantity of water retained by the material against the pull of gravity is termed the specific retention or field capacity and is again expressed as a percentage of the total volume of the material. A somewhat similar term, moisture equivalent,[1] is frequently used to represent the moisture retained by a saturated sample when subjected to an arbitrary centrifugal force. It is evident that the sum of the specific yield and the specific retention of a material is equal to its porosity.

If evaporation is prevented, the greater part of the water retained by a column of rock or sand and gravel, after draining for 24 hr, will be retained almost indefinitely as a film held by molecular adhesion on the walls of the interstices. The greater the amount of total interstitial surface in a rock or unconsolidated material the greater is its specific retention. As would be expected, it is found that, as the effective diameter of grain decreases, the specific retention generally increases because the total exposed surface area increases with decreasing grain size. (See page 233.)

Although the total porosity of a clay or fine sand might be equivalent to the total porosity of a coarse gravel, it follows from the above that the large specific retention of the clay would result in a very small specific yield, whereas the reverse would be true for the coarse gravel. For practical purposes, a water-bearing forma-

[1] L. J. Briggs and J. W. McLane, The Moisture Equivalent of Soils, *U. S. Department of Agriculture Bureau of Soils Bul.* 45, 1907.

tion of coarse gravel would supply large quantities of water to wells, whereas clay formations, although saturated and of high porosity, would be of little value in this respect. Accordingly, we find that specific yield is termed by some as the effective or practical porosity.

Determinations of the specific yield or specific retention by laboratory methods are limited by the difficulties of securing undisturbed and representative samples that were noted under permeability determinations by laboratory methods. In addition, the short sample columns used in the laboratory cannot duplicate the very long capillary tubes that probably exist in the thick sections found in the field. As for permeability, the most satisfactory determinations of specific yield are made in the field through the medium of pumping tests.

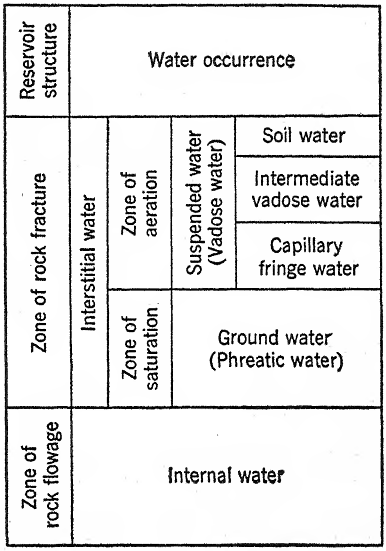


FIG. 70.

The Ground-Water Reservoir

The classification of the earth's crust with reference to its properties as a reservoir for the storage and transmission of percolating ground waters and the subdivision of this reservoir into its component parts is shown by Fig. 70. Interstices are probably absent in the zone of rock flowage, because the stresses are beyond the elastic limit and the rock is in a state of plastic flow. Water in this zone is classified as internal water and is not in the realm of the hydrologist. The depth at which rocks undergo permanent deformation is not known accurately but is generally estimated as many miles.

In the zone of rock fracture, the stresses are below the elastic limit, and interstices can exist in the rocks. Water in this zone is stored in the soil or rock interstices and accordingly is termed interstitial water. Although there is no direct relation between porosity and depth, in general, the porosity decreases with depth,

the large openings particularly being absent at great depths. In crystalline rocks most of the water is encountered within 300 ft of the surface.[1] In sedimentary rocks, such as limestone and sandstone, porous zones that yield water readily are encountered in some places at depths of more than 6000 ft, although most wells in these strata find little water below a depth of 2000 ft. The decrease in size of interstices with increased depth is caused in part by the increased pressure at great depth, which tends to close the pore spaces or crevices, and in part by the cementation of interstices by the heavier and more highly mineralized waters.

The zone of aeration is that part of the earth's crust where the water present is not under hydrostatic pressure, except temporarily, and for the most part the interstices are filled with atmospheric gases. Water retained in this zone is held by molecular attraction and is termed pellicular, suspended, or vadose water. The thickness of the zone of aeration varies considerably depending on the geology, hydrology, and topography of the area. It may be virtually nonexistent in lowland areas adjacent to bodies of surface water as in marsh lands, or it may be as much as 1000 ft thick as in arid regions of great topographic relief.

The belt of soil water consists of the soil and other unconsolidated materials in which the root systems of plants, grasses, and trees are developed and from which water is discharged to the atmosphere by evaporation or transpiration. Evaporation occurs largely at the surface except in tight clay soils under prolonged drying where shrinkage cracks develop and permit air circulation to some depth. Although water may be brought to the evaporation areas by capillarity, in general water is not discharged in appreciable quantities by evaporation below depths of a few feet. As to transpiration, note that, although the root penetration of most common grasses and field crops is seldom more than a few feet, records indicate root development for wheat to depths of 7 ft; for alfalfa as much as 30 ft[2]; and for some perennials in arid regions as much as 50 ft.[3]

[1] E. E. Ellis, Occurrence of Water in Crystalline Rocks, *U. S. Geological Survey Water-Supply Paper* 160, 1906, pp. 19–28.

[2] W. W. Burr, The Storage and Use of Soil Moisture, *Nebraska Univ. Research Bull.* 5, 1914, p. 9.

[3] O. E. Meinzer, Plants as Indicators of Ground Water, *U. S. Geological Survey Water-Supply Paper* 577, 1927, p. 77.

The capillary fringe is the belt overlying the zone of saturation and containing interstices, some or all of which are filled with water that is in connection with and is a continuation of the zone of saturation, being held above that zone by capillarity acting against the force of gravity. The thickness of the capillary fringe in granular material is a function of the effective particle size and generally increases as the grain size decreases. The fringe thickness may range from a few inches in coarse gravel to 8 ft in silty material and is probably much greater in very fine-grained sediments. The capillary fringe in a given material may vary slightly in thickness from summer to winter because of changes in water temperature. The surface tension of water increases as the temperature decreases, and within the range from 60° to 32° F it increases about 3 per cent. Although the density of water varies with temperature, this change is negligible. Accordingly, then, the thickness of the capillary fringe would be somewhat greater in late winter and spring, the period of lowered ground-water temperature.

Beneath the capillary fringe lies the zone of saturation. It is this zone that is of importance to the hydraulic engineer and well driller as the source of water for wells and springs. It is of importance to the hydrologist as the reservoir that provides the closing link in the hydrologic cycle by serving as the mechanism for the intake, transport, and return of underground waters from and to the surface and the atmosphere. The upper surface of the zone of saturation is called the water table.

A profile section of a drainage basin is shown by Fig. 71 in diagrammatic form with greatly contracted horizontal scale. In the highland area, water is discharged from the soil belt either by direct evaporation from the soil or by transpiration from the vegetal cover. During a prolonged drought the vegetation in this area is dependent on the moisture retained in the soil belt, which is determined by the specific retention of the material composing this belt. Plant root systems contain many fine rootlets that are capable of absorbing a large part of the water held on the soil particles by molecular attraction. Inasmuch as an adequate water supply is vital to continued growth, the plant develops an extensive network of rootlets to satisfy its moisture demands. Under conditions of gradually diminishing soil water supply, when the leaves of the plant first undergo permanent wilting, minimum soil water content

is expressed by the wilting coefficient[1] of the soil, which is defined as the ratio of the weight of water in the soil, under the above-noted conditions, to the weight of the soil when dry. Accordingly, then, the capacity of a soil for storing water available for plant growth is measured by the difference between its specific retention and its wilting coefficient when expressed in comparable units.

In areas where the soil belt is close to or in contact with the capillary fringe, certain types of plants may develop rootlets in the capillary fringe and with the aid of capillarity are able to

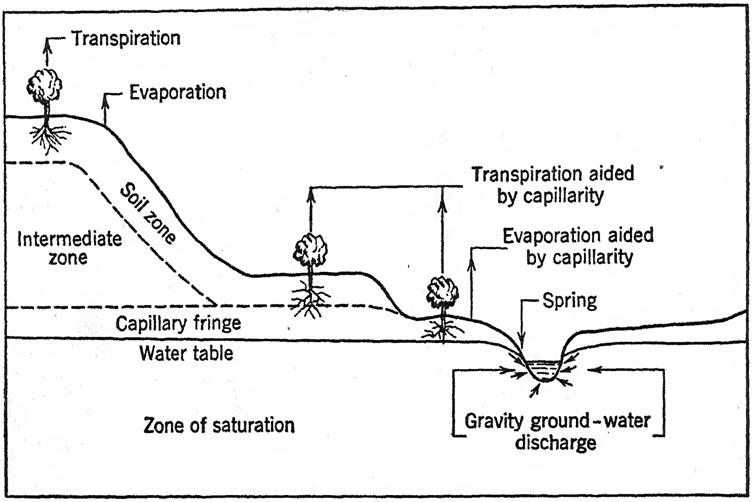


FIG. 71.

pump water from the zone of saturation. Such plants, which are called phreatophytes,[2] can maintain a continuous discharge of water from the water table to the surface and may contribute large quantities of water to the atmosphere.

In regions where the soil belt is very thin, the capillary fringe may be in close contact with the surface and permit direct evaporation discharge to the atmosphere with continuous replenishment from the water table through capillary lift by the fringe material.

[1] L. J. Briggs and H. L. Shantz, The Wilting Coefficient for Different Plants and Its Indirect Determination, *U. S. Department of Agriculture Bureau of Plant Industry Bul.* 230, 1912.

[2] O. E. Meinzer, Outline of Ground-Water Hydrology, *U. S. Geological Survey Water-Supply Paper* 494, 1923, p. 55.

The moisture discharge by evaporation under this condition may be considerable if a large part of the drainage area is represented by land of low relief and small elevation above the surface streams. The rate of evaporation varies greatly in response to the variations in conditions over the soil surface and depends upon the evaporation opportunity, i.e., the ratio of the actual evaporation to the evaporation from a free water surface under existing atmospheric conditions.

A portion of the record obtained from an automatic water-stage recorder in operation on a shallow well near Roscommon, Michigan, is reproduced as Fig. 72. Detailed examination of this diagram shows the consistent daily drawdown of water level through the

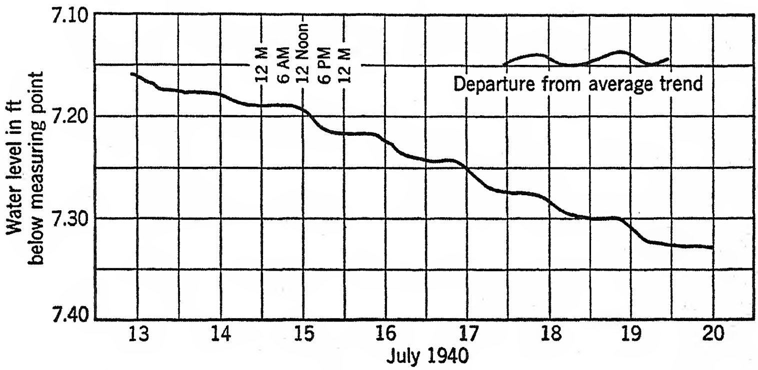


FIG. 72.

sunlight period. A time difference is noted at each end of the cycle, in that the drawdown period generally starts about 8:00 AM each day or about 2½ hr after sunrise, and the recovery of water level or cessation of drawdown starts shortly before 6:00 PM or more than 2 hr before sunset. The maximum rate of drawdown of water level occurs during the noon period when the sun is at the zenith. The periods of so-called recovery are represented by the intervals of decreased drawdown rate and in this example do not imply an actual rise in water level. During the growing season, the general trend of water level is downward, and the transpiration phenomenon is superimposed on this downward trend. To show the effect more clearly, a portion of the graph is reproduced by replotting the departure of the curve from the average downtrend. (See also Figs. 49 and 50, page 160.)

Water Table and Artesian Aquifers

Although the idealized conditions represented by Fig. 71 are found frequently in the field, usually an actual cross section of a valley is more complex than indicated by this diagram. Field reconnaissance may reveal more than one water-bearing formation with considerable variation in the character of each stratum. The many geologic processes involved, coupled with the great variations in intensity and duration of the forces in action during each stage of development leading up to the existing structures, have resulted in an infinite number of variations in the geologic and hydrologic dimensions of the ground-water reservoir. The "hodge-podge" assortment of the drift cover in glaciated areas typifies these complexities. However, the heterogeneous nature of the surficial materials does not invalidate the fundamental principles but merely complicates their application.

A stratum or formation of permeable material that will yield gravity ground water in appreciable quantities is termed an aquifer. The term "appreciable quantity" is relative because, where ground water is obtained with difficulty, even fine-grained, poorly productive materials may be classed as principal aquifers. If an aquifer is overlain by a confining bed of impervious material and if the water level in a tightly cased well penetrating the aquifer rises above the bottom of the confining bed, the aquifer is termed artesian. The overlying confining bed is an aquiclude. The artesian aquifer differs from the water-table aquifer in that the surface, formed by contouring or connecting the heights of the water level in tightly cased wells tapping the aquifer, is not a free surface exposed to the soil atmosphere but is an imaginary pressure surface standing above the body of the aquifer and consequently receives the name of piezometric surface. Although the term piezometric surface can be applied also to the water-table surface, the reverse is not true. Contours drawn on the piezometric surface are referred to as isopiestic lines. A diagrammatic cross section illustrating the application of the above terminology is shown in Fig. 73.

The water level in Well 1, which taps aquifer A, coincides with the water table or surface of the zone of saturation in this aquifer, and consequently Well 1 is a nonartesian or water-table well. The water levels in Wells 2, 3, and 4 stand above the base of the over-

lying aquiclude, and aquifers B and C are artesian. Region a–a is an area of artesian flow, and Well 4 is a flowing well. The lower static level in aquifer B indicates that water is moving from aquifer A or C into aquifer B through a distant break in aquiclude A or B or by vertical leakage through the aquicludes. The high head in Wells 3 and 4 indicates that the recharge or intake area of

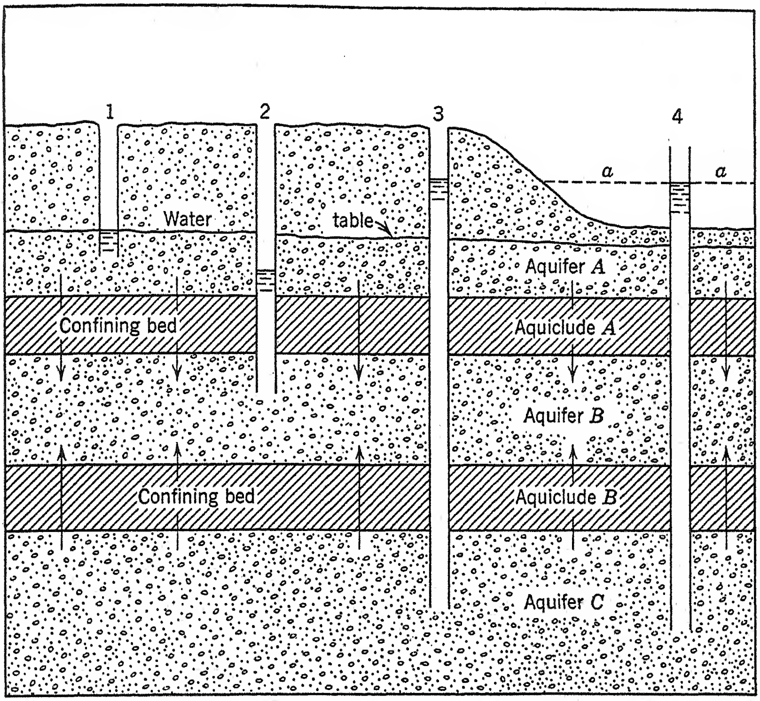


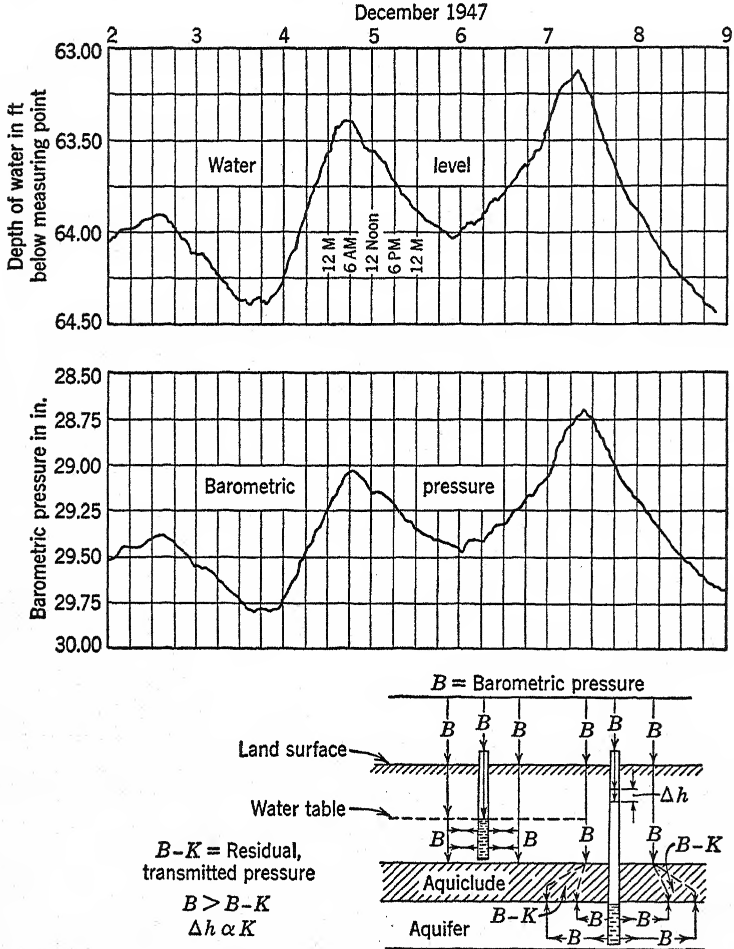
FIG. 73.

aquifer C is at a relatively high elevation, probably above the land surface shown in the cross section. Aquifers like B and C, which contain confined ground water, are pressure conduits and exhibit interesting elastic phenomena.

Elastic Properties of Artesian Aquifers

It is a common observation that wells in some areas will undergo changes of water level during periods of large fluctuation in barometric pressure. A representation of the mechanism causing the change of water level under varying air pressure and a hydrograph

from a well exhibiting this effect are shown in Fig. 74. As the barometric pressure increases, the water level in the well casing



Idealized cross section of aquifer showing how bridging action of overlying aquiclude sets up pressure differential in the underlying aquifer which is balanced by a change in hydrostatic head.

FIG. 74.

tends to be depressed. An equal effect is exerted on the soil column and on the shallow water table with a resultant balance of the pressure inside and outside the well tapping the shallow aquifer,

and consequently no net change in water level occurs. In the deeper aquifer, however, the overlying aquiclude is competent to some degree in resisting the load imposed by the rising barometer and does not transmit the full effect of the air pressure change. Consequently, the water level in the well tapping confined ground water is depressed an amount equal to the proportion of the barometric pressure change that is not transmitted by the aquiclude. The ratio of water-level change to the barometric change, in equivalent units, is termed the barometric efficiency of the well. Note that the effect is inverse, that is, as the barometric pressure rises the water level declines.

Inasmuch as the increased hydrostatic pressure in the aquifer, which accompanies a rise in barometric pressure, exceeds the residual pressure transmitted through the aquiclude, a net positive pressure is exerted on the aquiclude. As a result of this pressure the aquiclude is compressed slightly, or, conversely, the aquifer expands a small amount. The slight increase in aquifer volume accommodates the water displaced from the well casing by the increased pressure on the water surface. Changes in barometric pressure are generally of very short duration compared to the time required for the displaced water to move through the formation for any distance. Consequently, barometric fluctuations are recorded in confined aquifers if the overlying aquiclude is competent to resist pressure and extends over an appreciable area. Wells located near an outcrop area or near a break in the aquiclude, where contact with the surface or surface formations occurs within close proximity to the well, will not exhibit barometric effects.

Reports of blowing and sucking wells which exhibit pronounced updraft or downdraft of air at the well mouth may be explained by the above-noted barometric effects. These reports are especially prevalent where an extensive aquiclude occurs some distance above the water table, so that there is a body of air confined between the water table and the aquiclude, which communicates with the atmosphere only through wells. Also the frequent reference to noticeable cloudiness or color in the well water preceding a storm might be explained in part by the rapid rise of water level, which would accompany a barometric low and would bring into the well silty or fine material, as a result of the quick inrush of water through the screen. Some cloudiness may also be caused by gas that escapes from solution when the atmospheric pressure is lowered.

Superimposed loads on the earth's crust also produce changes of water level in wells tapping confined ground water, as indicated by Fig. 75, which shows an autographic record of water-level fluctuation caused by railroad trains passing within 100 ft of the observation well. The alternate loading and unloading of the earth's

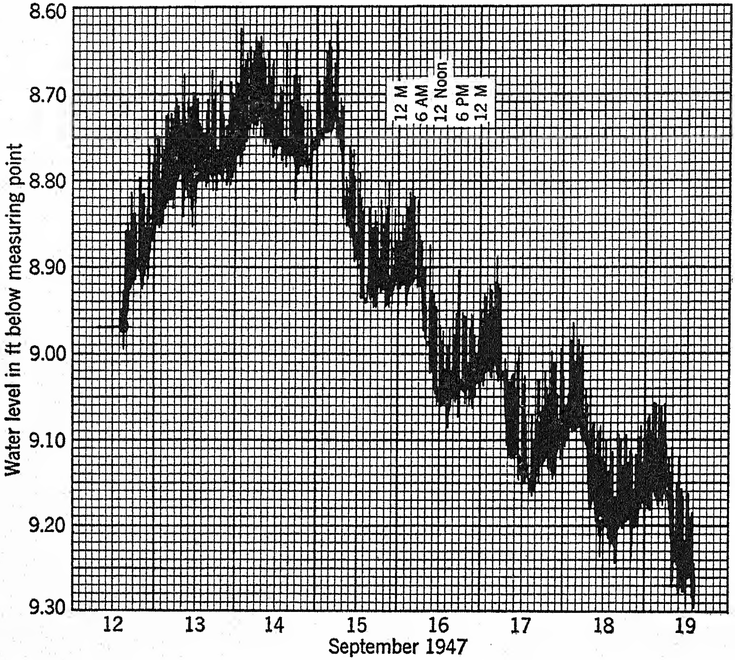


FIG. 75. Hydrograph from automatic water-stage recorder in operation on well tapping Marshall sandstone at Battle Creek, Michigan. Short-period vertical displacements superimposed on curve are water-level fluctuations caused by artesian loading from passing railroad trains.

crust by ocean tides in the coastal areas results in a corresponding cycle of water-level fluctuation in wells as shown by Fig. 76. In these cases, the resultant of the impressed pressure that is transmitted to the aquifer, because of the incompetency of the aquiclude to resist entirely the increase in pressure, causes a rise in water level in the well casing, and the ratio of this rise to the total load impressed is termed the tidal efficiency of the aquifer. Inasmuch as the tidal efficiency is a measure of the incompetency of the aquiclude and the barometric efficiency is a measure of its compe-

tency, it is evident that the sum of the barometric and tidal efficiencies of an aquifer must equal unity, as demonstrated mathematically by Jacob.[1]

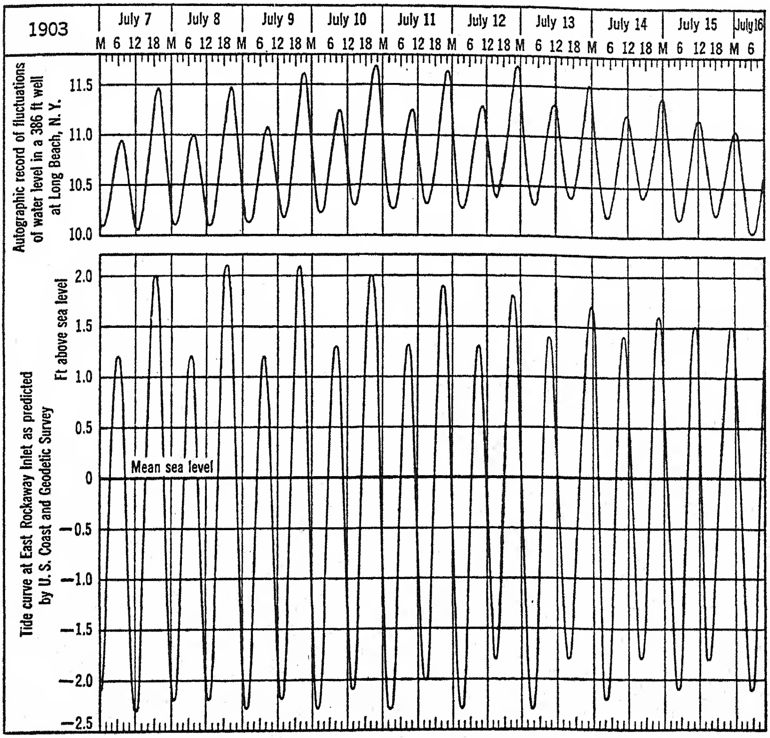


FIG. 76. Hydrograph showing fluctuations of water level in a 386-ft well at Long Beach, N. Y., as compared to the tide at East Rockaway Inlet, N. Y. From Plate 4 of *U. S. Geological Survey Water-Supply Paper* 155.

Subsurface Leakage

The presence of aquicludes or confining layers of considerable thickness and of dense, compact texture has probably served to further the somewhat popular but quite erroneous belief that the artesian aquifers are insulated strata containing connate waters. In this connection, it should be noted that many of our highly developed artesian aquifers would be dry today if such insulation

[1] C. E. Jacob, On the Flow of Water in an Elastic Artesian Aquifer, *Trans. Am. Geophys. Union*, 1940, p. 583.

were general. Fortunately, however, most aquifers receive recharge either through direct infiltration on outcrop areas, through permeable breaks in the confining aquicludes, or by means of leakage through the aquiclude itself. Like many physical terms, the word impervious is only relative and not absolute because air or water will permeate most materials if sufficient time and pressure are involved.

To demonstrate the possible magnitude of aquiclude leakage, there is represented in Fig. 77 an idealized cross section of a

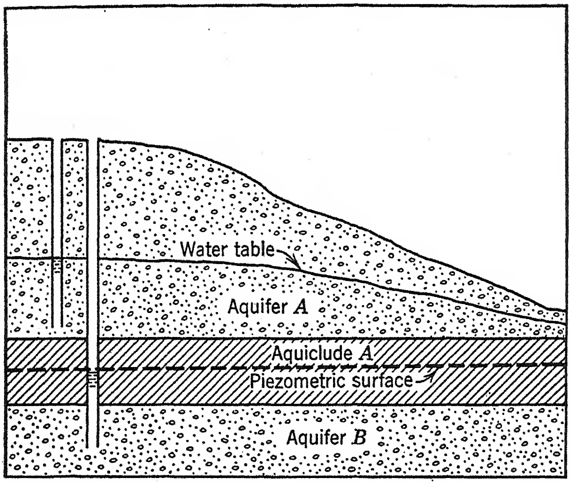


FIG. 77. Generalized cross section of shallow and deep aquifer showing differential hydrostatic head.

geologic condition that is found frequently in the field. It is assumed that the average coefficient of permeability is 2000 gal per day per sq ft for aquifer *B* and 0.2 gal per day per sq ft for aquiclude *A*, or a ratio of 10,000 to 1. The permeability value selected for the aquifer is representative of the average obtained for many sand and gravel formations. The value selected for the aquiclude corresponds to a sample of clayey silt tested in the hydrologic laboratory[1] of the U. S. Geological Survey. The mechanical analysis for this material indicates clay 49.3 per cent,

[1] L. K. Wenzel, Methods for Determining Permeability of Water-Bearing Materials, *U. S. Geological Survey Water-Supply Paper* 887, 1942, p. 13.

silt 45.3 per cent, and material larger than silt but less than 0.50 mm 5.4 per cent; the porosity was 55.5 per cent. Assume that the water table in the shallow aquifer has a head 50 ft greater than the piezometric surface in the deeper aquifer. Assign a thickness of 50 ft for aquiclude *A*. With the foregoing conditions, it is calculated that the leakage through the aquiclude from the shallow to the deep aquifer would occur at the rate of 0.2 gal per day per sq ft of aquiclude area. This seems a minor item at first consideration, but for each square mile the leakage totals 5.6 million gal per day or enough to supply a community of 56,000 people at an average rate of 100 gal per capita per day. When we consider the many square miles of contributing area available to most large aquifers, it is evident that the assumed aquiclude can contain even less pervious material and still pass appreciable quantities of water.

For the assumed conditions with a porosity of 56 per cent, the movement of water through the aquiclude would occur at the rate of 0.6 in. per day or require about 3 yr for a traverse through the 50-ft section. Accordingly, then, an aquifer recharged only by leakage from adjacent aquicludes will not show water-level fluctuations in response to short period changes in precipitation rate.

In addition to the dewatering problems in subsurface construction where aquifers are exposed by excavation, other difficult problems may arise in deep excavation into an overlying aquiclude. Prior to excavation, the stresses in an aquiclude would be in equilibrium with the total force exerted by the underlying aquifer. Assume that at the site the aquifer has a large hydrostatic head and a high transmissibility. The aquiclude over a long period of time has compacted to a thickness that provides the inherent stability to balance the upward pressure from the aquifer. Although detailed information is not available, it would seem probable that any excavation to appreciable depth in the aquiclude might disturb the force balance to an extent that might result in upheaval of the pit floor and general instability. If rupture of the aquiclude occurred or if permeable zones were exposed, large boils or springs might develop. A condition of this type might be remedied by a few properly spaced wells that penetrate the deep aquifer and are pumped at a rate sufficient to reduce the pressure and restore an equilibrium state.

Underflow

To all who are acquainted with the construction of blind drains the type of ground-water flow termed underflow will strike a familiar chord. The geologic "horse" in sedimentary rock and the buried kames, eskers, alluvium, and outwash channels in the drift mantle are examples of nature's large-scale underdrains. Under-

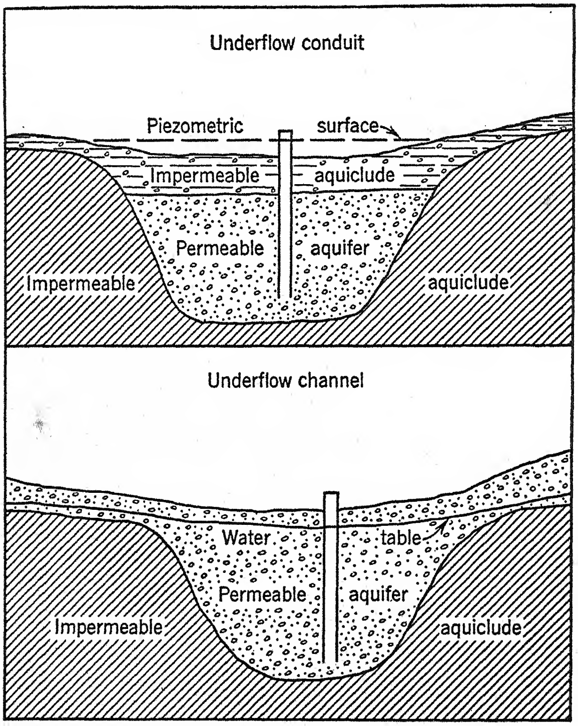


FIG. 78.

flow may occur under either water-table or artesian conditions as shown by Fig. 78. Inasmuch as the word channel is generally used in surface-water terminology for flow with a free surface, the term underflow channel can be assigned for the water-table condition because a free surface exists. In a similar manner, underflow conduit can be used for the artesian condition because the term conduit generally implies confined flow.

In periods of extended drought many stream channels, though

dry with reference to surface flow, may carry appreciable quantities of water as underflow. In view of the vast network of preglacial and interglacial stream channels throughout the glaciated areas of the United States, plus the evidence that many of these buried channels are very large, it becomes evident that the underflow in channels filled with very permeable material may be an appreciable part of the base flow from some drainage basins. Although the velocity of such underflow would be very much less than surface flow, the total discharge becomes appreciable if large areas are involved.

Seepage

The movement of water between ground-water aquifers and surface sources can be termed seepage and is further classified as influent seepage, which is recharge from surface bodies of water, and effluent seepage, which is discharge to surface bodies of water. Thus surface streams are influent streams if the stream contributes water to the ground-water reservoir and effluent streams if water is received from the water table. A sketch of conditions existing in each type is shown by Fig. 79. The local build-up of head on the water table underlying an influent stream is termed a ground-water mound. The so-called base flow of surface streams is the effluent seepage from the drainage basin. During periods of prolonged drought, when the total flow of a stream is restricted to the base flow, the stream is functioning solely as a drain. Accordingly, the collection of pertinent data concerning the volume of discharge and the time rate of water-table decline for base-flow periods, which is one phase of present field investigations conducted by the U. S. Geological Survey, will provide in time the basis for application of drain formulas and the ultimate forecast of base flow for major streams.

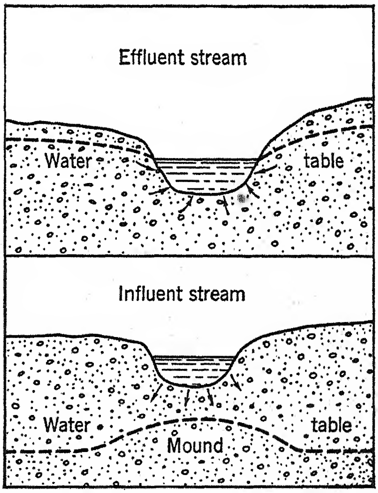


Fig. 79.

Ground-Water Hydraulics

Although the first studies of the flow of water through capillary tubes by Hagen[1] and Poiseuille[2] indicated that the rate of flow is proportional to the hydraulic gradient, it was Darcy[3] who confirmed and applied this law to the flow of water percolating through filter sands. Darcy's law is expressed as follows, in the system of units of general usage,

$$v = \frac{PI}{7.48p} \tag{8}$$

where v is velocity in feet per day, P coefficient of permeability in gallons per day per square foot, I hydraulic gradient in feet per foot, and p porosity in per cent.

In most ground-water problems, the total volume of flow is required, rather than the velocity, and consequently equation 8 is modified to the following form

$$Q_d = PIA \tag{9}$$

where Q_d is discharge in gallons per day, P coefficient of permeability in gallons per day per square foot, I hydraulic gradient in feet per foot, and A area of flow cross section in square feet.

This formula may be adapted for use with the more convenient coefficient of transmissibility by noting the distinction between its definition and that of the coefficient of permeability

$$Q_d = TIW \tag{10}$$

where Q_d and I are defined as above, T is the coefficient of transmissibility in gallons per day per foot, and W the width of flow cross section in feet.

Either equation 9 or 10 may be used for determining the discharge of ground water through underflow channels or conduits or for computing the discharge across a given length of a contour on the water table or the piezometric surface. Most underflow problems can be greatly simplified by assuming an idealized rectangular cross section that closely approximates the actual

[1] G. Hagen, Ueber die Bewegung des Wassers in engen cylindrischen Rohren, *Ann. Physik Chem.*, Leipzig, 1839, **46**, 423–442.

[2] J. L. M. Poiseuille, Recherches expérimentales sur le mouvement des liquides dans les tubes de tres petit diamètre, *Roy. Acad. Sci. Inst. France Math. Phys. Sci. Mem.*, 1846, **9**, 433.

[3] Henri Darcy, *Les Fontaines publiques de la ville de Dijon*, Paris, 1856.

section. An approximation of this type is shown in Fig. 80, with appropriate notations concerning the application of the above terminology. A sample computation is made using the following

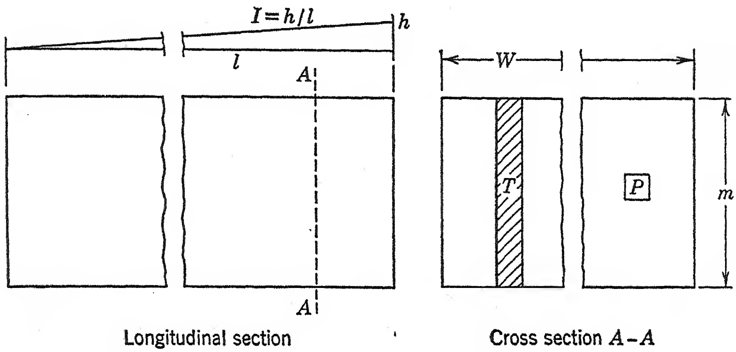


FIG. 80.

assumed values, which are representative of average conditions for a sand and gravel filled channel.

Assumptions:

$$P = 2000 \text{ gal per day per sq ft}$$

$$I = 10 \text{ ft per mile}$$

$$A = mW$$

where $m = 100$ ft, average thickness of aquifer, and $W = 5000$ ft, average width of aquifer; then by equation 9

$$Q_d = PIA$$

$$= 2000 \cdot \left[\frac{10}{5280}\right] \cdot (100 \times 5000)$$

$$Q_d = 1.9 \text{ mgd}$$

or by equation 10

$$Q = TIW$$

where

$$T = P \cdot m = 2000 \cdot 100 = 200{,}000$$

$$Q_d = 200{,}000 \cdot \left[\frac{10}{5280}\right] \cdot 5000$$

$$Q_d = 1.9 \text{ mgd}$$

Although the above formulas are of considerable value in underflow determinations, their application requires a knowledge of the permeability or transmissibility of the aquifer. The first application of Darcy's law to the analysis of the hydraulics of wells was made by Dupuit[1] in 1863. His equation was derived for a discharging well located at the center of a highly idealized circular island and thus quite limited in its application. A modification of the Dupuit analysis was developed by Thiem[2] in 1906 that utilized for the first time two or more observation wells to determine the

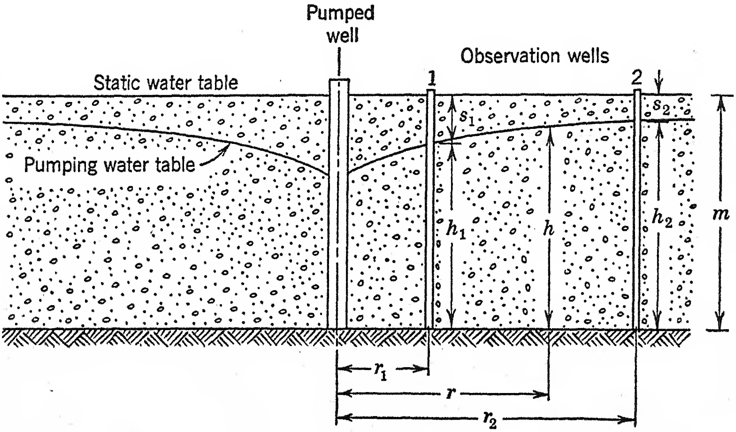


FIG. 81.

field coefficient of permeability. The derivation of Thiem's formula may be set up from the profile section through the cone of depression shown by Fig. 81. From Darcy's law, the flow through any concentric cylindrical section of the water-bearing material is given by equation 9

$$Q_d = PIA$$

Using cylindrical coordinates, we take r as the radius of any cylinder and h as the height of the cone of depression at the distance r from the well. Then

$$I = \frac{dh}{dr} \tag{11}$$

[1] Jules Dupuit, *Études théoretiques et pratiques sur le mouvement des eaux*, Paris, second edition, 1863.

[2] Gunter Thiem, *Hydrologische Methoden*, J. M. Gebhardt, Leipzig, 1906, 56 pp.

and the flow area

$$A = 2\pi rh \tag{12}$$

Therefore

$$Q_d = P \cdot \frac{dh}{dr} \cdot 2\pi rh \tag{13}$$

Rewriting and setting up the integral between the limits r_1 and r_2, the respective distances to observation wells 1 and 2

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi P}{Q_d} \int_{h_1}^{h_2} h\, dh \tag{14}$$

Integrating and inserting the limits

$$\log_e \frac{r_2}{r_1} = \frac{2\pi P}{2Q_d} [h_2^2 - h_1^2] = \frac{\pi P}{Q_d} [h_2^2 - h_1^2] \tag{15}$$

But

$$h_2^2 - h_1^2 = [h_2 + h_1][h_2 - h_1] \tag{16}$$

and

$$h_2 - h_1 = s_1 - s_2 \tag{17}$$

If the amount of drawdown is small compared to the saturated thickness of the water-bearing material then h_2 and h_1 are nearly equal and approximate the saturated thickness, m, or

$$h_2 + h_1 = 2m \tag{18}$$

$$\log_e \frac{r_2}{r_1} = \frac{\pi P}{Q_d} \cdot 2m(s_1 - s_2) = \frac{2\pi Pm}{Q} (s_1 - s_2) \tag{19}$$

but

$$T = Pm$$

therefore

$$\log_e \frac{r_2}{r_1} = \frac{2\pi T}{Q_d} (s_1 - s_2) \tag{20}$$

Converting the logarithm to the base 10, expressing the pumping rate in gallons per minute, and transposing to solve for T

$$T = \frac{527.7Q \log_{10} \frac{r_2}{r_1}}{s_1 - s_2} \tag{21}$$

where T is the coefficient of transmissibility in gallons per day per

foot, Q is discharge of pumped well in gallons per minute, r_1 and r_2 are the respective distances of observation well from pumped well in feet, and s_1 and s_2 are the respective drawdown or recovery in observation well in feet.

The derivation of the Thiem formula is based on the following assumptions and its successful application is dependent on the degree to which these qualifications are satisfied by the field conditions: (1) the aquifer is homogeneous, isotropic, and of infinite areal extent; (2) the discharging well penetrates and receives water from the entire thickness of the aquifer; (3) the coefficient of transmissibility is constant at all places and at all times; (4) pumping has continued at a uniform rate for a time sufficient for the hydraulic system to reach an equilibrium stage or a steady flow condition; (5) the flow lines are radial; and (6) flow is laminar. Despite the limiting assumptions, Thiem's formula is widely applicable to ground-water problems and, as will be demonstrated, many of the above limitations can be removed by appropriate adjustment.

As shown by Wenzel,[1] the equilibrium formulas used by Slichter, Turneaure and Russell, Israelson, and Muskat are essentially modified forms of Thiem's method. Furthermore, Jacob[2] demonstrated that Wenzel's "limiting formula" and "gradient formula" are also specialized forms of the Thiem method. Accordingly, all the above formulas are limited by the same assumptions used in deriving Thiem's formula. Several of the above equilibrium formulas entail the determination of R, the distance from the pumped well at which the drawdown is inappreciable, and necessarily assume that the drawdown cone has reached a state of equilibrium over the entire area of influence. The use of R arises when observation wells are not available, and the required two points for the equilibrium formula are (1) the pumped well where r and s are measurable and (2) the point of zero drawdown at the radius of influence. Arbitrary values have been assigned to R by several investigators: Slichter[3] gives 600 ft;

[1] L. K. Wenzel, Methods for Determining Permeability of Water-Bearing Materials, *U. S. Geological Survey Water-Supply Paper* 887, 1942, pp. 79–82.

[2] C. E. Jacob, Notes on Determining Permeability by Pumping Tests under Water-Table Conditions, U. S. Geological Survey, mimeographed report, June 1944.

[3] C. S. Slichter, Theoretical Investigation of the Motion of Ground Water, *U. S. Geological Survey* 19*th Ann. Rept.*, 1899, Part 2, p. 360.

Muskat[1] 500 ft; and Tolman[2] 1000 ft. Turneaure and Russell[3] calculate R by assuming that the cone of influence ceases development when it has intercepted a width of underflow that contributes a volume of flow equivalent to the discharge of the pumped well. When it is recognized that measurable drawdowns were observed by Leggette[4] at distances as great as 7.1 miles from a pumped well, it appears that under certain conditions the above estimates for R may be low. A ground-water reservoir tends toward a state of equilibrium with natural discharge balancing natural recharge. Consequently, the development of a well disturbs this balance and the new equilibrium state is reached by the propagation of the cone of depression to an extent where the natural recharge is increased or the natural discharge is decreased by an amount equal to the withdrawal by the well. In some instances the radius of the cone necessary to reach the areas of natural recharge or discharge may be many times greater than the values cited or estimated by the above-mentioned investigators.

The Nonequilibrium Formula

A major advancement in ground-water hydraulics was made by Theis[5] in 1935 with his development of the nonequilibrium formula which introduces the time factor and the specific yield or coefficient of storage. This formula was derived by analogy between the flow of ground water and the flow of heat by conduction. Later, Jacob[6] demonstrated the derivation of this formula using hydraulic concepts directly.

A generalized free-body diagram of the flow system in the vicinity of a discharging well is shown by Fig. 82. Assuming that impermeable planes bound the system on top and bottom and all

[1] Morris Muskat, *Flow of Homogeneous Fluids through Porous Media*, McGraw-Hill, 1937, p. 95.

[2] C. F. Tolman, Ground Water, McGraw-Hill, 1937, p. 387.

[3] F. E. Turneaure and H. L. Russell, *Public Water Supplies*, John Wiley, third edition, 1924, p. 258.

[4] R. M. Leggette, The Mutual Interference of Artesian Wells on Long Island, N. Y., *Trans. Am. Geophys. Union*, 1937, p. 493.

[5] C. V. Theis, The Relation between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-Water Storage, *Trans. Am. Geophys. Union*, 1935, pp. 519–524.

[6] C. E. Jacob, On the Flow of Water in an Elastic Artesian Aquifer, *Trans. Am. Geophys. Union*, 1940, pp. 574–586.

flow is radial, we find from the principle of the conservation of matter that the difference in the rate of flow through the inner and

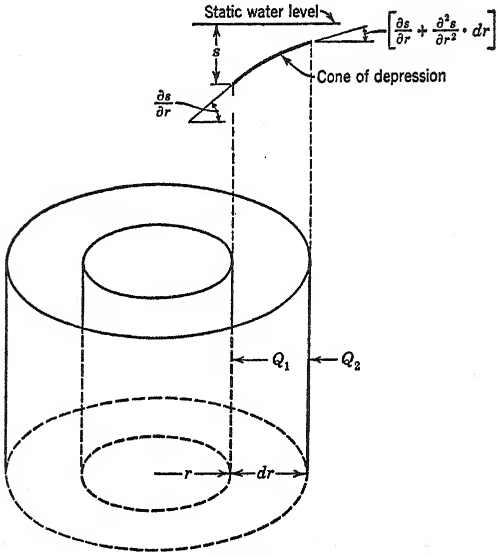


FIG. 82.

outer faces of the cylindrical shell must be drawn from storage within the shell.

$$Q_1 - Q_2 = \frac{dv}{dt} \tag{22}$$

From equation 10 the flow through the inner face is

$$Q_1 = TI_1W_1 = \frac{-T\,\partial s 2\pi r}{\partial r} \tag{23}$$

Since the second derivative defines the rate of change in slope, we can determine the slope or gradient of the piezometric surface at the outer face of the cylinder.

$$I_2 = I_1 + \frac{\partial^2 s}{\partial r^2}\,dr$$

$$= \frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}\,dr \tag{24}$$

Then the flow through the outer face is

$$Q_2 = -T\left(\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}\,dr\right)2\pi(r + dr) \tag{25}$$

The rate of change in volume within the cylindrical shell is expressed as

$$\frac{dv}{dt} = 2\pi r\,dr\,\frac{\partial s}{\partial t}\,S \tag{26}$$

where S is the coefficient of storage. For water-table conditions, S is equivalent to the specific yield of the materials de-watered by pumping. For artesian conditions, where water is drawn from storage by the compression of the aquifer, S is equal to the water obtained from a column of water-bearing material with a base one foot square and a height equal to the thickness of the aquifer.

Substituting the above values in equation 22

$$-T\frac{\partial s}{\partial r}2\pi r + T\left(\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}\,dr\right)2\pi(r + dr) = 2\pi r\,dr\,\frac{\partial s}{\partial t}\,S$$

$$-T\frac{\partial s}{\partial r}2\pi r + T\left[2\pi r\,\frac{\partial s}{\partial r} + 2\pi r\,dr\,\frac{\partial^2 s}{\partial r^2} + 2\pi\,dr\,\frac{\partial s}{\partial r} + 2\pi(dr)^2\,\frac{\partial^2 s}{\partial r^2}\right]$$
$$= 2\pi r\,dr\,\frac{\partial s}{\partial t}\,S$$

Dividing through by $2\pi r T\,dr$ and neglecting differentials higher than first order there follows

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t} \tag{27}$$

This is the differential equation for the radial flow of water in an elastic artesian aquifer. For a constant pumping rate Q, the solution of this equation is given by

$$s = \frac{Q}{4\pi T}\int_{r^2S/4Tt}^{\infty}\frac{e^{-u}}{u}\,du \tag{28}$$

Expressing Q in gallons per minute and T in gallons per day per foot, equation 28 is written thus

$$s = \frac{114.6Q}{T}\int_{1.87r^2S/Tt}^{\infty}\frac{e^{-u}\,du}{u} \tag{29}$$

$$\text{where } u = \frac{1.87r^2S}{Tt} \tag{30}$$

t = time since pumping started in days

Q = discharge of pumped well in gallons per minute

The expression in equation 29 is not directly integrable as an elementary function, but its value can be computed by the following series.

$$\int_{1.87r^2S/Tt}^{\infty} \frac{e^{-u}}{u}\,du$$
$$= W(u) = -0.577216 - \log_e u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} \cdots \tag{31}$$

As noted above, the exponential integral is written symbolically as $W(u)$, which in this usage is generally read "well function of u." Tables of the value of this exponential integral have been published.[1] Values of $W(u)$ for values of u from 10^{-15} to 9.9 as tabulated by Wenzel[2] are given in Table 11. Values in the table are values of $W(u)$ for different values u equal to N (Columns 1 and 18), multiplied by 10 to the various powers shown at the top of each remaining column. For example, when u has the value 5.0, $W(u)$ is determined from the line having $N = 5$ and Column 17 as 0.001148, or, when u has the value 0.005, $W(u)$ is 4.7261 (from the same line and Column 14).

From inspection of equations 29 and 30 it is seen that, if s can be measured for one or more values of r and for several values of t and if the discharge Q is known, S and T can be determined. However, the presence of two unknowns and the nature of the exponential integral makes it impossible to effect an exact analytical solution. Inasmuch as one of the unknowns, T, occurs twice in the equation, once in the argument of the function and again as a divisor of the exponential integral, solution by trial would be most laborious. However, a graphical method of superposition, devised by Theis, makes it possible to obtain a simple solution of

[1] *Smithsonian Physical Tables*, Table 32, eighth revised edition, 1933; the values to be used are those given for $Ei\ (-x)$, with the sign changed.

[2] L. K. Wenzel, Methods for Determining Permeability of Water-Bearing Materials, *U. S. Geological Survey Water-Supply Paper* 887, 1942, facing p. 89.

the equation. The first step in this method is the plotting of a "type curve," on logarithmic coordinate tracing paper, which represents the evaluation of the exponential integral or series of

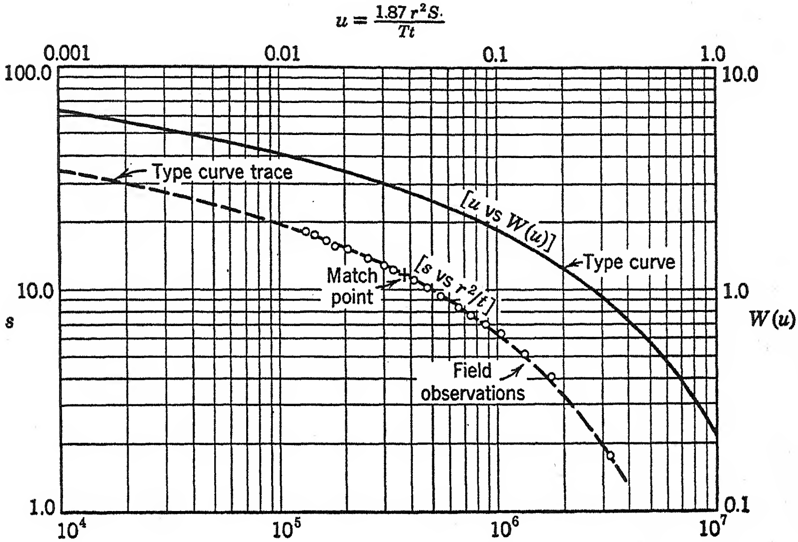


FIG. 83. Logarithmic graph of the exponential integral "type curve."

$Q = 250$ gpm

Match-point coordinates

$u = 0.1 \quad W(u) = 1.82$

$r^2/t = 3.75 \times 10^5 \quad s = 11.6$

$$T = \frac{114.6Q \cdot W(u)}{s} = \frac{114.6 \times 250 \times 182}{11.6} = 4500 \text{ gpd /ft}$$

$$S = \frac{uT}{1.87r^2/t} = \frac{0.1 \times 4500}{1.87 \times 3.75 \times 10^5} = 6.4 \times 10^{-4}$$

equation 31. From Table 11 values of $W(u)$ are plotted against the argument u to generate the type curve which is shown by a solid line in Fig. 83.

Rearranging equations 29 and 30 there follows

$$s = \left[\frac{114.6Q}{T}\right] W(u) \tag{32}$$

and

$$\frac{r^2}{t} = \left[\frac{T}{1.87S}\right] u \tag{33}$$

TABLE 11

Values of $W(u)$ for Nonequilibrium Formula

1	2	3	4	5	6	7	8	9
N	$N\times 10^{-15}$	$N\times 10^{-14}$	$N\times 10^{-13}$	$N\times 10^{-12}$	$N\times 10^{-11}$	$N\times 10^{-10}$	$N\times 10^{-9}$	$N\times 10^{-8}$
1.0	33.9616	31.6590	29.3564	27.0538	24.7512	22.4486	20.1460	17.8435
1.1	33.8662	31.5637	29.2611	26.9585	24.6559	22.3533	20.0507	17.7482
1.2	33.7792	31.4767	29.1741	26.8715	24.5689	22.2663	19.9637	17.6611
1.3	33.6992	31.3966	29.0940	26.7914	24.4889	22.1863	19.8837	17.5811
1.4	33.6251	31.3225	29.0199	26.7173	24.4147	22.1122	19.8096	17.5070
1.5	33.5561	31.2535	28.9509	26.6483	24.3458	22.0432	19.7406	17.4380
1.6	33.4916	31.1890	28.8864	26.5838	24.2812	21.9786	19.6760	17.3735
1.7	33.4309	31.1283	28.8258	26.5232	24.2206	21.9180	19.6154	17.3128
1.8	33.3738	31.0712	28.7686	26.4660	24.1634	21.8608	19.5583	17.2557
1.9	33.3197	31.0171	28.7145	26.4119	24.1094	21.8068	19.5042	17.2016
2.0	33.2684	30.9658	28.6632	26.3607	24.0581	21.7555	19.4529	17.1503
2.1	33.2196	30.9170	28.6145	26.3119	24.0093	21.7067	19.4041	17.1015
2.2	33.1731	30.8705	28.5679	26.2653	23.9628	21.6602	19.3576	17.0550
2.3	33.1286	30.8261	28.5235	26.2209	23.9183	21.6157	19.3131	17.0106
2.4	33.0861	30.7835	28.4809	26.1783	23.8758	21.5732	19.2706	16.9680
2.5	33.0453	30.7427	28.4401	26.1375	23.8349	21.5323	19.2298	16.9272
2.6	33.0060	30.7035	28.4009	26.0983	23.7957	21.4931	19.1905	16.8880
2.7	32.9683	30.6657	28.3631	26.0606	23.7580	21.4554	19.1528	16.8502
2.8	32.9319	30.6294	28.3268	26.0242	23.7216	21.4190	19.1164	16.8138
2.9	32.8968	30.5943	28.2917	25.9891	23.6865	21.3839	19.0813	16.7788
3.0	32.8629	30.5604	28.2578	25.9552	23.6526	21.3500	19.0474	16.7449
3.1	32.8302	30.5276	28.2250	25.9224	23.6198	21.3172	19.0146	16.7121
3.2	32.7984	30.4958	28.1932	25.8907	23.5881	21.2855	18.9829	16.6803
3.3	32.7676	30.4651	28.1625	25.8599	23.5573	21.2547	18.9521	16.6495
3.4	32.7378	30.4352	28.1326	25.8300	23.5274	21.2249	18.9223	16.6197
3.5	32.7088	30.4062	28.1036	25.8010	23.4985	21.1959	18.8933	16.5907
3.6	32.6806	30.3780	28.0755	25.7729	23.4703	21.1677	18.8651	16.5625
3.7	32.6532	30.3506	28.0481	25.7455	23.4429	21.1403	18.8377	16.5351
3.8	32.6266	30.3240	28.0214	25.7188	23.4162	21.1136	18.8110	16.5085
3.9	32.6006	30.2980	27.9954	25.6928	23.3902	21.0877	18.7851	16.4825
4.0	32.5753	30.2727	27.9701	25.6675	23.3649	21.0623	18.7598	16.4572
4.1	32.5506	30.2480	27.9454	25.6428	23.3402	21.0376	18.7351	16.4325
4.2	32.5265	30.2239	27.9213	25.6187	23.3161	21.0136	18.7110	16.4084
4.3	32.5029	30.2004	27.8978	25.5952	23.2926	20.9900	18.6874	16.3848
4.4	32.4800	30.1774	27.8748	25.5722	23.2696	20.9670	18.6644	16.3619
4.5	32.4575	30.1549	27.8523	25.5497	23.2471	20.9446	18.6420	16.3394
4.6	32.4355	30.1329	27.8303	25.5277	23.2252	20.9226	18.6200	16.3174
4.7	32.4140	30.1114	27.8088	25.5062	23.2037	20.9011	18.5985	16.2959
4.8	32.3929	30.0904	27.7878	25.4852	23.1826	20.8800	18.5774	16.2748
4.9	32.3723	30.0697	27.7672	25.4646	23.1620	20.8594	18.5568	16.2542
5.0	32.3521	30.0495	27.7470	25.4444	23.1418	20.8392	18.5366	16.2340
5.1	32.3323	30.0297	27.7271	25.4246	23.1220	20.8194	18.5168	16.2142
5.2	32.3129	30.0103	27.7077	25.4051	23.1026	20.8000	18.4974	16.1948
5.3	32.2939	29.9913	27.6887	25.3861	23.0835	20.7809	18.4783	16.1758
5.4	32.2752	29.9726	27.6700	25.3674	23.0648	20.7622	18.4596	16.1571
5.5	32.2568	29.9542	27.6516	25.3491	23.0465	20.7439	18.4413	16.1387
5.6	32.2388	29.9362	27.6336	25.3310	23.0285	20.7259	18.4233	16.1207
5.7	32.2211	29.9185	27.6159	25.3133	23.0108	20.7082	18.4056	16.1030
5.8	32.2037	29.9011	27.5985	25.2959	22.9934	20.6908	18.3882	16.0856
5.9	32.1866	29.8840	27.5814	25.2789	22.9763	20.6737	18.3711	16.0685
6.0	32.1698	29.8672	27.5646	25.2620	22.9595	20.6569	18.3543	16.0517
6.1	32.1533	29.8507	27.5481	25.2455	22.9429	20.6403	18.3378	16.0352
6.2	32.1370	29.8344	27.5318	25.2293	22.9267	20.6241	18.3215	16.0189
6.3	32.1210	29.8184	27.5158	25.2133	22.9107	20.6081	18.3055	16.0029
6.4	32.1053	29.8027	27.5001	25.1975	22.8949	20.5923	18.2898	15.9872
6.5	32.0898	29.7872	27.4846	25.1820	22.8794	20.5768	18.2742	15.9717
6.6	32.0745	29.7719	27.4693	25.1667	22.8641	20.5616	18.2590	15.9564
6.7	32.0595	29.7569	27.4543	25.1517	22.8491	20.5465	18.2439	15.9414
6.8	32.0446	29.7421	27.4395	25.1369	22.8343	20.5317	18.2291	15.9265
6.9	32.0300	29.7275	27.4249	25.1223	22.8197	20.5171	18.2145	15.9119
7.0	32.0156	29.7131	27.4105	25.1079	22.8053	20.5027	18.2001	15.8976
7.1	32.0015	29.6989	27.3963	25.0937	22.7911	20.4885	18.1860	15.8834
7.2	31.9875	29.6849	27.3823	25.0797	22.7771	20.4746	18.1720	15.8694
7.3	31.9737	29.6711	27.3685	25.0659	22.7633	20.4608	18.1582	15.8556
7.4	31.9601	29.6575	27.3549	25.0523	22.7497	20.4472	18.1446	15.8420
7.5	31.9467	29.6441	27.3415	25.0389	22.7363	20.4337	18.1311	15.8286
7.6	31.9334	29.6308	27.3282	25.0257	22.7231	20.4205	18.1179	15.8153
7.7	31.9203	29.6178	27.3152	25.0126	22.7100	20.4074	18.1048	15.8022
7.8	31.9074	29.6048	27.3023	24.9997	22.6971	20.3945	18.0919	15.7893
7.9	31.8947	29.5921	27.2895	24.9869	22.6844	20.3818	18.0792	15.7766
8.0	31.8821	29.5795	27.2769	24.9744	22.6718	20.3692	18.0666	15.7640
8.1	31.8697	29.5671	27.2645	24.9619	22.6594	20.3568	18.0542	15.7516
8.2	31.8574	29.5548	27.2523	24.9497	22.6471	20.3445	18.0419	15.7393
8.3	31.8453	29.5427	27.2401	24.9375	22.6350	20.3324	18.0298	15.7272
8.4	31.8333	29.5307	27.2282	24.9256	22.6230	20.3204	18.0178	15.7152
8.5	31.8215	29.5189	27.2163	24.9137	22.6112	20.3086	18.0060	15.7034
8.6	31.8098	29.5072	27.2046	24.9020	22.5995	20.2969	17.9943	15.6917
8.7	31.7982	29.4957	27.1931	24.8905	22.5879	20.2853	17.9827	15.6801
8.8	31.7868	29.4842	27.1816	24.8790	22.5765	20.2739	17.9713	15.6687
8.9	31.7755	29.4729	27.1703	24.8678	22.5652	20.2626	17.9600	15.6574
9.0	31.7643	29.4618	27.1592	24.8566	22.5540	20.2514	17.9488	15.6462
9.1	31.7533	29.4507	27.1481	24.8455	22.5429	20.2404	17.9378	15.6352
9.2	31.7424	29.4398	27.1372	24.8346	22.5320	20.2294	17.9268	15.6243
9.3	31.7315	29.4290	27.1264	24.8238	22.5212	20.2186	17.9160	15.6135
9.4	31.7208	29.4183	27.1157	24.8131	22.5105	20.2079	17.9053	15.6028
9.5	31.7103	29.4077	27.1051	24.8025	22.4999	20.1973	17.8948	15.5922
9.6	31.6998	29.3972	27.0946	24.7920	22.4895	20.1869	17.8843	15.5817
9.7	31.6894	29.3868	27.0843	24.7817	22.4791	20.1765	17.8739	15.5713
9.8	31.6792	29.3766	27.0740	24.7714	22.4688	20.1663	17.8637	15.5611
9.9	31.6690	29.3664	27.0639	24.7613	22.4587	20.1561	17.8535	15.5509

TABLE 11—(*Continued*)

Values of $W(u)$ for Nonequilibrium Formula

10	11	12	13	14	15	16	17	18
$N\times10^{-7}$	$N\times10^{-6}$	$N\times10^{-5}$	$N\times10^{-4}$	$N\times10^{-3}$	$N\times10^{-2}$	$N\times10^{-1}$	N	N
15.5409	13.2383	10.9357	8.6332	6.3315	4.0379	1.8229	0.2194	1.0
15.4456	13.1430	10.8404	8.5379	6.2363	3.9436	1.7371	.1860	1.1
15.3586	13.0560	10.7534	8.4509	6.1494	3.8576	1.6595	.1584	1.2
15.2785	12.9759	10.6734	8.3709	6.0695	3.7785	1.5889	.1355	1.3
15.2044	12.9018	10.5993	8.2968	5.9955	3.7054	1.5241	.1162	1.4
15.1354	12.8328	10.5303	8.2278	5.9266	3.6374	1.4645	.1000	1.5
15.0709	12.7683	10.4657	8.1634	5.8621	3.5739	1.4092	.08631	1.6
15.0103	12.7077	10.4051	8.1027	5.8016	3.5143	1.3578	.07465	1.7
14.9531	12.6505	10.3479	8.0455	5.7446	3.4581	1.3098	.06471	1.8
14.8990	12.5964	10.2939	7.9915	5.6906	3.4050	1.2649	.05620	1.9
14.8477	12.5451	10.2426	7.9402	5.6394	3.3547	1.2227	.04890	2.0
14.7989	12.4964	10.1938	7.8914	5.5907	3.3069	1.1829	.04261	2.1
14.7524	12.4498	10.1473	7.8449	5.5443	3.2614	1.1454	.03719	2.2
14.7080	12.4054	10.1028	7.8004	5.4999	3.2179	1.1099	.03250	2.3
14.6654	12.3628	10.0603	7.7579	5.4575	3.1763	1.0762	.02844	2.4
14.6246	12.3220	10.0194	7.7172	5.4167	3.1365	1.0443	.02491	2.5
14.5854	12.2828	9.9802	7.6779	5.3776	3.0983	1.0139	.02185	2.6
14.5476	12.2450	9.9425	7.6401	5.3400	3.0615	0.9849	.01918	2.7
14.5113	12.2087	9.9061	7.6038	5.3037	3.0261	.9573	.01686	2.8
14.4762	12.1736	9.8710	7.5687	5.2687	2.9920	.9309	.01482	2.9
14.4423	12.1397	9.8371	7.5348	5.2349	2.9591	.9057	.01305	3.0
14.4095	12.1069	9.8043	7.5020	5.2022	2.9273	.8815	.01149	3.1
14.3777	12.0751	9.7726	7.4703	5.1706	2.8965	.8583	.01013	3.2
14.3470	12.0444	9.7418	7.4395	5.1399	2.8668	.8361	.008939	3.3
14.3171	12.0145	9.7120	7.4097	5.1102	2.8379	.8147	.007891	3.4
14.2881	11.9855	9.6830	7.3807	5.0813	2.8099	.7942	.006970	3.5
14.2599	11.9574	9.6548	7.3526	5.0532	2.7827	.7745	.006160	3.6
14.2325	11.9300	9.6274	7.3252	5.0259	2.7563	.7554	.005448	3.7
14.2059	11.9033	9.6007	7.2985	4.9993	2.7306	.7371	.004820	3.8
14.1799	11.8773	9.5748	7.2725	4.9735	2.7056	.7194	.004267	3.9
14.1546	11.8520	9.5495	7.2472	4.9482	2.6813	.7024	.003779	4.0
14.1299	11.8273	9.5248	7.2225	4.9236	2.6576	.6859	.003349	4.1
14.1058	11.8032	9.5007	7.1985	4.8997	2.6344	.6700	.002969	4.2
14.0823	11.7797	9.4771	7.1749	4.8762	2.6119	.6546	.002633	4.3
14.0593	11.7567	9.4541	7.1520	4.8533	2.5899	.6397	.002336	4.4
14.0368	11.7342	9.4317	7.1295	4.8310	2.5684	.6253	.002073	4.5
14.0148	11.7122	9.4097	7.1075	4.8091	2.5474	.6114	.001841	4.6
13.9933	11.6907	9.3882	7.0860	4.7877	2.5268	.5979	.001635	4.7
13.9723	11.6697	9.3671	7.0650	4.7667	2.5068	.5848	.001453	4.8
13.9516	11.6491	9.3465	7.0444	4.7462	2.4871	.5721	.001291	4.9
13.9314	11.6289	9.3263	7.0242	4.7261	2.4679	.5598	.001148	5.0
13.9116	11.6091	9.3065	7.0044	4.7064	2.4491	.5478	.001021	5.1
13.8922	11.5896	9.2871	6.9850	4.6871	2.4306	.5362	.0009086	5.2
13.8732	11.5706	9.2681	6.9659	4.6681	2.4126	.5250	.0008086	5.3
13.8545	11.5519	9.2494	6.9473	4.6495	2.3948	.5140	.0007198	5.4
13.8361	11.5336	9.2310	6.9289	4.6313	2.3775	.5034	.0006409	5.5
13.8181	11.5155	9.2130	6.9109	4.6134	2.3604	.4930	.0005708	5.6
13.8004	11.4978	9.1953	6.8932	4.5958	2.3437	.4830	.0005085	5.7
13.7830	11.4804	9.1779	6.8758	4.5785	2.3273	.4732	.0004532	5.8
13.7659	11.4633	9.1608	6.8588	4.5615	2.3111	.4637	.0004039	5.9
13.7491	11.4465	9.1440	6.8420	4.5448	2.2953	.4544	.0003601	6.0
13.7326	11.4300	9.1275	6.8254	4.5283	2.2797	.4454	.0003211	6.1
13.7163	11.4138	9.1112	6.8092	4.5122	2.2645	.4366	.0002864	6.2
13.7003	11.3978	9.0952	6.7932	4.4963	2.2494	.4280	.0002555	6.3
13.6846	11.3820	9.0795	6.7775	4.4806	2.2346	.4197	.0002279	6.4
13.6691	11.3665	9.0640	6.7620	4.4652	2.2201	.4115	.0002034	6.5
13.6538	11.3512	9.0487	6.7467	4.4501	2.2058	.4036	.0001816	6.6
13.6388	11.3362	9.0337	6.7317	4.4351	2.1917	.3959	.0001621	6.7
13.6240	11.3214	9.0189	6.7169	4.4204	2.1779	.3883	.0001448	6.8
13.6094	11.3068	9.0043	6.7023	4.4059	2.1643	.3810	.0001293	6.9
13.5950	11.2924	8.9899	6.6879	4.3916	2.1508	.3738	.0001155	7.0
13.5808	11.2782	8.9757	6.6737	4.3775	2.1376	.3668	.0001032	7.1
13.5668	11.2642	8.9617	6.6598	4.3636	2.1246	.3599	.00009219	7.2
13.5530	11.2504	8.9479	6.6460	4.3500	2.1118	.3532	.00008239	7.3
13.5394	11.2368	8.9343	6.6324	4.3364	2.0991	.3467	.00007364	7.4
13.5260	11.2234	8.9209	6.6190	4.3231	2.0867	.3403	.00006583	7.5
13.5127	11.2102	8.9076	6.6057	4.3100	2.0744	.3341	.00005886	7.6
13.4997	11.1971	8.8946	6.5927	4.2970	2.0623	.3280	.00005263	7.7
13.4868	11.1842	8.8817	6.5798	4.2842	2.0503	.3221	.00004707	7.8
13.4740	11.1714	8.8689	6.5671	4.2716	2.0386	.3163	.00004210	7.9
13.4614	11.1589	8.8563	6.5545	4.2591	2.0269	.3106	.00003767	8.0
13.4490	11.1464	8.8439	6.5421	4.2468	2.0155	.3050	.00003370	8.1
13.4367	11.1342	8.8317	6.5298	4.2346	2.0042	.2996	.00003015	8.2
13.4246	11.1220	8.8195	6.5177	4.2226	1.9930	.2943	.00002699	8.3
13.4126	11.1101	8.8076	6.5057	4.2107	1.9820	.2891	.00002415	8.4
13.4008	11.0982	8.7957	6.4939	4.1990	1.9711	.2840	.00002162	8.5
13.3891	11.0865	8.7840	6.4822	4.1874	1.9604	.2790	.00001936	8.6
13.3776	11.0750	8.7725	6.4707	4.1759	1.9498	.2742	.00001733	8.7
13.3661	11.0635	8.7610	6.4592	4.1646	1.9393	.2694	.00001552	8.8
13.3548	11.0523	8.7497	6.4480	4.1534	1.9290	.2647	.00001390	8.9
13.3437	11.0411	8.7386	6.4368	4.1423	1.9187	.2602	.00001245	9.0
13.3326	11.0300	8.7275	6.4258	4.1313	1.9087	.2557	.00001115	9.1
13.3217	11.0191	8.7166	6.4148	4.1205	1.8987	.2513	.000009988	9.2
13.3109	11.0083	8.7058	6.4040	4.1098	1.8888	.2470	.000008948	9.3
13.3002	10.9976	8.6951	6.3934	4.0992	1.8791	.2429	.000008018	9.4
13.2896	10.9870	8.6845	6.3828	4.0887	1.8695	.2387	.000007185	9.5
13.2791	10.9765	8.6740	6.3723	4.0784	1.8599	.2347	.000006439	9.6
13.2688	10.9662	8.6637	6.3620	4.0681	1.8505	.2308	.000005771	9.7
13.2585	10.9559	8.6534	6.3517	4.0579	1.8412	.2269	.000005173	9.8
13.2483	10.9458	8.6433	6.3416	4.0479	1.8320	.2231	.000004637	9.9

If a constant withdrawal rate is maintained, that is, if Q is constant, the bracketed portions of equations 32 and 33 are constant for a given pumping test. Note that s is related to r^2/t in a manner that is similar to the relation of $W(u)$ to u. Consequently, if values of the drawdown or recovery, s, are plotted against r^2/t on logarithmic coordinate tracing paper, to the same scale as the type curve, $W(u)$ versus u, the curve of observed data will be similar to the type curve.

The circles on Fig. 83 represent the successive values of drawdown as computed from periodic measurements of the water-level decline in an observation well 48 ft from a well that was pumped at the constant rate of 250 gpm. In practice, the computed values of s and r^2/t are plotted on a separate sheet of logarithmic tracing paper and this graph of observed data is superimposed on the graph of the type curve. When the coordinate axes of the two curves are held parallel, the data curve is translated to a position which represents the best fit of the field data to the type curve. With both graph sheets at the best match position, an arbitrary point on the top curve is selected and pricked through or otherwise marked on the lower curve. The coordinates of this common point are noted for the upper and the lower graph. The trace of the type curve, for the match position, is shown as a dashed line through the field data, on Fig. 83. The coordinates of the match point are recorded, and the use of these data with equations 32 and 33 to solve for T and S is also shown.

The determination of the coefficients of transmissibility and storage for an aquifer, by the discharging-well method, is somewhat similar to the testing procedure of measuring beam deflections under a given load to determine the elastic properties of a structural material. As in the case of the beam, we can predict the drawdown or deflection of the water level at any point or for any load from a knowledge of its behavior under a known load at known points of observation.

Assume that a well of 300-gpm capacity is to be drilled in the comparatively poor aquifer covered by the pumping test of Fig. 83 and that the following conditions apply for this example.

Total depth of well	= 200 ft
Screen setting	= 170 to 200 ft
Diameter of well	= 12 in.
Proposed yield	= 300 gpm
Static depth to water	= 5 ft below grade

The problem is to predict the performance of this well for 30 days of continuous operation at peak capacity with total withdrawal from storage, that is, no recharge from rainfall or other sources. From the above conditions we may set up the known quantities as follows:

$$Q = 300 \text{ gpm} \qquad r = 6 \text{ in.} = 0.5 \text{ ft}$$
$$t = 30 \text{ days} \qquad T = 4500 \text{ gpd/ft}$$
$$S = 6.4 \times 10^{-4}$$

Then from equation 30

$$u = \frac{1.87(0.5)^2 6.4}{4.5 \times 10^3 \times 30 \times 10^4} = 2.2 \times 10^{-9}$$

The value of $W(u)$ corresponding to the above value of u is read from Table 11 as

$$W(u) = 19.4$$

From equation 32

$$s = \frac{114.6 \times 300 \times 19.4}{4500} = 148 \text{ ft}$$

The pumping level at the end of the 30-day period is

Static level	=	5 ft below grade
Drawdown	=	148 ft
Pumping level	=	153 ft below grade

The estimated pumping level is only 17 ft above the top of the well screen by the end of the 30-day period if the well is pumped continuously at the maximum rate. Consequently, it is advisable to examine the performance of this proposed well over longer periods of pumping. In the manner outlined above for the computation of the 30-day pumping level, the levels for other periods are computed and plotted as the lowermost curve on Fig. 84. As indicated by this curve, the pumping level will reach the top of the well screen after 9 mo of continuous pumping at the 300-gpm rate. We assume that the aquifer is completely penetrated[1] and that the top of the aquifer coincides with the top of the well screen. Accordingly, the lowering of the water level in the pumped well below the top of the aquifer results in partial dewatering of the

[1] The discharge Q will be smaller for a partially penetrating well. For a discussion of the effect of partial penetration on well yield see reference 30 at the end of this chapter.

aquifer. This reduction in saturated thickness proportionately reduces the transmissibility and thereby increases the drawdown and furthers the decline of pumping level for the given discharge rate. Extensive dewatering of an aquifer develops a "vicious cycle" of increased drawdown followed by decreased aquifer thickness, a condition which occasions additional drawdown and further reduction in aquifer capacity until the well fails at the excessive pumping rate. If the rate of pumping is reduced, the water level in the pumped well will recover in proportion to the reduction in discharge. For the above example reducing the pumping rate to

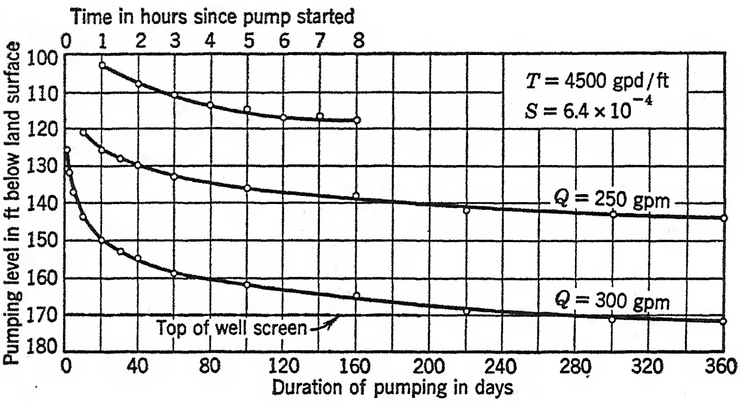


FIG. 84.

250 gpm raises the pumping level to 28 ft above the top of the well screen at the time when the 300-gpm rate would start dewatering the aquifer.

In the above example, the original assumption that all water would be withdrawn from storage without replenishment results in a progressive decline of water level to infinite time. Such calculations are of value in estimating minimum performance of wells under adverse conditions of extended drought. Under normal conditions, as pumping continues, the cone of depression deepens and expands until (1) it intercepts a surface stream which is adequate to support the well discharge under the given conditions, (2) it encompasses an area that will support the well yield under the prevailing rate of surface infiltration, or (3) it intercepts areas of discharge and reduces this discharge by an amount equal to the well withdrawal. Most frequently, the well yield is obtained from a

combination of two or more of these sources. If the total water available from the several sources is less than the pumping rate, progressive decline of water level may occur to a degree determined by the excess of discharge over recharge. Ultimately the net discharge cannot be greater than the available recharge from all sources.

As indicated in the above example, excessive dewatering of an aquifer is to be avoided, but it does not follow that all dewatering is objectionable. In any water-table aquifer it is of course necessary to partially dewater the aquifer to induce flow toward the well. Sometimes it may be advisable to reduce the saturated thickness to provide storage capacity for the intake of percolating waters when infiltration occurs. If the cone of depression is maintained at a high stage and a shallow depth below land surface, then, when recharge occurs, all water in excess of the limited storage capacity will be rejected. The problem of determining proper pumping levels and well yields is one of engineering economics, which must be solved for each particular well field.

It is of interest to note that the majority of well installations to date were rated or sized without benefit of long-term drawdown predictions. The most general and widespread method in use for determining well capacity is to make a brief and generally inadequate pumping test. Measurements are made of the well discharge and drawdown through this short pumping period. Water levels are measured by an air line and altitude gage and are accurate to perhaps the nearest foot. If the pumping level remains relatively steady during the latter part of the test period, as it generally does when the pumping rate is constant because the decline is rather slow after several hours, it is quite common practice to assume that the well will continue to pump at the observed rate for an infinite time. The pumping levels for the above example are computed for an 8-hr period and a discharge of 300 gpm, and plotted as the uppermost curve on Fig. 84. The level from the sixth to the eighth hour inclusive does not decline more than 2 ft and air-line readings would probably show little or no change in drawdown for this period. Thus, in accord with the above-mentioned practice, it is assumed that this well will deliver 300 gpm or more for any period. The fallacy of this reasoning is better understood after examination of the modified form of the nonequilibrium formula.

Modified Nonequilibrium Formula

It was recognized by Jacob[1] that the sum of the terms in the series of equation 31 beyond $\log_e u$ is not of appreciable magnitude when u becomes small. From the form of equation 30 it is noted that u decreases as the time, t, increases. Accordingly, for large values of t, the terms beyond $\log_e u$ in the exponential series may be neglected and equation 32 may be written

$$s = \frac{114.6Q}{T} W(u) = \frac{114.6Q}{T}[-0.5772 - \log_e u]$$

or

$$s = \frac{114.6Q}{T}\left[\log_e\left(\frac{1}{u}\right) - 0.5772\right] \tag{34}$$

but

$$u = \frac{1.87r^2S}{Tt} \qquad \text{and} \qquad \frac{1}{u} = \frac{Tt}{1.87r^2S}$$

then

$$s = \frac{114.6Q}{T}\left[\log_e\left(\frac{Tt}{1.87r^2S}\right) - 0.5772\right] \tag{35}$$

In applying the above equation to measurements of the draw-down or recovery of water level in a particular observation well, the distance r will be constant and there follows

at time t_1

$$s_1 = \frac{114.6Q}{T}\left[\log_e\left(\frac{Tt_1}{1.87r^2S}\right) - 0.5772\right]$$

at time t_2

$$s_2 = \frac{114.6Q}{T}\left[\log_e\left(\frac{Tt_2}{1.87r^2S}\right) - 0.5772\right]$$

then the change in drawdown from time t_1 to t_2

$$s_2 - s_1 = \frac{114.6Q}{T}\log_e\left(\frac{t_2}{t_1}\right) \tag{36}$$

Converting to logarithms to the base 10

$$s_2 - s_1 = \frac{264Q}{T}\log_{10}\left(\frac{t_2}{t_1}\right) \tag{37}$$

[1] C. E. Jacob, Drawdown Test to Determine Effective Radius of Artesian Well, *Proc. A.S.C.E.*, vol. 72, No. 5 (May 1946), pp. 629–646.

where Q and T are as previously defined, t_1 and t_2 are time in days since pumping started, and s_1 and s_2 are respective drawdowns at noted times, in feet.

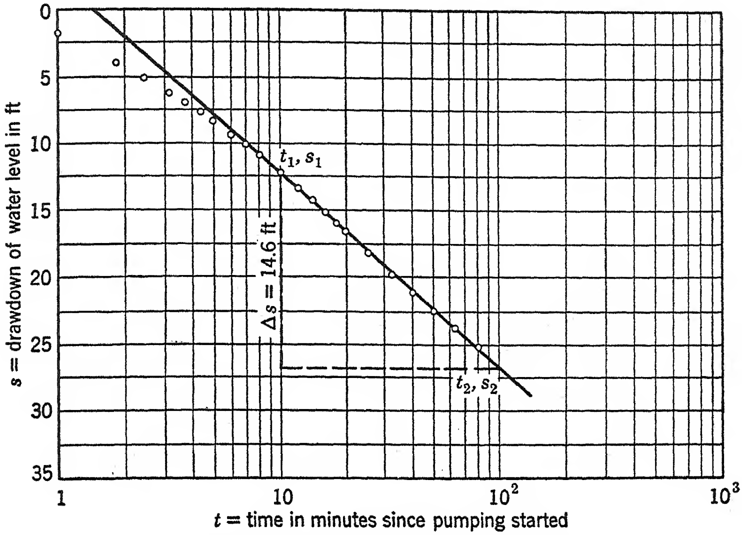


FIG. 85. Semilog graph of pumping test data for application of modified Theis formula.

$$Q = 250 \text{ gpm}$$

$$r = 48 \text{ ft}$$

$$T = \frac{264Q \cdot \log_{10} t_2/t_1}{s_2 - s_1}$$

$$T = \frac{264 \times 250 \times \log_{10} 100/10}{26.8 - 12.2}$$

$$T = 4500 \text{ gpd/ft}$$

$$S = \frac{0.3Tt_0}{r^2}$$

$$S = \frac{0.3 \times 4500 \times \frac{1.46}{1440}}{48^2}$$

$$S = 6.0 \times 10^{-4}$$

The most convenient procedure for application of the above equation is to plot the observational data for each well on semilogarithmic coordinate paper as shown by Fig. 85, which is based on the same test data used in Fig. 83. From this curve make an arbitrary choice of t_1 and t_2 and note the corresponding values of

s_1 and s_2. For convenience t_1 and t_2 are chosen one log cycle apart; then

$$\log_{10}\left(\frac{t_2}{t_1}\right) = 1$$

and

$$s_2 - s_1 = \Delta s = \frac{264Q}{T} \tag{38}$$

or

$$T = \frac{264Q}{\Delta s} \tag{39}$$

where Δs is the drawdown difference per log cycle, in feet.

Although the absolute value of the drawdown increases as the logarithm of the time of pumping, it follows from equation 38 that the drawdown per log cycle of time varies directly with the discharge Q, and inversely as the coefficient of transmissibility T.

Extrapolating the straight line of the semilog curve to its intersection with the zero-drawdown axis permits computation of S, the storage coefficient, as follows.

When $s = 0$, equation 24 yields

$$s = 0 = \frac{114.6Q}{T}\left[\log_e\left(\frac{Tt_0}{1.87r^2S}\right) - 0.5772\right]$$

$$\log_e\left(\frac{Tt_0}{1.87r^2S}\right) = 0.5772 \qquad \text{or} \qquad \frac{Tt_0}{1.87r^2S} = e^{0.5772}$$

and

$$S = \frac{Tt_0}{1.87r^2e^{0.5772}} = \frac{0.3Tt_0}{r^2} \tag{40}$$

where S, T, and r are as previously defined and t_0 is time intercept on zero-drawdown axis, in days.

The curvature of the semilog graph where time t is small indicates that the approximation is not valid in this region. Caution should be exercised in the use of this method to make certain that pumping has continued until t becomes large and all plotted points fall on a straight line. For moderate distances from the pumped well, this condition is generally satisfied within an hour or less for artesian conditions, but for water-table aquifers 12 hr or more may be required because of the time lag due to slow draining of the interstices.

It is advisable for any test to plot both the log-log graph for the type curve application and the semilog plot for the modified formula. The semilog plot, as indicated above, is an approximation method which yields a straight-line graph in the region where this approximation is justified. As shown by Fig. 83, when t is small and u is appreciable, the field observations fall beneath the selected line. As the distance, r, increases, the time required for the field data to become asymptotic to the limiting value of the slope also increases. Thus, for an observation well at a considerable distance from the pumped well, there is a greater risk in selecting the tangent prematurely. Assurance is gained by comparison with the result given by the log-log plot and by mutual agreement with the results from other observation wells. The interference effect of other pumping wells, the limitation of geologic boundaries, and other extraneous effects may distort the plotted data in a manner that makes the straight-line selection unreliable. The semilog plot does, however, present certain advantages in pumping-test analysis and its use is well justified as long as proper caution is observed. If analysis of the data is correct, the values of T and S determined by either method should agree within a small percentage.

In general, it is not possible to determine the storage coefficient, S, from observations within the pumping well, by either of the above methods because the effective radius of the well is not known. For a well finished in rock, the effective radius may approach the nominal radius, but not necessarily because the existence of large crevices, the overreaming action of the drill bit, or local cementation of the bore face may result in an effective radius greater or smaller than the nominal size. In a well finished in unconsolidated materials, the water level in the pumped well is lower than the water level in an equivalent uncased hole by the amount of friction loss through the screen. If development of the well is incomplete, the packing of fine material in the formation adjacent to the screen can greatly reduce the permeability and result in an effective radius which is considerably less than the nominal drilled size. A method of determining the effective radius for any well has been developed by Jacob.[1]

Noting from equation 35 that the drawdown varies with the

[1] C. E. Jacob, Drawdown Test to Determine Effective Radius of Artesian Well, *Proc. A.S.C.E.*, vol. 72, No. 5 (May 1946), pp. 629–646.

logarithm of the time of pumping we can recognize the fallacy in determining well capacity from the short-duration type of pumping test which was outlined previously. Any linear plot of drawdown level versus pumping time, on a time scale extended to fit an 8-hr pumping test as shown by the uppermost curve of Fig. 83 will invariably give the deceptive picture of the approach to a fixed pumping level. This flattening of slope is representative of a logarithmic relation when plotted to rectangular coordinates. If the pumping levels for the 8-hr test were plotted to the compressed time scale of the lower graph, there would appear to be a nearly vertical decline of operating level for the 8-hr period. When one considers that a well user may plan to pump a well continuously for the total of 8760 hr per year and for many years, it is evident that the 8-hr or other short-term acceptance test is not only inadequate but quite misleading unless supported by quantitative methods such as those outlined above.

Adjustment of Test Data for Thin Aquifers

One of the basic assumptions in the derivation of the Thiem and the Theis formulas is the stipulation of a constant value of transmissibility. However, under water-table conditions the drawdown of water level by a discharging well reduces the saturated thickness of the aquifer, and, if this reduction in thickness is appreciable, the transmissibility is not constant but decreases with time. The following method, described by Jacob,[1] permits the correction of observed data to compensate for this effect. Appreciable reduction in saturated thickness voids the relation expressed by equation 18, and consequently equation 15 must be reduced to the drawdown form in some other manner. From Fig. 81 note that

$$h = m - s$$

then

$$h_2^2 = m^2 - 2ms_2 + s_2^2$$

$$h_1^2 = m^2 - 2ms_1 + s_1^2$$

and

$$h_2^2 - h_1^2 = -2ms_2 + 2ms_1 + s_2^2 - s_1^2$$

$$= 2ms_1 - s_1^2 - (2ms_2 - s_2^2)$$

[1] C. E. Jacob, Notes on Determining Permeability by Pumping Tests under Water-Table Conditions, U. S. Geological Survey, mimeographed report, June 1944.

$$h_2^2 - h_1^2 = 2m\left[\left(s_1 - \frac{s_1^2}{2m}\right) - \left(s_2 - \frac{s_2^2}{2m}\right)\right] \tag{41}$$

Substituting the above expression in equation 15 there follows

$$\log_e \frac{r_2}{r_1} = \frac{\pi P}{Q_d}\, 2m\left[\left(s_1 - \frac{s_1^2}{2m}\right) - \left(s_2 - \frac{s_2^2}{2m}\right)\right]$$

but

$$T = Pm$$

then

$$\log_e \frac{r_2}{r_1} = \frac{2\pi T}{Q_d}\left[\left(s_1 - \frac{s_1^2}{2m}\right) - \left(s_2 - \frac{s_2^2}{2m}\right)\right] \tag{42}$$

Converting to logarithms to the base 10, replacing Q_d by Q, and transposing equation 42, there follows

$$T = \frac{527.7Q \log_{10} r_2/r_1}{[(s_1 - s_1^2/2m) - (s_2 - s_2^2/2m)]} \tag{43}$$

The above equation should be used in lieu of equation 21 if the saturated thickness of the aquifer is appreciably diminished by the declining water level. Note that if the drawdowns s_1 and s_2 are very small compared to the original saturated thickness, m, the correction fraction may be omitted, and equation 43 reduces to equation 21. Compensation of the drawdown data by subtracting the factor $(s^2/2m)$ should result in a straight-line graph for the semilog plottings of the Thiem or modified Theis method.

The Method of Images

The assumption of infinite areal extent for an aquifer, which is necessary for the development of either the equilibrium or the nonequilibrium formula, is essentially fulfilled by a few major aquifers of sedimentary rock, such as the Dakota sandstone described by Meinzer.[1] However, in most areas the existence of formation boundaries or of folds and faults or the dissection by surface streams serves to limit the continuity of consolidated strata to distances of a few miles or more. In the unconsolidated materials and particularly in the glaciated areas the prerequisite

[1] O. E. Meinzer and H. A. Hard, The Artesian-Water Supply of the Dakota Sandstone in North Dakota, with Special Reference to the Edgeley Quadrangle, *U. S. Geological Survey Water-Supply Paper* 520, 1925, pp. 73–95.

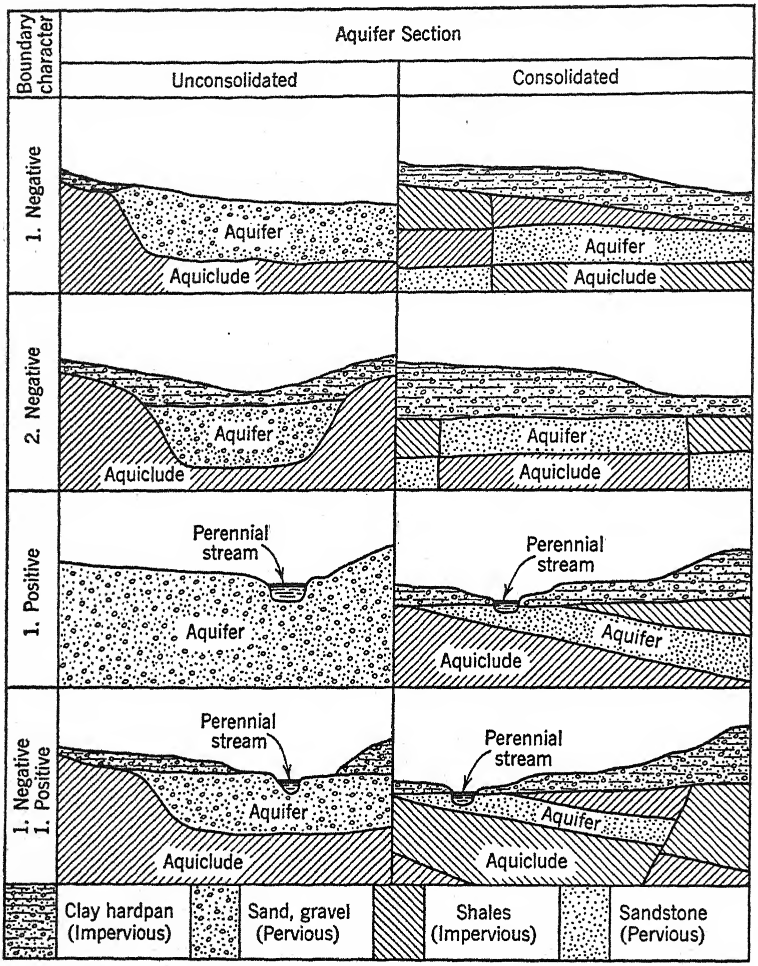


Fig. 86. Idealized examples showing possible geologic structures which form definite bounding conditions for the flow systems in aquifers. The term negative is used for boundaries formed by impermeable materials which do not contribute flow to the system. Positive boundaries refer to the intersection of the aquifer by sources of water which sustain the hydrostatic head and stop the growth of the drawdown cone.

of infinite areal extent is seldom satisfied. Consequently, it is necessary to make appropriate adjustment for the effect of these geologic boundaries before the above formulas can be applied to problems of flow in areally limited aquifers.

Inasmuch as an impervious formation detracts from the contributing area of the aquifer it bounds, we refer to its contact as a negative boundary. In a similar manner, we use the term positive boundary where an aquifer is intersected by a perennial stream or other body of surface water with sufficient flow to prevent development of the cone of depression beyond the surface source. Several possible types of geologic boundaries are shown in generalized form by Fig. 86.

It is recognized that, except for some faulted structures, most geologic boundaries do not occur as abrupt straight-line demarcations but rather as tapered and irregular terminals. In general, however, the area covered by a well-field or pumping-test site is relatively small compared to the area of even the limited aquifers, and it is often possible to treat the geologic boundary as an abrupt discontinuity. The greater the distance to the boundary from the well site the smaller would be the error involved by this approximation. Where conditions permit the assumption of a straight-line demarcation for a geologic boundary, it is possible to solve the flow problem by the substitution of a hypothetical system that satisfies the limits of the real system.

The method of images devised by Lord Kelvin in his work on electrostatic theory is a convenient tool for the solution of boundary problems. An idealized section of an aquifer that is intersected and bounded by a surface stream is shown by Fig. 87. To be effective as a boundary, the stream flow must equal or exceed the withdrawal of the well because any flow below the well yield would result in drying up of the stream and elimination of the boundary. Assume the stream to be of infinitesimal width or the equivalent of a line source. In a rigid analysis it would be necessary also that the stream extend the full depth of the aquifer to justify fully the use of unidimensional method. However, reasonable estimates can be made by this method when observation wells are available at distances that are sufficient to minimize the effect of vertical flow components. If the stream cannot be depleted, the boundary limit requires that there shall be zero drawdown at the line source. Any system that can satisfy this boundary limit is a solution of the real

problem. As shown by the central diagram of Fig. 87, the real and bounded aquifer is replaced by an imaginary aquifer of infinite areal extent and an imaginary recharging well is placed on the opposite side of and equidistant from the boundary. As illustrated,

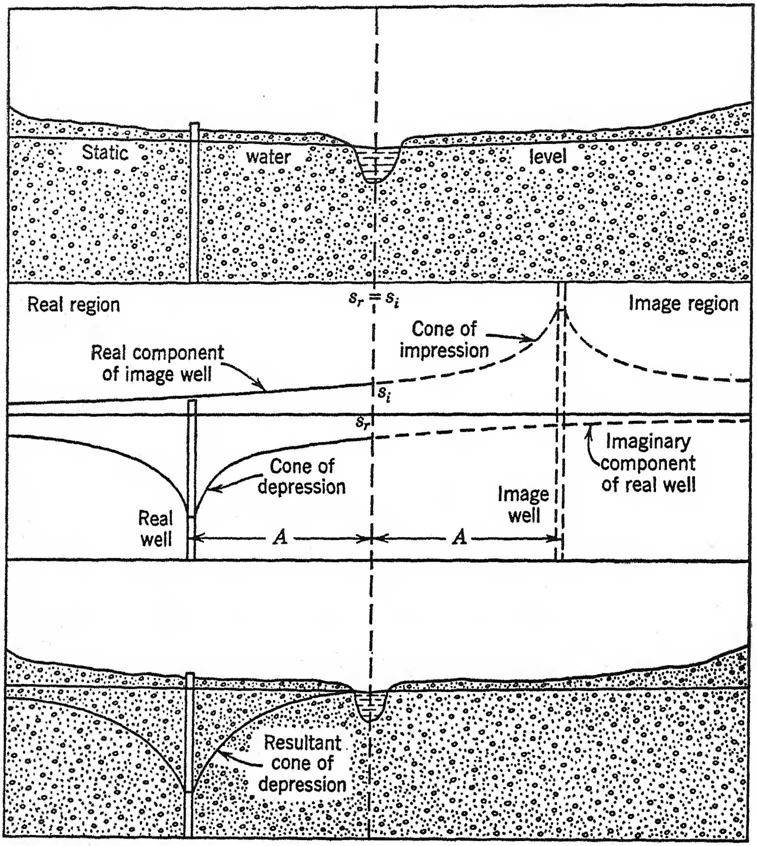


Fig. 87. Idealized section of an aquifer which is intersected by a stream together with a hypothetical well system for the solution of this type of flow problem.

the imaginary recharge well returns water to the aquifer at the same rate as it is withdrawn by the real discharge well. Consequently, the image well produces a build-up of water level at the boundary that is exactly equal to and cancels the drawdown of the real well. This system results in zero drawdown at the boundary which satisfies the limit of the real problem.

The real components of the cone of depression of the real well and the cone of impression of the image well are shown as solid lines in the region of real values. To secure the resultant cone of depression or to evaluate the drawdown at any point in the real

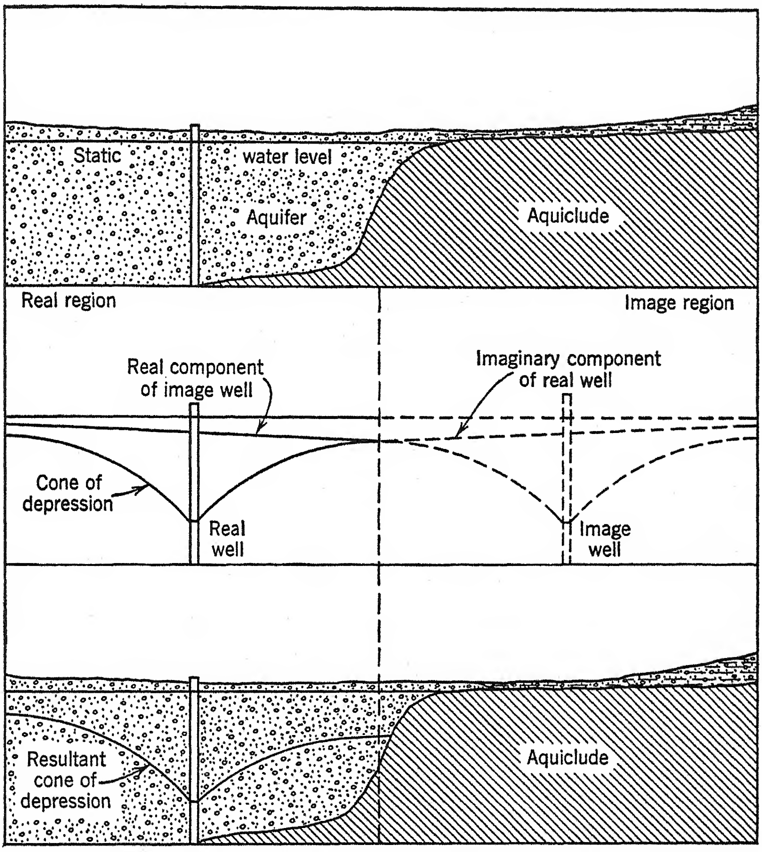


FIG. 88. Idealized section of an aquifer bounded by an impermeable formation together with a hypothetical well system for the solution of this type of flow problem.

region, it is necessary to add algebraically the real components of the several depression cones. The resultant cone of depression is steepened on the riverward side of the well and flattened on the landward side. This point should be recognized in drawing or examining contour maps of the water-table or piezometric surface for aquifers of this type.

An aquifer bounded by impervious strata is shown in idealized section by Fig. 88. The boundary is approximated as before by a sharp line of demarcation. Inasmuch as the impervious strata cannot contribute water to the pumped well, the limit imposed by the barrier is that there shall be no flow across the boundary line. As shown by the image setup in the central diagram, an imaginary discharging well is placed across the boundary at a right angle to and equidistant from the boundary. The drawdown of the image well at the boundary is equal to the effect of the real well, and the symmetrical drawdown cones produce a ground-water divide everywhere along the boundary. The image system produces a normal derivative equal to zero along the divide, and, as there can be no flow across a divide, this image system satisfies the limit of the real problem and is therefore a solution.

The real components of the cones of depression of the image well and the real well are shown as solid lines in the region of real values. Again, the resultant cone of depression or the drawdown at any point in the aquifer is determined by adding the real component of each depression cone. The resultant cone of depression for this example is flattened on the side adjacent to the boundary and steepened on the opposite side of the well.

An example of an aquifer bounded by impervious strata on two sides is shown by the idealized section of Fig. 89. The setup of the primary images to balance the effect of the real well at each boundary is similar to the previous examples. The real component of each cone of depression is shown as a solid line in the region of real values. Although these primary images balance the effect of the real well at their respective boundaries, each image produces an unbalanced drawdown at the farther boundary. These unbalanced drawdowns at the boundaries theoretically produce a gradient and consequent flow across the boundary. Thus it is necessary to add a secondary set of image wells at the appropriate distances to balance the residual effect of the primary images. Each image well in the secondary set will again disturb the balance at the farther boundary and all successive sets of images to infinity will leave residuals at the boundaries. In practice it is necessary only to add image pairs until the residual effects are negligible in comparison with the total effect.

The modified nonequilibrium formula illustrated in Fig. 85 is particularly advantageous for the analysis of image or boundary

effects because it is easier to recognize changes in slope of a straight line than to detect changes in curvature of a log-log curve. Equation 38 indicates that $\Delta s/1$, the slope of the semilog graph, is dependent only on the pumping rate and the transmissibility of the aquifer.

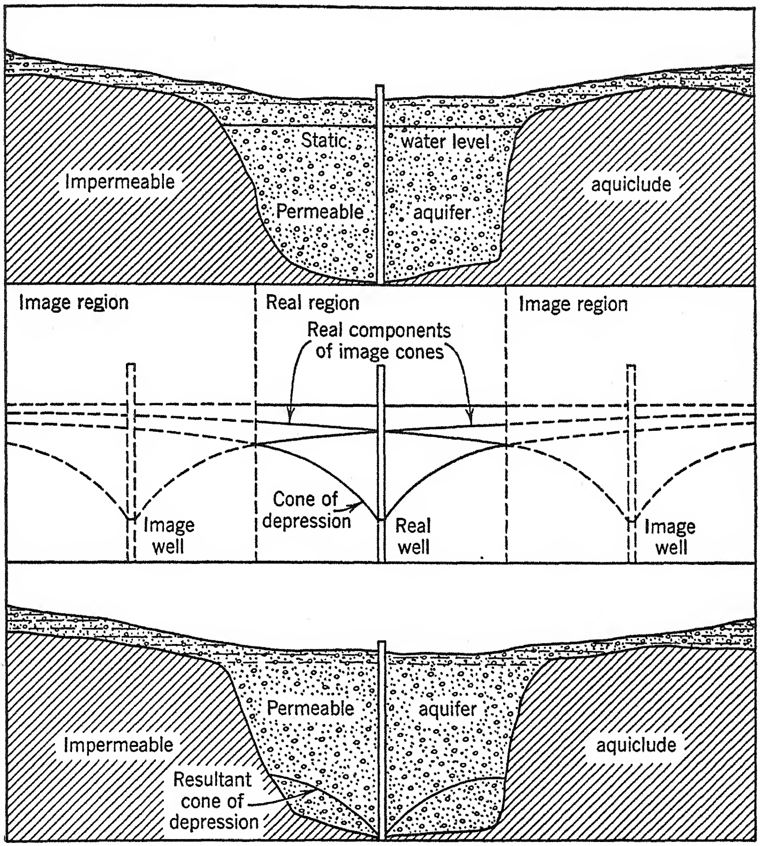


FIG. 89. Idealized section of underflow channel bounded by impermeable formation and setup of hypothetical well system used for solution of flow problems under this condition.

For a specific aquifer the transmissibility is a constant and the rate of pumping may be held constant. Under these conditions the graph of drawdown versus the logarithm of time since the pump started will show successive changes in slope as pumping continues. The water level will draw down at an initial rate under the influence

of the real well that is nearest to the observation well. When the cone of depression of the nearer image well affects the observation well, the rate of drawdown will be doubled after r^2/t becomes small, because the total rate of withdrawal is then equal to the rate of the real well plus one image well, or twice the rate of the real well. The effect of the farther image triples the slope of the semilog graph after r^2/t becomes small for this image, because the total rate for the real well plus two image wells is three times as great as the rate of the real well.

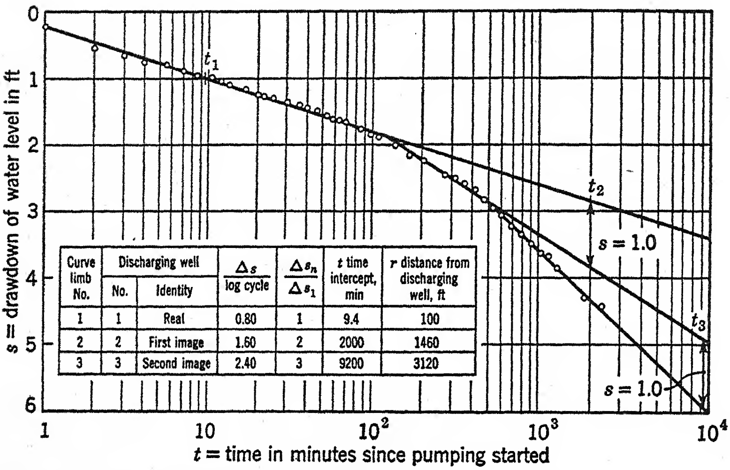


FIG. 90. Graph showing drawdown of water level in observation Well E-1 by pumping tests on Test Well 36 at Hemphill Road and Saginaw Street, at southern city limits of Flint, Mich.

As previously mentioned, the use of Jacob's approximate method should be restricted to small values of u, which occur when the distance r is small and the time t is large. Frequently image distances are large in relation to the distance from an observation well to a pumping well. If a single image is involved, the semilog plotting of drawdown versus time shows a two-limbed graph. The transition from the first limb to the second limb follows a curved path through the region where the values of u for the image well are large. When t becomes sufficiently large in relation to r^2, the value of u for the image well becomes small and the observed data follow the straight line of the second segment of the graph.

If more than one image well occurs and if the image wells are at comparable distances, then the effect of second-, third-, or higher-order image wells may reach an observation well before sufficient time has elapsed for u to become small enough to warrant the application of this method to the effect of the first image well. Under these circumstances, the observed data follow a path of increasing curvature. An example of a semilog graph for an observation well affected by second- and higher-order image wells is shown as Fig. 90.

Approximate values for the location of the image wells are obtained by drawing tangents to the observed-data graph at the appropriate slope values. If the divergence of each line of Fig. 90 were noted, the graph could be separated and replotted as three separate lines with each component having the same slope and intercepting the zero-drawdown axis at its respective value of time. The time intercept on this axis permits the calculation of the storage coefficient by equation 40. Inasmuch as the coefficient of storage, S, is constant for a given aquifer the following relation obtains.

$$S = \frac{0.3Tt_1}{r_1^2} = \frac{0.3Tt_2}{r_2^2} = \frac{0.3Tt_3}{r_3^2}$$

or

$$\frac{t_1}{r_1^2} = \frac{t_2}{r_2^2} = \frac{t_3}{r_3^2} \tag{44}$$

It follows from the above equation that, if the time intercepts are known for all wells, real or imaginary, and if the distance from the observation well to the real well is known, the distance to any image well can be determined from the data on the semilog graph.

The determination of the image distance from the time intercept of the semilog graph on the zero-drawdown axis may involve considerable error if the observational data are dispersed and if the slope is small because the intercept is poorly defined for small slopes. In addition, this intercept generally occurs at very small values of time, and consequently small errors in the intercept locus result in appreciable errors in the distance values.

The following method avoids the objections of the intercept method. Assume two observation wells at distances r_1 and r_2 from a discharging well, and from equation 35 the drawdown in each

well is calculated as follows.

$$s_1 = \frac{114.6Q}{T}\left[\log_e\left(\frac{Tt_1}{1.87r_1^2S}\right) - 0.5772\right]$$

$$s_1 = \frac{114.6Q}{T}\left[\log_e\left(\frac{Tt_2}{1.87r_2^2S}\right) - 0.5772\right]$$

From the semilog graph for one well record the value of the time for a particular value of s, and from the graph for the second well record the time value for the same value of s. Then when $s_1 = s_2$

$$\log_e\left(\frac{Tt_1}{1.87r_1^2S}\right) = \log_e\left(\frac{Tt_2}{1.87r_2^2S}\right)$$

or

$$\frac{t_1}{r_1^2} = \frac{t_2}{r_2^2} \tag{45}$$

This relation is identical with equation 44. From the form of equations 44 and 45 it follows that for a given aquifer the times of occurrence of zero drawdown or of equal drawdown vary directly as the squares of the distances from the observation well to the discharging well and are independent of the rate of pumping. The equal-drawdown method was used for the data on Fig. 90.

The time values at the tangent intersections determine only the approximate distances to the image wells. The time intercepts so determined will be too small and the calculated image distances will therefore be smaller than the correct distances. These preliminary estimates of the image distances may be corrected by trial. The method is to assume locations of the image wells based on the values computed by the first approximation. Using equations 32 and 33, we can determine the drawdown component resulting from each image well. The time-drawdown graph is obtained by adding these components. Successive adjustments may be made in the image distances until the computed and observed graphs are in agreement.

If a pumping test is run without prior knowledge of geologic boundaries and if the semilog graphs for all observation wells show evidence of image reflections, it would be possible to locate such boundaries by calculating the distance from each observation well to each image well. By scribing this distance as an arc from

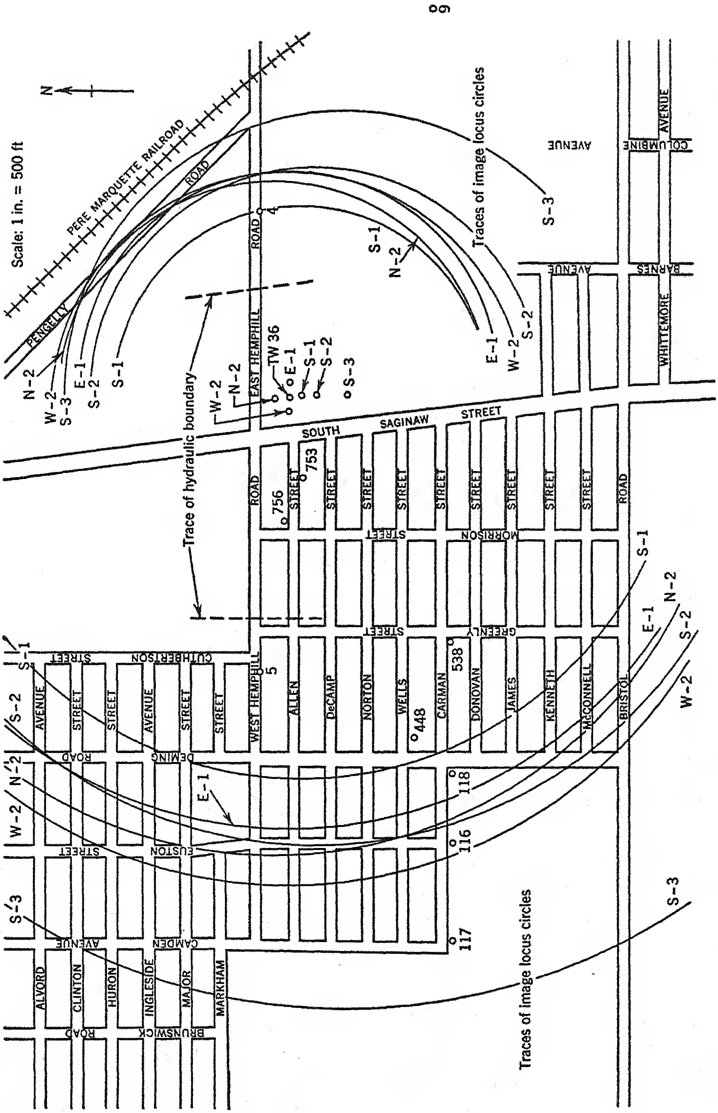


FIG. 91. Map showing location of test wells and pumping-test site near intersection of East Hemphill Road and South Saginaw Street at southern city limits of Flint, Michigan.

the respective observation well, the image well is located at the intersection of the arcs. The boundary is located at the midpoint of a line joining the real well and the image well.

The field layout and the image arc intersections for a test of the above type are shown in Fig. 91. Theoretically the arcs should intersect at a common point, but deviations of the real aquifer from the vertically bounded aquifer assumed in the derivation of the method of images result in a dispersion of the arcs and their intersections. As shown in Fig. 91, the east image well is quite definitely located by the arc intersections in the vicinity of East Hemphill Road, notwithstanding the fact that the observation wells are concentrated in a rather small area. With few or no arc inter-

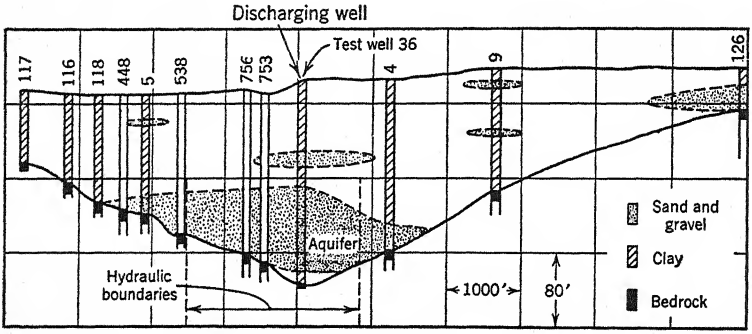


FIG. 92.

sections, the location of the image well may be taken at the center of the arc band where arcs are most closely grouped. It therefore appears from Fig. 91 that the west image well is near the intersection of the arc band with West Hemphill Road. Without a knowledge of the geology of the area, it may be necessary to describe the complete circles in order to determine the location of the narrowest portion of the arc band. The dispersion of the arcs for the west image suggests that the west boundary may be less abrupt than the east boundary. For comparison, Fig. 92 shows the geologic cross section of the aquifer as determined by test drilling and the trace of the computed boundaries are shown thereon as dashed lines. Recall that the boundaries determined by the pumping-test data represent a rectangular aquifer section which is equivalent hydraulically to the real aquifer.

Under favorable conditions, which justify the several assump-

tions embodied in the above methods, these principles properly applied permit the location of geologic boundaries within sufficient limits to reduce greatly the number of test wells required for their confirmation. In any problem, the pumping-test analysis must conform with the geologic evidence if both are sound. Any disagreement between the hydraulic data and the geologic data is untenable and points to incorrect analysis of either or both sets of data. It should also be recognized that long-term extrapolation of pumping-test results hinges on the loose assumption that the conditions observed over a short period of pumping will continue to prevail over the extrapolated period. Consequently, it is advisable for the purpose of long-term predictions to reinforce the data by a number of pumping tests at several sites, to obtain an adequate sampling of the area that will be affected by long-continued pumping.

Application of the Storage Equation to an Aquifer

The general hydrologic equation groups all parts of ground-water movement under the broad term of ground-water increment. With the knowledge of ground-water occurrence and mechanics now available, it is feasible to set up the storage equation for an aquifer as follows,

$$F + R_S + R_U + R_L + R_W = E + D_S + D_U + D_L + D_W + \Delta S \tag{46}$$

where F = recharge from infiltration
R_S = recharge from surface bodies of water
R_U = recharge from lateral underflow
R_L = recharge by leakage through an aquiclude
R_W = recharge by wells, trenches, or other infiltration devices

and

E = discharge by evapo-transpiration
D_S = discharge to surface bodies of water
D_U = discharge by lateral underflow
D_L = discharge by leakage through an aquiclude
D_W = discharge by wells
ΔS = change in storage volume

Quantitative field investigations to evaluate the magnitude of the several terms in equation 46 for our major aquifers represent one phase of the work of the U. S. Geological Survey and its cooperat-

ing agencies. The extent of these surveys to date, however, has been so limited that very few areas are adequately covered by records of sufficient length to permit appraisal of all the above factors.

Estimates of the portion of rainfall that may enter an aquifer are generally made by an analysis of fluctuations in ground-water level as the result of specific storms or on the basis of long-term correlations between water-level hydrographs and precipitation records. The intensity, frequency, and time of occurrence of the rainfall greatly influence the amount of ground-water recharge. Short periods of heavy precipitation may result in considerable overland runoff, but their duration may be so short that the rain may do no more than wet the upper part of the soil. Conversely, a rain of light intensity but long duration is conducive to slow, continued percolation that ultimately saturates the soil and permits considerable recharge to the underlying water table. Inasmuch as the soil-moisture deficiency may absorb a large part of the percolating waters as they pass through the soil zone to the underlying water table, it follows that light rainfalls of short duration are of little benefit unless they occur with sufficient frequency to overcome the depletion of soil moisture.

The most favorable period for recharge from precipitation is the nongrowing season, extending from the first killing frosts in fall to the last killing frosts in spring. During this period, the moisture demands of vegetation are generally negligible, and the soil evaporation is greatly reduced. Accordingly, then, a large portion of the precipitation penetrates to the water table until the soil belt is frozen. In this connection, it is of interest to note that snow cover provides relatively good insulation from frigid temperatures. Consequently, early snowfall of sufficient depth may protect the soil zone from frost formation and may permit appreciable percolation into the soil during periods of snow melt. Especially favorable is a spring period of gradual snow melt over unfrozen soil with precipitation occurring at intensities that limit surface runoff.

Topography is also a controlling factor in determining the opportunity and amount of recharge to an aquifer from infiltration. In areas of great relief the steep slopes accelerate the rate of overland runoff. However, in areas having flat slopes the surface runoff is sluggish and there is appreciable ponding or surface storage for lengthy periods and thus greater opportunity for ground-water recharge. In regions where the water table is close to

the surface, the volume of storage space available for recharge intake is limited by the shallow depth to the water table. When the reservoir is filled, the excess recharge is rejected and the water is discharged as surface runoff.

The amount of water-table rise for a given rainfall may vary considerably within any area because of differences in porosity of the aquifer, in both vertical and horizontal directions. In material of low porosity, a moderate rainfall may result in very large rises of water level, although the total volume of water recharged is quite small. Generally, these high heads result in rapid discharge of the

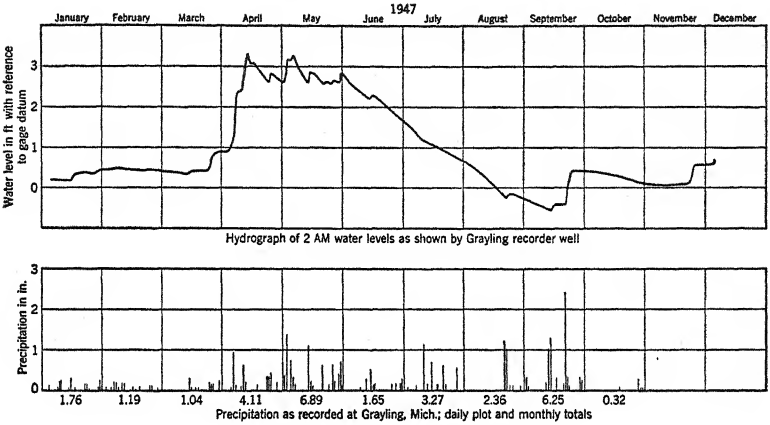


FIG. 93.

stored water after the rain ceases. Throughout a period of recharge, there is continued discharge by the aquifer. Consequently, in correlating records of ground-water level with precipitation, it should be recognized that the recorded rise in water level represents the net difference between the simultaneous recharge and discharge.

A portion of the hydrograph for a shallow observation well located near Grayling, Michigan, is reproduced as Fig. 93. This well is finished in sand and gravel at a depth of 8 ft and is located in an area that is densely covered by small oak and pine trees. It is of interest to note that the water level in this well recovered from a near-record low stage in January 1947 to a near-record high stage by May 1947 as the result of a short period of above-normal precipitation, which occurred during the latter part of the nongrowing season.

The determination of recharge from or discharge to surface bodies of water is generally made by stream-discharge measurements at selected cross sections. For areas where stream-gaging stations are established, it may be possible to secure continuous records of discharge and to utilize these data during base-flow periods. The amount of seepage per foot of channel may be small in comparison with the total discharge of the stream. Consequently, discharge measurements must be made with great precision and repeated on numerous occasions at various stages. It may be necessary to gage the stream flow at several widely distant sections to introduce sufficient seepage length to detect measurable differences in discharge. If gaging sections must be spaced at long intervals, the geologic reconnaissance must be made in sufficient detail to evaluate the effect of underflow channels, faults, or other discontinuities that may intersect the stream between gaging sections.

Considerable attention has been directed to methods of determining and forecasting effluent seepage, or discharge to surface bodies of water, because it represents the base flow of our surface streams. Inasmuch as the water table throughout a drainage basin is a hydraulic unit, it seems probable that a definite law could be developed that would properly weight the geologic characteristics of the basin and would express the rate of effluent seepage in terms of the water-table height at known points in the basin. With such a relationship, it would be feasible to forecast accurately the effluent seepage for any position of the water table at a given point. By selection of appropriate observation wells, which introduce time lag in proportion to their distance from the stream, these forecasts could be made for prolonged periods. A contribution toward this analysis was made by Jacob,[1] who outlined an experimental technique for research into this field. A formula developed by Theis[2] for the flow of water to a drain holds considerable promise for the prediction of effluent seepage from water-table elevation and forms the basis for interpretation of experimental evidence now being collected.

The development of agricultural drainage to lower regional

[1] C. E. Jacob, Correlation of Ground-Water Levels and Precipitation, Long Island, N. Y., *Trans. Am. Geophys. Union*, 1944, p. 939.

[2] C. V. Theis, Ground-Water in the Middle Rio Grande Valley of New Mexico, *U. S. Geological Survey Rio Grande Joint Investigation*, 1937, p. 44.

water tables by increasing effluent seepage may lead to detrimental conditions through a part of the growing season. There are shown in Table 12 the average hydrologic conditions that prevail during

TABLE 12

SUMMARY OF AVERAGE HYDROLOGIC CONDITIONS THAT PREVAIL IN MANY AREAS

	Average Condition Prevailing in the Season Noted	
Factor	Winter–Spring	Summer–Fall
Soil moisture content	High	Low
Capillary fringe	Thick and at high stage	Thin and at low stage
Water table	High stage	Low stage
Vegetation demand	Minimum	Maximum
Ground-water flow	Cold and viscous water retards flow	Warm and less viscous water accelerates flow
Surface water	High stages and peak flows	Low stages and base flows

the spring and summer seasons. It is evident that the spring season is one in which all or nearly all the factors contributing to poor drainage are at their maximum or worst condition. Drains designed to meet this extreme condition are not only more than adequate through the balance of the year but may even cause overdrainage through the growing season. Through the summer and fall seasons such overdrainage may deplete the regional ground-water body to such an extent that the capillary zone is drawn beyond the reach of the plant rootlets, and large soil-moisture deficiencies may develop. This practice may account for the large loss of top soil evidenced in some overdrained muckland areas. It would seem reasonable that in some areas drainage ditches should be blocked and the water should be used for irrigation in the season of low water table.

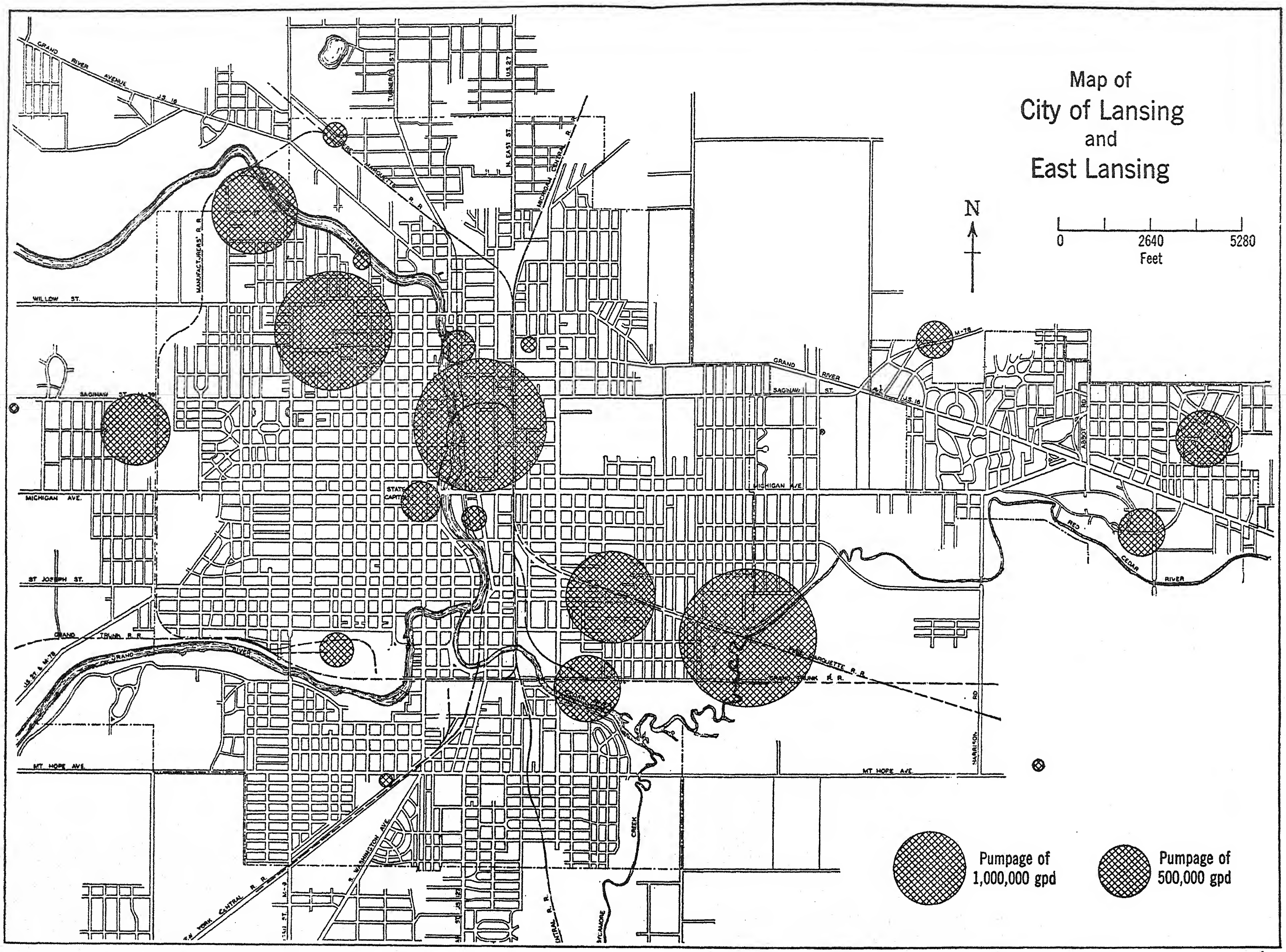
The measurement of recharge or discharge by underflow is based on observation of ground-water gradients that prevail across key cross sections of the underflow channel or conduit. From drilling records it may be possible to estimate the size and shape of the underflow channel at several points. If feasible, pumping tests should be conducted to determine the channel capacity at the control section. Observation wells, strategically located, provide the basis for making long-term measurements of the changes in water-table gradients across the control sections. From a knowledge of the channel area, transmissibility, and water-table gradient it is

possible to determine the quantity of underflow at each time of water-level observation.

The studies of recharge and discharge from adjacent aquicludes are thus far of limited scope. Considerable research in this field is required, both in the collection of field data and in the development of applicable mathematical techniques. As previously mentioned, the rates of ground-water movement through the aquicludes are small and the collection of experimental data may be handicapped by long time lags. It would seem desirable, however, to obtain water-level records for wells tapping the aquicludes at varying depths and particularly to observe the fluctuations in areas of heavy withdrawal or reduction in aquifer pressure. For the present, the leakage factor can only be estimated and such estimates would be difficult to defend.

The withdrawal of water by wells in any area can be estimated by a comprehensive canvass of all well owners to determine their rate of pumping and the extent to which this rate fluctuates daily, weekly, and annually. Many well installations are not metered, but reasonable estimates can usually be made by correlation of water use with a convenient unit of production that is measured. Where water use is not related to a particular unit of output, it may be correlative with total pay rolls, total operating expenses, or total net or gross income. It is generally possible to secure fair to good records of total pumpage by the above methods. From the pumpage inventory, a map of the type shown by Fig. 94[1] may be drawn to show the amount and distribution of pumping at the time of the inventory. This map serves as a guide to the interpretation of the water-level contour map, as regions of large withdrawal should correspond with the areas of low water level and regions of little or no pumping should be areas of high water level. Deviations from the general correspondence between the pumpage-distribution map and the water-level contours may indicate the presence of zones of natural recharge or discharge or may reveal the existence of geologic boundaries or inhomogeneities. In addition to this areal correlation of pumpage and water level, it is necessary to plot pumpage versus time to determine the seasonal and long-term trends in withdrawal and correlate these data with the water-level hydrographs as shown by Fig. 95.[1]

[1] W. T. Stuart, Ground-Water Resources of the Lansing Area, Michigan, *Progress Report* 13, Michigan Department of Conservation, June 1945.



Map of
City of Lansing
and
East Lansing
N
0
2640
5280
Feet
Pumpage of 1,000,000 gpd
Pumpage of 500,000 gpd

Although artificial recharging of ground-water reservoirs is not a widespread practice at present, it is most probable that the rising trend in the number of recharge installations will steepen as the development of larger ground-water supplies continues. Recharge is effected either by water spreading or by diffusion wells. Water

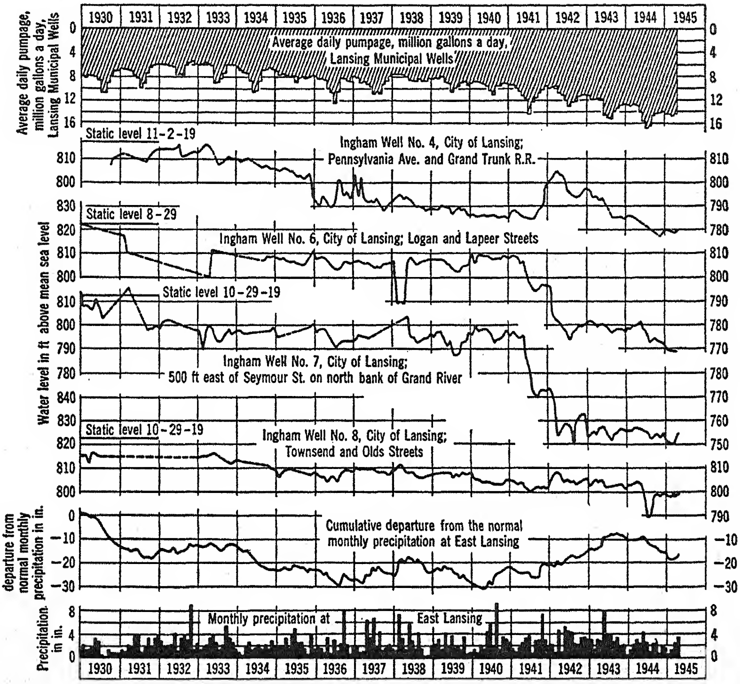


FIG. 95. Graphs showing pumpage, fluctuations of water levels, and precipitation, Lansing, Mich.

spreading may be accomplished by (1) the flooding method, (2) the basin method, or (3) the ditch or furrow method. The flooding method is restricted to topography which lends itself to surface ponding under conditions that permit prolonged retention of the surface waters. The basin and ditch methods require periodic maintenance to remove the accumulated silt from the percolation areas. A plan of the Canyon Basin spreading ground[1] in the San

[1] A. T. Mitchelson and Dean C. Muckel, Spreading Water for Storage Underground, *U. S. Department of Agriculture Tech. Bul.* 578, 1937, pp. 26–31.

Gabriel Valley, California, is shown in Fig. 96. Well hydrographs showing the effect of this spreading operation are shown in Fig. 97.

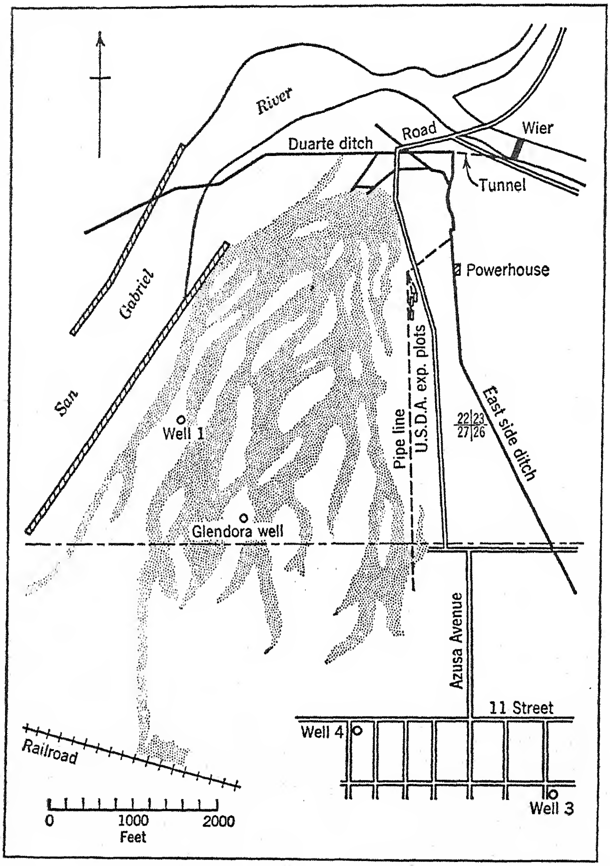


FIG. 96. Spreading ground of Canyon Basin, San Gabriel River, near Azusa, Calif. The general outline of the debris cone overlying the basin is shown. The Duarte ditch which supplies water for artificial spreading, the location of the experimental plots of the Bureau of Agricultural Engineering, and the location of key wells are also shown.

Recharge of underground reservoirs by diffusion wells is practiced on a large scale in Kings and Queens Counties on Long Island, New York, as the result of conservation legislation which requires that new air-conditioning and cooling wells with a capacity

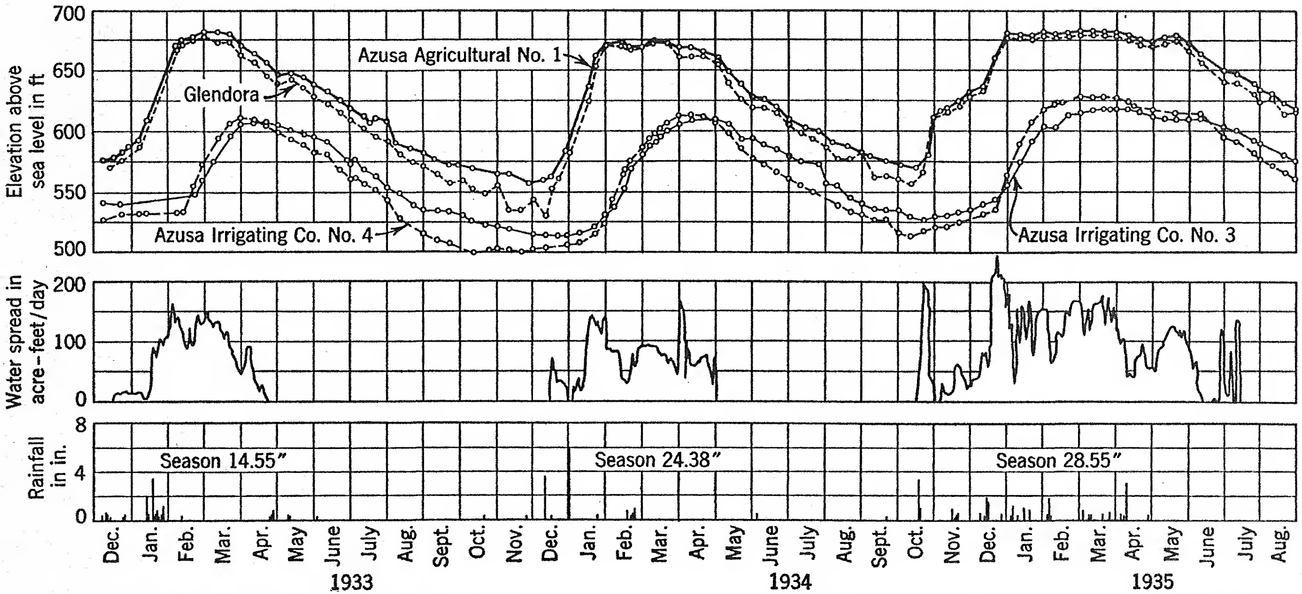


FIG. 97. Hydrographs of wells near San Gabriel River spreading grounds showing effect of spreading on elevation of underground water table, also showing daily total amount of water spread during seasons of 1932–1933, 1933–1934, and 1934–1935, and daily rainfall for these seasons.

greater than 100,000 gpd must return the water to the aquifer from which it is drawn. During the summer of 1944 more than 200 recharge wells and basins returned water at a combined rate of 60,000,000 gpd.[1] Diffusion wells require periodic surging and development to remove the accumulation of silt and other fine materials. Although the unit volume cost of recharging with diffusion wells may greatly exceed that of the spreading method, in the highly industrialized and urbanized areas this may be the only possible method. An example of the solution of a problem of critical water shortage by recharging through diffusion wells is found in the experience of several large distillers in Louisville, Kentucky, during the summer of 1944.[2] Faced with an imminent water shortage the distillers, at the suggestion of the U. S. Geological Survey, converted to city water during cold weather, permitted the wells to rest, and used the cold surface water from the city supply to recharge the aquifer. Recharging started on March 10, 1944, and continued until the latter part of May 1944. By utilizing city water and ground water it was possible for the distilleries to continue full operation during the summer.

Provision is made in the storage equation, page 259, to add the contribution by artificial recharge. This quantity is determined by a pumpage inventory in the case of diffusion wells and by surface-water measurement in the case of water spreading.

The changes in storage volume within an aquifer are based on periodic observations of changes in water level for a selected network of observation wells. Contours of the water table or piezometric surface are drawn and used for estimates of change in storage volume. An example of a map showing contours on the piezometric surface of an artesian aquifer with extensive well development is shown by Fig. 98.[3]

The collection of detailed records of the above nature is a function of the U. S. Geological Survey and many cooperating state agencies. Although the scope of these surveys is still quite limited, the upward trend of investigatory surveys of ground-water conditions in recent years indicates a gradual realization by

[1] M. L. Brashears, Artificial Recharge of Ground Water on Long Island, New York, *Econ. Geol.*, vol. XLI, No. 5, August 1946, pp. 503–516.

[2] W. F. Guyton, Artificial Recharge of Glacial Sand and Gravel with Filtered River Water at Louisville, Kentucky, *Econ. Geol.*, vol. XLI, No. 6, September-October 1946, pp. 644–658.

[3] W. T. Stuart, Ground-Water Resources of the Lansing Area, Michigan, *Progress Report* 13, Michigan Department of Conservation, June 1945.

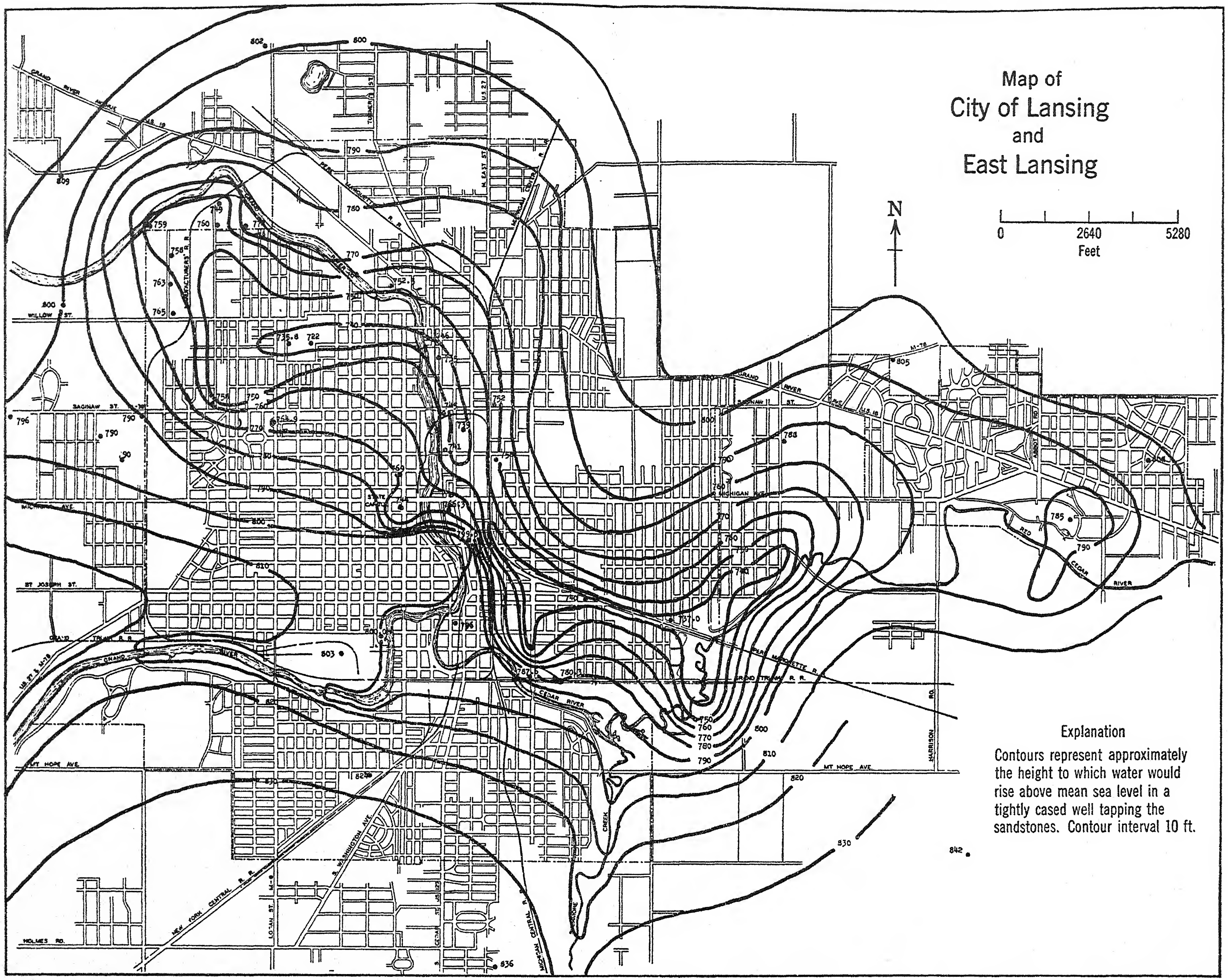

Map of
City of Lansing
and
East Lansing
N
0
2640
5280
Feet
Explanation
Contours represent approximately the height to which water would rise above mean sea level in a tightly cased well tapping the sandstones. Contour interval 10 ft.

Fig. 98.

engineers, geologists, and hydrologists that a real and practical solution of ground-water problems can be made.

It remains a common practice to adjudge the merits of a ground-water supply on the basis of a short capacity test on a single test well, no observational data being recorded except the discharge and drawdown of the well. Frequently, these measurements are made at random intervals and under widely varying conditions. Interruptions in pumping, which permit recovery of the water table, may invalidate the meager data collected and may lead to quite erroneous conclusions, but they seem of little importance to the untrained observer. The influence of geologic boundaries is oftentimes not recognized, and as a result unexpected well failure may occur.

Considerable optimism is warranted in facing future problems when one recognizes the advances made in ground-water hydrology within the short span of years since the development of the non-equilibrium formula by Theis in 1935. Although the mathematical tools are still limited, the awakened interest in the field will in time develop new methods and techniques. The collection of observational data will uncover the required evidence to interpret more clearly many ground-water phenomena that are now recorded but not understood.

When one recognizes that the great majority of our ground-water developments in this country are based on meager and obsolete knowledge of the principles of ground-water hydrology, it is evident that a large field of opportunity lies ahead for adequately trained hydrologists.

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CHAPTER VIII

RUNOFF

The practical objective of the science of hydrology is to provide a means for determining the characteristics of the hydrograph that may be expected for a stream draining any particular basin. The basic principles have been explained in the preceding pages. It remains to be shown how those principles can be best used for the determination of (1) the maximum flood flow that may be expected to occur with any stated frequency for a given basin; (2) the minimum flow that may be anticipated under a given set of conditions; and (3) the monthly, annual, or average long-term yield.

Before the solution of these several problems is taken up, attention should be called to the fact that the first is concerned almost entirely with surface runoff; in the second, ordinarily only ground-water flow is involved; and in the third, it is the sum of the two, without reference to the source.

SURFACE RUNOFF

In the determination of the maximum flood flow or any of the other characteristics of surface runoff, the very nature of the available data depends to a considerable extent upon the size of the area drained. We will, therefore, first consider the surface runoff from small plots because they serve to provide a simple basic understanding of the runoff process. The methods developed for small plots may be used for determining the runoff from subdivisions of drainage systems where no runoff data are available on the larger areas. Then the surface runoff of small watersheds having areas varying from a few acres to perhaps 10 sq miles will be studied. These are of the size encountered in the design of culverts, storm sewers, airports, and small bridges. Finally, methods will be discussed for determining the characteristics of the flood hydrographs that may be expected from larger drainage basins such as those involved in flood studies, power development, irrigation, and water supply.

Runoff from Small Plots

Overland flow occurs after the rate of precipitation exceeds the infiltration capacity for sufficient time to fulfill the demands of depression storage and establish an initial quantity of surface detention. In order to produce surface runoff it is necessary that areas in which surface detention and overland flow exist be connected with the stream by means of surface channels. It is possible, for example, to have surface runoff from a relatively impervious portion of the ground surface pass over a more pervious soil and be completely absorbed before reaching the stream. When rain falls on a ground surface over which surface runoff has not recently occurred, there are obstacles that hinder the formation of the small ephemeral channels which exist during fully established overland flow. It is often necessary for surface detention at local points to be built up to a considerable depth before it can by-pass or remove twigs, grass, and other litter from these tiny stream paths. The presence of dusty earth will in itself cause the formation of large globules of water held in place by surface tension. After this preliminary phase has been completed, surface runoff will be established in a definite network of rills and rivulets that will carry the water toward the larger drainage channels. During fully developed surface runoff, the storage equation may be written for a watershed. If a very small runoff plot is selected, fairly detailed information on conditions existing during surface runoff can be obtained. Tests on small plots are usually made with rain simulators. (See page 187.) An analysis of rainfall and runoff is first made to determine the shape of the infiltration capacity curve and the relation between surface detention and rate of runoff. Secondly, the information gained from the analysis may be used to synthesize a hydrograph of runoff that would occur from any other precipitation pattern. This second step is the important practical result that may be obtained from any such study. It is, for example, the operation which on a larger scale is required for designing spillways, storm sewers, culverts, or other structures that must permit the safe passage of storm water.

An example of such an analysis and synthesis will be presented here to illustrate the procedure. The experiment selected for presentation was conducted by the U. S. Department of Agriculture

Soil Conservation Service.[1] Figure 99*a* shows the hydrograph resulting from a 60-min application of rain at the uniform rate of

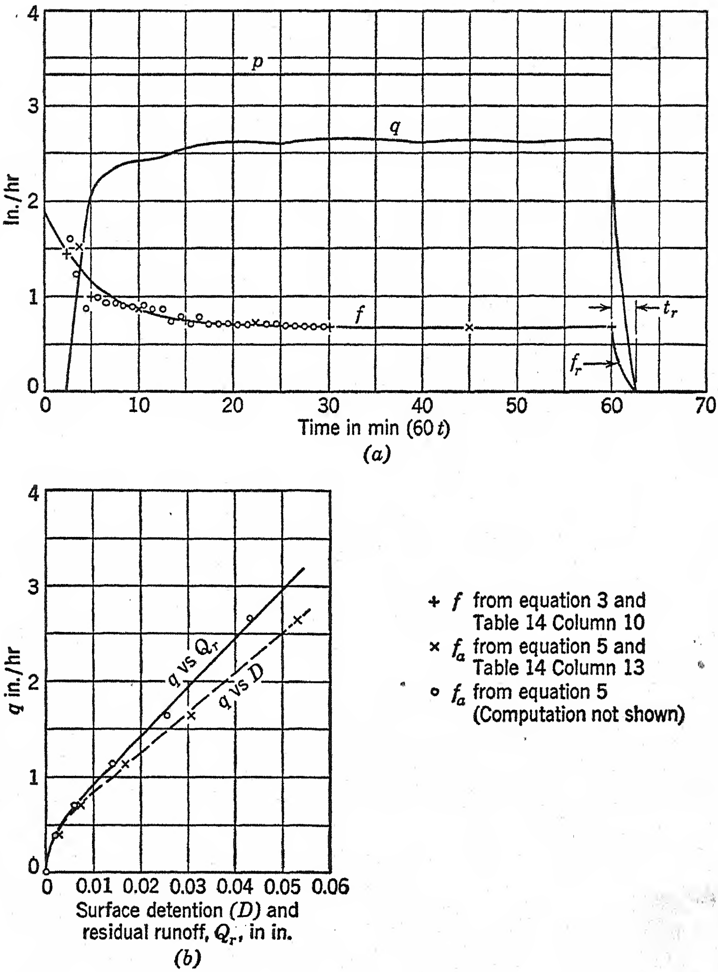

FIG. 99.

3.333 in. per hr to a 6 by 24 ft plot. This test is listed as run 90, site 18. The plot has a slope in its long dimension of 3.32 per cent. The

[1] E. L. Beutner, R. R. Gaebe, and R. E. Horton, Sprinkled Plat Runoff and Infiltration Experiments on Arizona Desert Soils, SCS-TP-38, September 1940.

soil is Mohave sandy clay loam which supported a sparse weed cover at the time of the test.

The storage equation may be written for any time increment of Δt hr during this rain as follows, neglecting evaporation,

$$D_1 + p_a\,\Delta t - q_a\,\Delta t - \frac{(f_1 + f_2)}{2}\,\Delta t = D_2 \tag{1}$$

In this equation the subscripts 1 and 2 represent the beginning and end, respectively, of a time interval. D is the average depth of surface detention in inches on the plot; p_a is the average rate of precipitation during an interval, in inches per hour; q_a is the average rate of surface runoff from plot during an interval, in inches per hour; f is the infiltration capacity at any time, t, in inches per hour.

When precipitation ends, the surface detention or storage existing on the plot is disposed of partly as residual surface runoff and partly as residual infiltration. The portion of surface detention that becomes residual runoff may be evaluated from the recession side of the hydrograph as follows: Starting at the end of the runoff and working backward in small time increments, the volume of residual runoff, Q_r, in inches may be determined for each increment as shown in Table 13, Columns 1 to 6. Values of Q_r were then plotted against rate of runoff, q, as shown in Fig. 99*b*. From this curve the volume of residual runoff corresponding to any rate of runoff during the recession side of this hydrograph can be determined. The relation of q to Q_r is precisely true only for the particular value of minimum infiltration capacity, f_c, for which it was derived. It must be recognized that, since the total surface detention is disposed of by the combination of overland flow and infiltration, their periods must end simultaneously. A variation in f_c may have a noticeable effect on the time required for the storage to be dissipated. The condition is comparable to a tank of water emptying through two orifices. If the size of one orifice is increased, the total time required for a given quantity of water to discharge will be decreased, and the relation between storage and discharge will change for each orifice. In most cases f_c is relatively small compared with q, so that the effect of a change in f_c is not of great importance.

The portion of the surface detention that becomes residual infiltration may be taken (see page 191) as $\frac{1}{3}f_c t_r$, and the total volume of surface detention, existing at the end of precipitation is,

TABLE 13

1 Time before End of SRO, min.	2 Δt min.	3 q in./hr	4 q_{av} in./hr	5 ΔQ_r in.	6 Q_r in.	7 f_r in./hr	8 f_r(av) in./hr	9 ΔF_r in.	10 $\Delta Q_r + \Delta F_r = \Delta D$ in.	11 D in.
0		0			0	0				0
	0.57		0.195	0.00185			0.030	0.00028	0.00213	
0.57		0.39			0.00185	0.060				0.00213
	0.50		0.545	0.00455			0.095	0.00079	0.00534	
1.07		0.70			0.00640	0.130				0.00747
	0.50		0.915	0.00763			0.190	0.00158	0.00921	
1.57		1.13			0.0140	0.250				0.0167
	0.50		1.390	0.0116			0.315	0.00263	0.0142	
2.07		1.65			0.0256	0.380				0.0309
	0.50		2.150	0.0179			0.530	0.00442	0.0223	
2.57		2.65			0.0435	0.680				0.0532

TABLE 14

1 Time from Beginning of Precipitation min ($60t$)	2 $\Delta t \times 60$ min	3 q in./hr	4 q_a in./hr	5 $q_a\,\Delta t$ in.	6 $p_a\,\Delta t$ in.	7 Q_r in.	8 $t_r \times 60$ min	9 $\frac{t_r}{3} \times 60$ min	10 f (Eq.3) in./hr	11 $\frac{f\,\Delta t}{2}$ in.	12 D in.	13 f_a (Eq.5) in.
60		2.65				0.044	2.57	0.857	0.68	0.170	0.053	
	30		2.65	1.325	1.665							0.68
30		2.65				0.044	2.57	0.857	0.68	0.085	0.054	
	15		2.61	0.651	0.833							0.72
15		2.56				0.043	2.54	0.847	0.77	0.064	0.055	
	10		2.40	0.400	0.555							0.87
5		2.08				0.033	2.35	0.783	0.98	0.021	0.046	
	2.58		1.04	0.045	0.143							1.51
2.42		0				0	0	0	1.44	0.029	0	

therefore, $D = Q_r + \frac{1}{3}f_c t_r$. The value of D at the end of surface runoff in Fig. 93 is then

$$0.0435 + \frac{0.68 \times 2.57}{3 \times 60} = 0.0435 + 0.0097 = 0.0532 \text{ in.}$$

In order to determine the relationship between D and q for the recession side of the hydrograph, a residual infiltration curve, f_r, was sketched on Fig. 99*a* from a value of 0.68 at a time of 60 min to zero at a time of 62.57 min. This curve was sketched by trial so that the area beneath it is $\frac{1}{3}f_c t_r$. Increments of residual infiltration may then be determined in the same manner as for residual runoff as shown in Table 13, Columns 7 to 9. These increments are then added to corresponding increments of Q_r to give increments D which when added cumulatively give the values of D shown in Columns 10 and 11. Values of D are plotted against q in Fig. 93*b*. The relationship represented by these points will be required in the process of synthesizing a hydrograph and will be discussed in more detail later.

The relations of Q_r to q, and of D to q, derived from the recession side of the hydrograph, must now be assumed to apply to the rising side as well in order to derive the infiltration capacity curve. The same assumption must be made again later in synthesizing a hydrograph. There is little doubt that these relations for the rising side of the hydrograph are somewhat different from the ones derived from the falling side. There is some question of the magnitude of this difference. Tests made by Izzard and Augustine[1] show reasonably close agreement of the relation between D and q for the two legs of the hydrograph. Actually, this difference may be unimportant in the final result since the curve determined on the basis of this assumption is also used in synthesizing a hydrograph from rainfall.

On the basis of the assumption that $D_1 = Q_{r1} + \frac{1}{3}f_1 t_{r1}$ equation 1 may now be written in the following form:

$$Q_{r1} + \tfrac{1}{3}f_1 t_r + p_a\,\Delta t - q_a\,\Delta t - \tfrac{1}{2}f_1\,\Delta t - \tfrac{1}{2}f_2\,\Delta t = D_2 \qquad (2)$$

The above equation may be solved for values of f_1 by starting at the end of precipitation and working backward through successive

[1] C. F. Izzard and M. T. Augustine, Preliminary Report on Analysis of Runoff Resulting from Simulated Rainfall on a Paved Plot, *Trans. Am. Geophys. Union*, 1943, Part II, p. 500.

time intervals. It must be assumed that at the end of precipitation the infiltration capacity has become practically constant so that f_2 for the first interval (from $t = 30$ min to $t = 60$ min) is equal to $p_2 - q_2$. It is also assumed that at any time the value of t_r may be determined from the recession side of the hydrograph and the value of Q_r from Fig. 93*b*. To facilitate the solution, the equation may be rearranged as follows:

$$f_1 = \frac{D_2 + q_a\,\Delta t + \frac{1}{2}f_2\,\Delta t - Q_{r1} - p_a\,\Delta t}{\frac{1}{3}t_{r1} - \frac{1}{2}\,\Delta t} \tag{3}$$

In this form all the quantities in the right member are known and the value of f_1 may be determined. This will be illustrated by an application to the hydrograph shown in Fig. 99*a*.

The solution is summarized in Table 14, Columns 1 to 12. Arbitrary time intervals are selected and indicated in Column 2. Care should be used to insure that values of $t/2$ and $\frac{1}{3}t_r$ are of entirely different magnitude to avoid extremely small and sensitive denominators. Values of q and corresponding values of Q_r and t_r are determined from Figs. 99*a* and 99*b*. The value of f_2 for the first time interval is 0.68 in. per hr, and the corresponding value of D_2 is 0.053 in., as shown in Table 13, Column 11. The solution then becomes

$$f_1 = \frac{0.053 + 1.325 + 0.0170 - 0.044 - 1.665}{(0.857/60) - (15/60)}$$

$$= \frac{-0.161}{-0.236} = 0.68 \text{ in./hr}$$

and D_1 is then computed as follows:

$$D_1 = Q_{r1} + \tfrac{1}{3}f_1 t_{r1} = 0.044 + 0.010 = 0.054 \text{ in.}$$

The values of f so determined are shown in Column 10 of Table 14 and are plotted in Fig. 99*a*.

If it is assumed that residual infiltration is unimportant and may be neglected, then $D_2 = Q_{r2}$, $D_1 = Q_{r1}$, and equation 2 becomes

$$Q_{r1} + p_a\,\Delta t - q_a\,\Delta t - f_a\,\Delta t = Q_{r2} \tag{4}$$

where $f_a = (f_1 + f_2)/2$. Equation 4 may be rearranged as follows to permit the determination of f_a for successive time intervals.

$$f_a = \frac{Q_{r2} - Q_{r1} + p_a\,\Delta t - q_a\,\Delta t}{\Delta t} \tag{5}$$

Equation 5 is obviously easier to use than equation 3. It is the one suggested by Horton[1] and used by the Soil Conservation Service in *Tech. Paper* 38. The simplifying assumptions made in connection with equations 4 and 5 will only influence the values of f_a so obtained where f is changing rapidly. Even then the difference in the results is small, and the additional work involved when using equation 3 may not be justified. Values of f_a for the time increments used in the previous numerical example have been computed by means of equation 5. These values are shown in Table 14, Column 13, and are plotted in Fig. 99*a*. Also shown in Fig. 99*a* are values of f_a obtained by means of equation 5 using 1-min intervals during the first 30 min.

In order to extend the f curve to time $t = 0$, and thus determine the initial value of infiltration capacity, f_0, it is necessary to include initial detention in the storage equation for the time interval extending from beginning of precipitation to the beginning of surface runoff. The problem is further complicated by the fact that, although the values of f probably will continue to rise as a continuous curve, if this curve rises above the precipitation curve, it becomes discontinuous as far as application of the storage equation is concerned. Specifically, the sum of the end values of f divided by two would no longer approximate the average value of infiltration rate for the interval. One method of overcoming this difficulty would be to determine an equation for the known portion of the curve and then extend this curve to a time of zero. Horton[1] found that the points agree very closely with an equation of the form

$$f = f_c + (f_0 - f_c)e^{-kt} \tag{6}$$

where k is a constant and t is the time from the beginning of precipitation, in hours.

By choosing any two sets of values of f and t from the f curve and placing them in the above equation, two equations with two unknowns, namely f_0 and k are obtained. The equations may then be solved by successive approximations. The numerical values obtained in this manner depend a great deal on what particular sets of values are taken from the f curve. For example, in Fig. 99*a*, for $f = 1.44$, $t = 2.42$ min, and $f = 0.77$, $t = 15$ min, the value of

[1] R. E. Horton, Analysis of Runoff Plot Experiments with Varying Infiltration Capacity, *Trans. Am. Geophys. Union*, 1939, Part IV, p. 693.

k is 10.2 and the value of f_0 is 1.85 min per hr; whereas, for $f = 1.44$, $t = 2.42$ min, and $f = 0.98$, $t = 5$ min., $k = 21.6$ and $f_0 = 2.51$ min per hr.

Another method of solving for the constants in this equation, which leads to more definite results, is suggested when the equation is rearranged as follows:

$$f - f_c = (f_0 - f_c)e^{-kt}$$

and written in logarithmic form,

$$\log (f - f_c) = \log (f_0 - f_c) - kt \log_e$$

and solving for t,

$$t = \frac{1}{k \log_e} \log (f_0 - f_c) - \frac{1}{k \log_e} \log (f - f_c) \tag{7}$$

It may be seen that in this form the equation is that of a straight line in which t and $\log (f - f_c)$ are the variables. The slope of the line is $-1/k \log_e$. Therefore, if an equation of this type correctly represents the form of the f curve, a straight line should be obtained when values of $\log (f - f_c)$ are plotted against t. The values derived for the example under discussion have been plotted in this manner in Fig. 100. A straight line is shown that represents the general trend of the values. The slope of this line is -0.212; therefore, $-1/k \log_e = -0.212$, and $k = 10.85$. When $t = 0$, f is equal to f_0. The value of $(f - f_c)$ at $t = 0$ is 1.20 and therefore $f_0 = 1.20 + 0.68 = 1.88$ in. per hr. The solid line plotted through the f values in Fig. 99a is based on these values of k and f_0. If all but the very highest values of f are ignored, a much larger value of f_0 will be obtained. For example, the values of f_0 and k derived in *Soil Conservation Service Tech. Paper* 38,[1] for this hydrograph are 5.49 and 29.2 respectively. The curve determined by these values is shown by the dashed line on Fig. 100.

When a value of f_0 is decided upon, the value of initial detention, D_i, may be computed. In this example, if f_0 is taken as 1.88 in. per hr, the value of D_i is

$$(p_a - f_a)t_i = \left(3.33 - \frac{1.88 + 1.47}{2}\right)\frac{2.42}{60} = 0.067 \text{ in.}$$

[1] F. L. Beutner, R. R. Gaebe, and R. E. Horton, Sprinkled Plat Runoff and Infiltration Experiments on Arizona Desert Soils.

Although the value of f_0 and, therefore, the value of D_i is subject to some uncertainty due to the personal judgment involved in

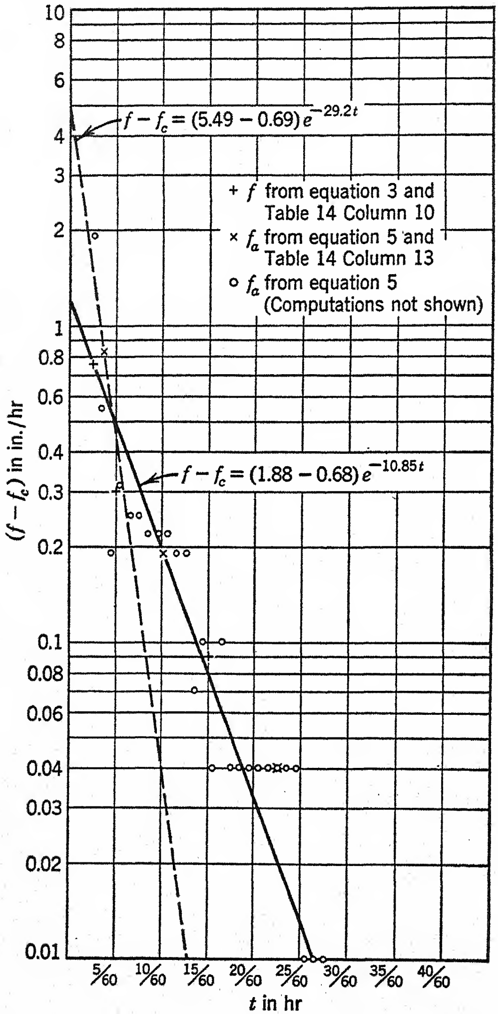

FIG. 100.

plotting the line on Fig. 100, the error introduced in utilizing these values to predict runoff from another rain is small because, when f_0 is large, D_i is correspondingly small, and vice versa, so that the

sum of initial infiltration and initial detention remains about the same.

The Relation between Surface Dentention and Discharge. There has been determined from the recession side of the hydrograph a series of corresponding values of q and D as shown in Table 13 and plotted in Fig. 99*b*. Such a relationship may usually be represented by an equation having the form

$$q = KD^m \tag{8}$$

When plotted logarithmically, an equation of this type is represented by a straight line having the slope m and the position on the q scale determined by K.

The value of m in equation 8 depends upon whether flow is laminar or turbulent. For laminar flow it may be shown both theoretically and experimentally that the value of m is 3. For turbulent flow experimental evidence indicates that m is approximately 5⁄3. Typical experimental results obtained for uniform steady flow on smooth concrete[1] are plotted in Fig. 101*a*. Lines having slopes of 3 and 5⁄3 respectively are drawn on the figure to illustrate that the plotted points fall very nearly along these slopes. In the transition zone it may be noted that the slope changes rapidly from 3 to 1 or less and then changes gradually to 5⁄3. Instead of steady uniform flow, runoff from rainfall is unsteady, nonuniform, and spatially variable. Furthermore, the velocity distribution is likely to differ from normal flow because of the incoming raindrops and outgoing infiltration. Consequently one cannot safely conclude that runoff from rainfall will have exactly the same characteristics exhibited by the data plotted in Fig. 101*a*. However, experimental results obtained by Izzard[2] for runoff resulting from the application of artificial rainfall on paved and turfed surfaces show that m is 3 for the laminar range. Until additional experimental evidence is secured it can only be assumed that flow in the transition and turbulent ranges will also be similar to that shown in Fig. 101*a*. It must be recognized that the Reynolds number, based on the average depth and velocity, for turfed plots is quite likely not comparable with a Reynolds number similarly

[1] Studies of River Bed Materials and their Movement, with Special Reference to the Lower Mississippi River, *Paper* 17, U. S. Waterways Exp. Sta., Vicksburg, Miss., January 1935.

[2] C. F. Izzard, The Surface-Profile of Overland Flow, *Trans. Am. Geophys. Union*, 1944, Part VI, p. 959.

determined for pavements. This is because the average depth and velocity may occur over only a small percentage of a turfed runoff area. The major portion of the area is likely to have depths and

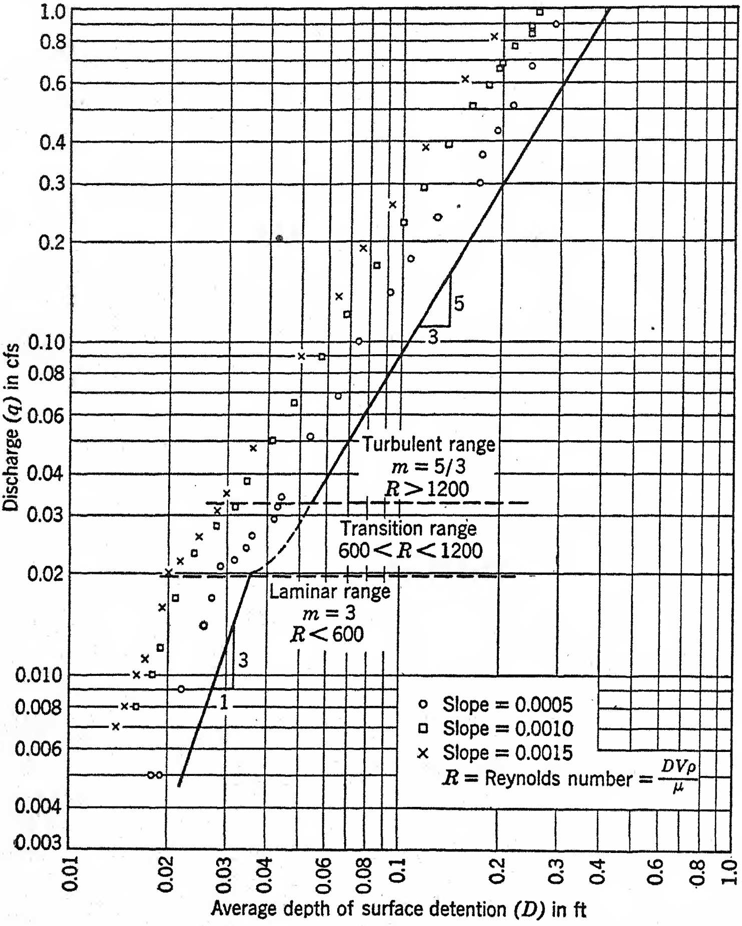

FIG. 101(a).

velocities less than the average, whereas the bulk of the runoff occurs in small rivulets having a depth and velocity greater than the average.

The value of K in equation 8 varies with the surface roughness, the Reynolds number, the slope of the plot, and the units of D and q. Its value may be readily determined from a set of experimental

data, either analytically or by extending the straight line in a logarithmic plotting to $D = 1$. It has been suggested that K might be determined from the Manning formula.[1,2] The Manning formula arranged for flow in thin sheets is shown in equation 9, Q_1 being discharge in cubic feet per second per foot of width.

$$Q_1 = \frac{1.486}{n} D^{5/3} S^{1/2} \tag{9}$$

When the value of m in equation 8 is $5/3$, equations 8 and 9 are comparable, and K is seen to be equal to $1.486 S^{1/2}/n$. It may be

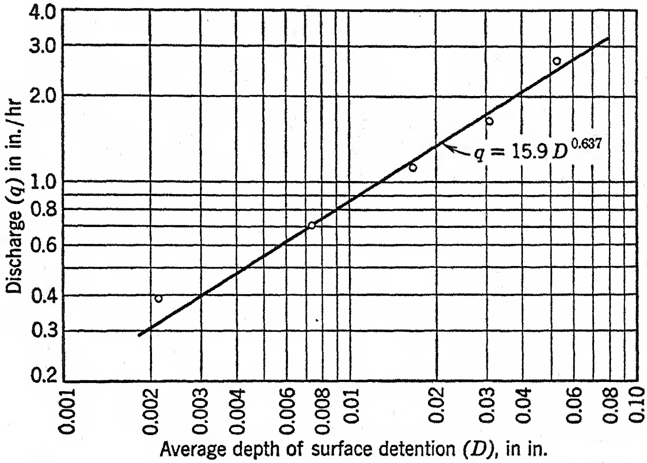

FIG. 101(*b*)

assumed that the slope of the energy gradient, S, is nearly equal to the slope of the ground surface. Values of K so obtained may be expected to give fairly adequate results for turbulent flow but could not be used for laminar flow or for flow in the transition range. For laminar flow, the exponent of S is 1 instead of $\frac{1}{2}$ and n is no longer a function of the roughness but of Reynolds number. The values of K and m for the plot used in the preceding illustrative example are found from a plotting of values of q and D in Fig. 101*b* to be 15.9 and 0.637 respectively. The fact that m has the

[1] R. E. Horton, Hydrologic Interrelations of Water and Soils, *Proc. Soil Science Soc. of Am.*, 1937, **1**, 401.

[2] R. E. Horton, The Interpretation and Application of Runoff Plot Experiments with Reference to Soil Erosion Problems, *Proc. Soil Science Soc. of Am.*, 1938, **3**, 340.

value 0.637 shows that discharge is in the transition range between laminar and turbulent flow.

Synthesis of a Hydrograph. Having discussed the derivation of the f curve and the relation between q and D from measured rates of precipitation and runoff it remains to consider methods of utilizing this information to predict runoff hydrographs from other rainfall occurring on this or similar plots. This will be done by means of a numerical example in which it will be assumed that the precipitation rates shown in Fig. 102 are applied to the plot used in the preceding numerical work. The previously derived f curve is

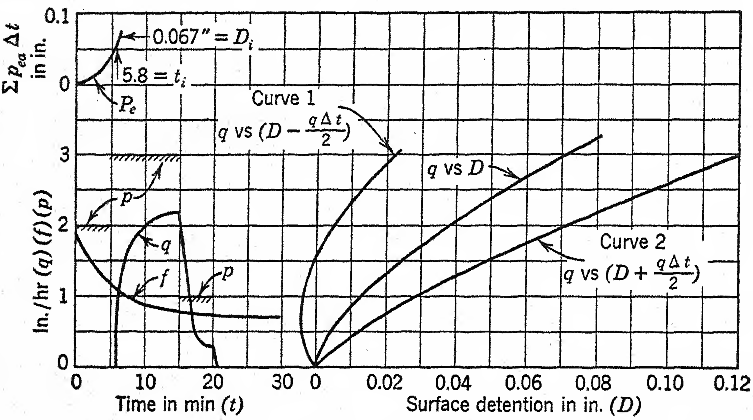

FIG. 102.

shown superimposed upon the rainfall graph in Fig. 96. Values of rainfall excess, p_e, at any time may be obtained by subtracting ordinates of the f curve from corresponding values of p, or, similarly, the average value of rainfall excess, p_{ea}, for any time interval may be obtained. This has been done for selected time intervals and values recorded in Columns 1 to 5 of Table 15. In Column 6 increments of supply have been determined. The increments have been added accumulatively in Column 7 to determine the time at which initial detention, D_1, is filled. The initial detention for this plot was found to be 0.067 in. (page 281). A plotting of the values given in Column 7 (see Fig. 102) shows that this quantity will be accumulated at t = 5.8 min, which will then be the time at which runoff begins. The storage equation in a form suitable for this

procedure is given as equation 10.

$$D_1 + p_{ea}\,\Delta t - \left(\frac{q_1 + q_2}{2}\right)\Delta t = D_2 \tag{10}$$

If this equation is applied to a time increment for which the values of D_1 and q_1 are known, q_2 and D_2 are the only unknown terms. The additional equation that permits the determination of these two quantities is equation 8 with the derived constants included as shown in equation 11.

$$q = 15.9D^{0.637} \tag{11}$$

Values of q_2 and D_2 may now be determined by the simultaneous solution of equations 10 and 11. Consider for example the interval from $t = 5.8$ to $t = 6$. Into equation 10 may be inserted $D_1 = 0$, $q_1 = 0$, $\Delta t/2 = 0.0017$ hr, and $p_{ea}\,\Delta t = 0.006$ in. Then

$$0.006 - 0.0017q_2 = D_2$$

A number of solutions of equation 11 for trial values of D_2 will show that the above relation is satisfied when $q_2 \doteq 0.55$ in. per hr. However, this procedure involves successive approximations and time can be saved by adapting a graphical method of solution. For this purpose equation 10 may be written in the following form

$$\left(D_1 - \frac{q_1\,\Delta t}{2}\right) + p_{ea}\,\Delta t = \left(D_2 + \frac{q_2\,\Delta t}{2}\right) \tag{12}$$

Equation 11 is then plotted in Fig. 102 together with the auxiliary curves, 1 and 2, Curve 1 being the relation between q and $\left(D - \frac{q\,\Delta t}{2}\right)$ and Curve 2 the relation between q and $\left(D + \frac{q\,\Delta t}{2}\right)$. Curves 1 and 2 were prepared for a time interval, Δt, of 2 min. The value of $\left(D_2 + \frac{Q_2\,\Delta t}{2}\right)$ in equation 12 may be obtained graphically by adding the value of $p_{ea}\,\Delta t$ for any interval to the abscissa of Curve 1, corresponding to q_1. The value of q_2 may then be obtained by going in a vertical direction to Curve 2. For example, consider the interval from $t = 6$ to $t = 8$. At $t = 6$ the value of q was found to be 0.55 in. per hr. The value of $p_{ea}\,\Delta t$ is found from Column 6, Table 15, to be 0.066 in. If this value is added to the abscissa of Curve 1 (-0.004 in.) the value of q_2 is found from Curve

2 to be 1.78 in. per hr. Other values obtained in the same manner are recorded in Table 15, Column 8, and plotted in Fig. 102.

TABLE 15

1 Time (t) min.	2 Precip-itation (p) in./hr	3 Inf.Cap. (f) in./hr	4 Precip.Exc. $(P - f) =$ (p_e) in./hr	5 p_{ea} in./hr	6 $p_{ea} \cdot \Delta t$ in.	7 Σ $p_{ea} \cdot \Delta t$ in.	8 Runoff (q) in./hr
0	2.00	1.88	0.12				
				0.21	0.003		
1	2.00	1.70	0.30			0.003	
				0.39	0.007		
2	2.00	1.52	0.48			0.010	
				0.545	0.009		
3	2.00	1.39	0.61			0.019	
				0.675	0.011		
4	2.00	1.26	0.74			0.030	
				0.785	0.013		
5	2.00	1.17	0.83			0.043	
	3.00		1.83				
				1.87	0.031		
6	3.00	1.09	1.91			0.074	0.55
				1.98	0.066		
8	3.00	0.95	2.05				1.78
				2.085	0.069		
10	3.00	0.88	2.12				1.99
				2.15	0.072		
12	3.00	0.82	2.18				2.11
				2.20	0.073		
14	3.00	0.78	2.22				2.16
				1.235	0.041		
16	1.00	0.75	0.25				1.47
				0.26	0.009		
18	1.00	0.73	0.27				0.35
				0.28	0.009		
20	1.00	0.71	0.29				0.26

Other Methods of Determining Runoff. A method of developing an equation for the rising side of the hydrograph has been suggested by Horton.[1,2] This method involves writing equation 10 in the

[1] R. E. Horton, Hydrologic Interrelations of Water and Soils, *Proc. Soil Science Soc. of Am.*, 1937, **1**, 401.

[2] R. E. Horton, The Interpretation and Application of Runoff Plot Experiments with Reference to Soil Erosion Problems, *Proc. Soil Science Soc. Am.*, 1938, **3**, 340.

following differential form,

$$p_e\,dt - q\,dt = dD \tag{13}$$

If equation 13 is solved for dt and the value of q from equation 8 is introduced, the following expression is obtained:

$$dt = \frac{dD}{p_e - Kd^m} \tag{14}$$

By assuming p_e to be constant, equation 14 may be readily integrated for selected whole number values of m, although the form of the resulting equations would be mathematically incorrect because p_e should be included as a function of time. For the particular case of $m = 2$, the resulting equation is

$$q = p_e \tanh^2 \tfrac{3}{2}\sqrt{Kp_e}\,t \tag{15}$$

For values of m of 1 and 3, an equation of a different form is obtained whereas for other exponents of D the difficulties of integration, even assuming p_e to be constant, preclude the development of an equation similar to equation 15. Horton suggested that it might be possible to find an equation similar to equation 15 which could be expected to give good results for all values of m. Until that is done it is believed that a direct approach to the problem such as that given on the preceding pages must be used.[1]

A basic analytical approach to the problem of surface runoff from a plane surface has been made by Keulegan.[2] Methods of designing urban and airport drainage facilities have been presented by Horner and Jens.[3,4] Many other noteworthy publications will be brought to the reader's attention by references given in the articles cited in this chapter. No discussion of this subject should omit reference to the publication by Horton of his *Surface Runoff Phenomena*,[5] which was one of the important contributions to the advancement of the science of hydrology.

[1] For a more detailed discussion of equation 15 see S. W. Jens, Drainage of Airport Surfaces—Some Basic Design Considerations, discussion by E. F. Brater in *Trans. A.S.C.E.*, 113, 785, 1948.

[2] G. H. Keulegan, Spatially Variable Discharge over a Sloping Plane, *Trans. Am. Geophys. Union*, 1944, Part VI, p. 956.

[3] W. W. Horner and S. W. Jens, Surface Runoff Determination from Rainfall Without Using Coefficients, *Trans. A.S.C.E.*, 1942, **107**, 1039.

[4] S. W. Jens, Drainage of Airport Surfaces—Some Basic Design Considerations, *Trans. A.S.C.E.*, 113, 785, 1948.

[5] R. E. Horton, *Surface Runoff Phenomena*, Part I—Analysis of the Hydrograph, Publication 101, Horton Hydrological Lab., Voorheesville, N. Y., February 1935.

Small Watersheds

The term "small watersheds" is here used to designate drainage basins varying in area from approximately 4 acres to 10 sq miles. Most of the watersheds for which culverts and storm sewers are designed fall within this category. The subject will be treated by first determining unit hydrographs and infiltration-capacity curves for several watersheds, followed by a demonstration of the usefulness of these tools in predicting runoff from rainfall. Some data will also be presented for the purpose of permitting the reader to evaluate the reliability of the unit-hydrograph principle when applied to small watersheds.

The unit hydrograph was introduced by L. K. Sherman in 1932.[1] In 1935 Bernard[2] suggested the distribution graph and the pluviagraph as refinements in the use of the unit hydrograph. Although originally devised for use on large watersheds where "unit storms" could be determined from daily rainfall records, it has been shown that the principle is applicable to small watersheds as well.[3,4] The authors' concept of the principles involved in this method are stated in Chapter II, page 31. (See also page 308 of this chapter.) The usefulness of the unit hydrograph and the distribution graph lies in the fact that, having determined them for a particular basin, we can construct, with satisfactory accuracy, the runoff hydrograph that would occur from any other unit or series of units of rainfall excess that might be applied to that basin.

The data necessary for determining unit hydrographs and f curves are continuous rainfall and runoff records. From such records, hydrographs resulting from intense storms of short duration are selected. Five such hydrographs from a watershed having an area of 76.5 acres are shown in Fig. 103.[5] Unit hydrographs result from short intense storms whose duration is less than the period of rise.

[1] L. K. Sherman, Streamflow from Rainfall by the Unit–Graph Method, *Eng. News-Record*, 1932, pp. 501–505.

[2] Merrill M. Bernard, An Approach to Determinate Stream Flow, *Trans. A.S.C.E.*, 1935, **100**, 347.

[3] W. W. Horner and F. L. Flynt, Relation between Rainfall and Run-off from Small Urban Areas, *Trans. A.S.C.E.*, 1936, **101**, 140.

[4] E. F. Brater, The Unit Hydrograph Principle Applied to Small Watersheds, *Trans. A.S.C.E.*, 1940, **105**, 1154.

[5] Data for these graphs were obtained from Hydrologic Data, *Hydrologic Bulls.* 1 and 2, North Appalachian Experimental Watershed, Coshocton, Ohio, Soil Conservation Service, U. S. Department of Agriculture.

The first step in the analysis of the graphs is to separate surface runoff from ground-water discharge. In the examples shown in Fig. 103, straight lines were drawn from the point where the hydrographs began to rise to arbitrarily selected points on the recession curve. Usually there is no difficulty involved in selecting the point of the beginning of surface runoff because of the abrupt fashion in which hydrographs rise. The end of surface runoff may be indicated by a break in the slope of the recession curve.[1] If not, some arbitrary point must usually be selected. Some uniformity among hydrographs from the same watershed may be achieved by assuming that the ratio between successive discharge rates, separated by uniform time increments, will be of a different order of magnitude when the graph represents only ground-water discharge than when channel storage resulting from surface runoff is being discharged. For example, the value of the discharge at the end of surface runoff divided by the discharge ½ hr earlier might be determined from some particular graph and then used as a criterion for determining the end point of other graphs.

Although the actual length of the base of the surface-runoff hydrograph may be changed considerably by various reasonable selections of the end point, the more important properties of the hydrograph, namely, the total volume of surface runoff and the magnitude of the larger ordinates, will vary but little. It will become evident during the following discussion that the actual method of selecting a base line separating surface runoff from ground-water flow is not as important as being consistent in its use, that is, using the same method in the synthesis of a hydrograph as was used in the analysis of the data. It is sometimes desirable to separate two overlapping hydrographs into single graphs as illustrated in Fig. 103*d*. The separating line (*a*–*b*) was determined by assuming that the recession curve of the first hydrograph would be the same as the one for the second graph.

Having separated base flow from surface runoff, the next step is to divide the surface-runoff graph into convenient time units, usually at least ten in number, and determine the average rate of surface runoff during each interval. It is well to select the largest interval that will still give a reasonably accurate integration of the

[1] C. R. Hursh and E. F. Brater, Separating Storm-Hydrographs from Small Drainage-Areas into Surface—and Subsurface—Flow, *Trans. Am. Geophys. Union*, 1941, Part III, p. 863.

hydrograph in order to keep the number of computations at a minimum. Ten-minute intervals were used in the hydrographs shown in Fig. 103. The values are tabulated in Table 16, Column 2. Each individual average rate could be converted to quantity discharge, but time is saved by converting only the sum of the individual averages to total volume. The precentage of the total discharge that appeared during each time interval is obtained by

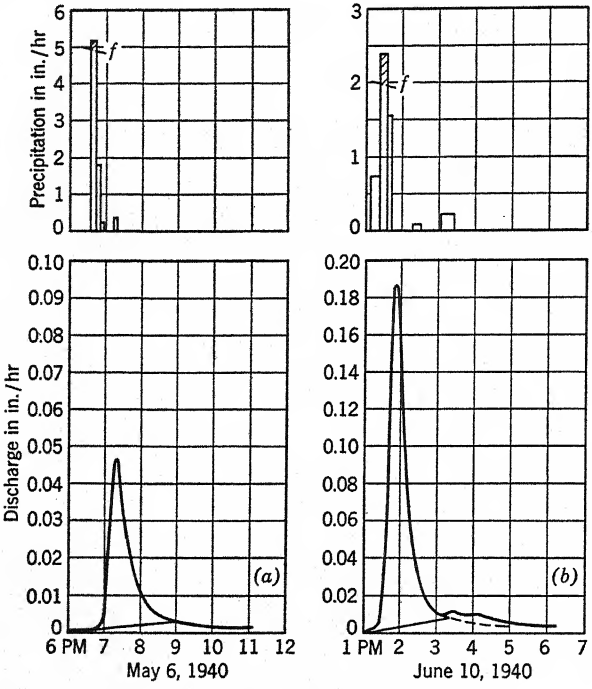

FIG. 103.

dividing each 10-min average successively by the sum of the averages, and multiplying by 100. These values, shown in Table 16, Column 3, are the ordinates of the distribution graphs. Sometimes the ratios rather than the percentages are used. The ratios would be obtained by dividing all values in Column 3 by 100, and might be thought of as the runoff in inches during each interval if the total runoff were 1 in. In addition to the average values, the peak surface runoff must be recorded and the corresponding peak percentage determined. This is the most important value of all because it is

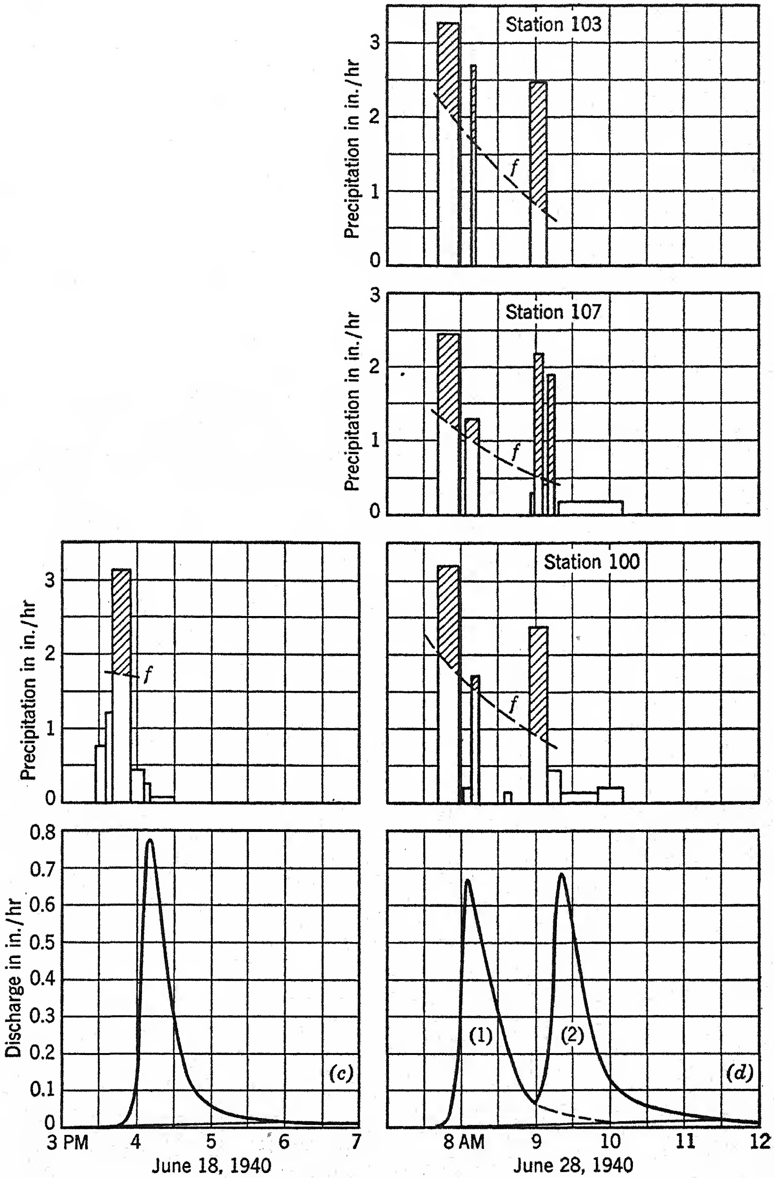

FIG. 103. *Continued.*

TABLE 16[1]

	May 6		June 10		June 18		June 28 (1)		June 28 (2)	
1	2	3	2	3	2	3	2	3	2	3
Number of 10-min Intervals	Discharge in./hr	Distribution Graph percentages	Discharge in./hr	Distribution Graph percentages	Discharge in./hr	Distribution Graph percentages	Discharge in./hr	Distribution Graph percentages	Discharge in./hr	Distribution Graph percentages
1	0.001	0.6	0.003	0.5	0.02	0.9	0.06	2.7	0.07	3.2
2	0.013	7.7	0.042	7.2	0.31	14.4	0.43	19.3	0.47	21.3
							(0.67)	(30.0)	(0.65)	(29.6)
3	0.040	23.8	0.118	20.1	.72 (0.77)	33.3 (35.7)	0.59	26.5	0.59	26.8
	(0.045)	(26.8)	(0.186)	(31.8)						
4	0.040	23.8	0.176	30.0	0.53	24.5	0.43	19.3	0.40	18.2
5	0.027	16.1	0.120	20.5	0.26	12.0	0.31	13.9	0.23	10.5
6	0.017	10.1	0.056	9.6	0.13	6.0	0.17	7.6	0.14	6.4
7	0.011	6.4	0.032	5.5	0.07	3.2	0.08	3.6	0.09	4.1
8	0.008	4.8	0.018	3.1	0.05	2.3	0.05	2.2	0.06	2.7
9	0.005	3.0	0.010	1.7	0.03	1.4	0.04	1.8	0.05	2.3
10	0.003	1.8	0.006	1.0	0.02	0.9	0.03	1.3	0.04	1.8
11	0.002	1.2	0.003	0.5	0.01	0.5	0.02	0.9	0.03	1.4
12	0.001	0.6	0.001	0.2	0.01	0.5	0.01	0.5	0.02	0.9
							0.01	0.4	0.01	0.5
Σ	0.168	100.0	0.585	99.9	2.16	99.9	2.23	100.0	2.20	100.1

[1] Values in parentheses are peak values.

used to compute the maximum rate of runoff to be expected from another rain.

To complete the analysis of the hydrograph, it is necessary to determine values of infiltration capacity. The two hydrographs of Fig. 103*d* provide an excellent opportunity to determine the effect of previous precipitation upon the infiltration capacity and will be used as an example to illustrate the procedure. The total surface runoff from these two storms is $2.23 \times {}^{10}/_{60} = 0.37$ in., and $2.20 \times {}^{10}/_{60} = 0.37$ in., respectively. The values 2.23 and 2.20 were obtained from Table 16, Column 2. It is now necessary to establish infiltration-capacity curves that cut the rain intensity graphs in such a manner that the shaded portion of the graphs have an area of 0.37 in. in each case. This is an approximate procedure. More refined methods are discussed in Chapter VI. There are shown in Fig. 103*d* the records from three intensity gages that are applicable to this watershed. The f curves that satisfy the above conditions are shown for each precipitation record. When the f curve cuts off more than a single segment, the location of the curve can best be determined by trial. Since the three f curves differ somewhat, it is necessary to determine an average value. In doing so, the several curves may be weighted according to the Thiessen method (page 86).

Infiltration capacities determined by the above method usually differ from those obtained from small plots for several reasons. In the first place, observed surface runoff from small watersheds may include subsurface storm flow (see page 20), whereas, from small plots, it does not. Part of the area of every basin is impermeable because of pavements, buildings, rock outcrops, and so on. Furthermore the portion of a drainage basin that is covered by the streams and connecting lakes also acts as if it were an impervious surface. In the above determinations no deduction was made for such areas, whereas small plots are usually so chosen that they have no such areas. Also in the above method no consideration is given to the fact that infiltration continues on an ever-decreasing area after the period of excess rainfall ends. On small plots this subsequent period of infiltration is negligible. Furthermore, because of the variation in rainfall intensity and distribution and because of the usually great distances between gages, the correct average rainfall pattern cannot ordinarily be determined for a large basin. The variation of the three intensity graphs shown in Fig. 103*d* is

not at all unusual. In more mountainous regions greater variations than this are common. Obviously this factor can cause computed values of f to be either too high or too low.

The Variation in Distribution Graphs from a Watershed. In the following pages it is suggested that distribution graphs of the type

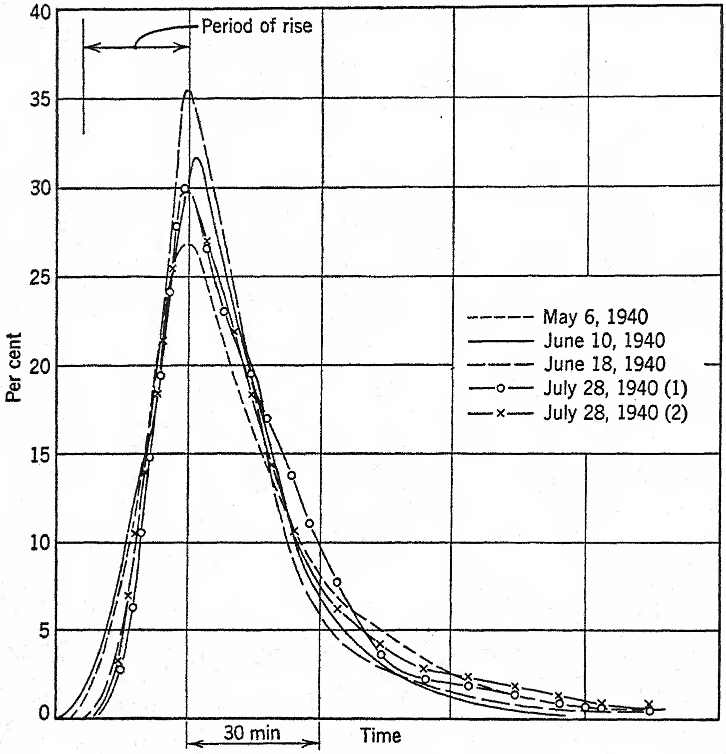

FIG. 104.

derived in the previous example may be used to predict runoff not only from the basin for which they were derived but also for other watersheds having similar physical characteristics. It has been previously stated (Chapter II, page 32) that from a purely theoretical viewpoint the principle of the unit hydrograph cannot be strictly true. To use a procedure of this type with confidence it is desirable to determine the degree of accuracy that can be expected from its application. An indication of this accuracy may be observed from the variations between distribution graphs obtained

from any particular watershed. The data for the five distribution graphs presented in Table 16, Column 3, are typical. These graphs are shown superimposed on each other in Fig. 104. The values of peak percentage provide the most sensitive basis of comparison. The ratio of the largest peak percentage to the smallest is 1.34, as compared with a corresponding ratio of maximum to minimum peak discharge of 16. It may be noted from Fig. 103 that the duration of rainfall excess varied from 32 to 8 min, giving a ratio of 4.

Various characteristics of nine distribution graphs determined for Watershed 97,[1] which has an area of 4581 acres, are shown in Table 17. Rain intensity and infiltration capacity values were determined from averages of from three to five recording gages. In order to give an indication of the variation within each column, the ratio of the largest value to the smallest is given at the bottom of many of the columns. It may be noted that the smallest variations occur for the values of peak percentage and period of rise, these ratios being 1.4 and 1.75 respectively, whereas the ratio of largest to smallest values of total precipitation, total surface, runoff, and peak rate of surface runoff are 2.8, 19.1, and 18.3 respectively. Attention may also be called to the duration of effective precipitation shown in Column 10, the largest being 92 min and the smallest 13 min with a ratio of 7. Some indication of the timing and distribution of the rainfall for each of the nine storms is given by data in Columns 11 and 12. It is believed that the variations in distribution graphs resulting from unit storms on any particular watershed are mainly the result of rainfall distribution and timing. Especially in long narrow watersheds, rains moving up the valley produce much lower peaks than rains moving down the valley. This is strikingly illustrated by two points plotted in Fig. 105*a* (see note on Fig. 105*a*).

In each of the two groups of distribution graphs presented, the maximum variation of the peak percentage from the average for the group is approximately 20 per cent. Similar variations have been noted on many other streams. In predicting maximum flood flow for design purposes, the effect of this variation is eliminated to some extent by choosing the higher peaked unit hydrographs in making the estimate.

[1] Data for these graphs were obtained from Hydrologic Data, *Hydrologic Buls.* 1 and 2, U. S. Department of Agriculture Soil Conservation Service, North Appalachian Experimental Watershed, Coshocton, Ohio.

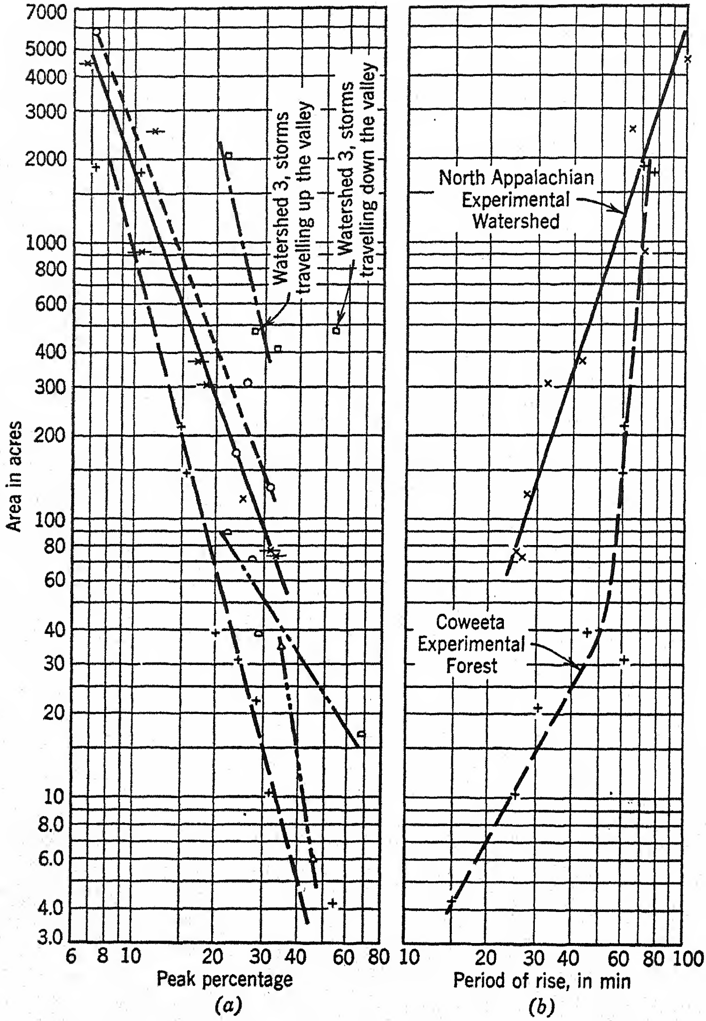

FIG. 105.

Comparison of Distribution Graphs from Similar Watersheds. Average values of the various properties of the distribution graphs from eight watersheds having areas varying from 74.2 acres to 4581 acres are shown in Table 18. These watersheds are located adjacent to each other on the North Appalachian Experimental Watershed, and consequently they are of a similar type. It may be

LEGEND FOR FIG. 105.

North Appalachian Experimental Watershed, Coshocton, Ohio. (Horizontal lines show range of individual values.)

Data taken from Hydrologic Data, *Hydrologic Bul.* 2, 1942, U. S. Department of Agriculture Soil Conservation Service,[1] Blacklands Experimental Watershed, Waco, Texas.

Data taken from Hydrologic Data, *Hydrologic Bul.* 3, 1942, U. S. Department of Agriculture Soil Conservation Service,[1] Central Great Plains Experimental Watershed, Hastings, Nebr.

Data taken from Hydrologic Studies, Compilation of Rainfall and Run-Off from the Watersheds of the Red Plains Conservation Experiment Station, Guthrie, Oklahoma, U. S. Department of Agriculture Soil Conservation Service.[1]

Coweeta Experimental Forest, Appalachian Forest Experimental Station, Asheville, N. C., U. S. Department of Agriculture Forest Service.[2]

Bent Creek Experimental Forest, Appalachian Forest Experimental Station, Asheville, N. C., U. S. Department of Agriculture Forest Service.[2]

[1] Computations made by R. E. Snell, a student in the Univ. of Mich., Horace H. Rackham School of Graduate Studies.

[2] E. F. Brater, The Unit Hydrograph Principle Applied to Small Watersheds, *Trans. A.S.C.E.* **105**, 1154, 1940.

noted from Column 7, that when peak percentages are based on an interval equal to one tenth of the base length, they are nearly of the same magnitude. On the other hand, values in Column 5 show that when peak percentages are based on a uniform time interval,

TABLE 17

1	2	3	4	5	6	7	8
Date	Precipitation in.	Surface Runoff in.	Maximum Rate of Surface Runoff in./hr	Peak Percentage of Distribution Graph	Period of Rise min	Average Infiltration Capacity in./hr	Per Cent of Precipitation Appearing as Surface Runoff
July 8, 1939	0.80	0.080	0.039	23.0	90	1.45	10.0
July 8, 1939	0.60	0.172	0.079	20.8	110	1.54	28.7
May 6, 1940	0.50	0.025	0.009	18.25	90	2.00	5.0
June 10, 1940	1.065	0.168	0.061	18.2	100	2.40	15.8
June 11, 1940	0.67	0.214	0.088	20.5	100	0.96	31.9
June 18, 1940	1.05	0.268	0.112	20.9	110	2.00	25.5
June 23, 1940	0.87	0.066	0.024	17.95	95	1.95	7.6
June 28, 1940	1.39	0.476	0.165	17.35	140	0.93	34.2
Aug. 28, 1940	1.40	0.181	0.088	24.3	80	1.53	12.9
Ratio of Maximum to Minimum	2.8	19.1	18.3	1.4	1.75	2.6	6.8

10 min in this case, they vary inversely with the area. These values of peak percentage are plotted against area in Fig. 105*a*, where their trend is represented by the solid straight line. Similar values for five other groups of watersheds are also shown in Fig. 105*a*. All of the groups exhibit a similar trend. Although size is by no means the only characteristic of a watershed that influences the shape of the unit hydrograph, Fig. 105*a* indicates that it is one of the principal ones. It may be expected that as more experimental data become available, it will be possible to evaluate the effect of shape, stream density, slope, and other factors in a similar manner. Curves of the type shown in Fig. 105*a* will serve as a basis for the determination of peak runoff from small watersheds by methods which will be discussed presently.

TABLE 17 (*Continued*)

9	10	11	12	13	14	15	16
Duration of Precipitation							
Average Duration of Total Precipitation min	Average Duration of Effective Precipitation min	Time of Beginning and End of Effective Precipitation[1]	Distribution of Precipitation Approximate Percentage of Average Precipitation[1]	Time of Beginning of Surface Runoff	Time of Peak of Hydrograph	Time from Beginning of Effective Precipitation to Peak min	Time from End of Effective Precipitation to Peak min
		7:30–7:39	50				
36	9–24[2]	7:40–7:50	160	7:40	9:10	97	82
		7:30–7:54	120				
		12:20–12:30	100				
35–120[2]	9–36[2]	12:45–12:54	135	12:50	2:40	130	102
		12:24–1:00	80				
		None	35				
50	19	6:33–6:53	100	6:35	8:05	94	75
		6:34–6:52	175				
		1:10–1:34	81				
99–311[2]	18	1:06–1:22	100	1:20	3:00	110	92
		1:06–1:32	118				
		1:14–1:28	78				
77	13	1:05–1:20	112	1:15	2:55	100	87
		1:17–1:26	100				
		3:35–3:45	87				
76	14	3:33–3:42	109	4:00	5:50	131	117
		3:43–3:58	95				
		8:14–8:35	92				
99	15	8:20–8:35	103	8:40	10:15	117	102
		8:18–8:32	100				
		7:40–9:08	72				
158	92	7:39–9:15	111	7:50	10:10	150	58
		7:40–9:16	125				
		10:05–10:16	72				
147–421[2]	17	9:53–10:14	100	10:00	11:20	83	68
		9:57–10:11	111				
3.2	7.0					1.8	2.8

[1] The upper value was taken from a typical station near the upper end of the basin, the next two from the central and lower portions, respectively.

[2] In some cases the variation was so great that the range of values rather than the average is given.

Critical Rainfall Duration. In order to predict runoff from any watershed, it is first necessary to determine the maximum precipitation that may be expected. However, because both magnitude and intensity of precipitation vary with the duration, it is necessary to estimate the critical rainfall period for each basin. The data shown in Table 17 as well as other similar studies on *small watersheds* indicate that a volume of rainfall excess may be converted to runoff by means of a single application of the distribution graph, if its duration is no longer than the period of rise. The graph resulting from a longer rain must be derived by successive applications of the distribution graph to unit durations of rainfall

excess (see page 312). However, because rains of a given frequency decrease in intensity with duration, the maximum discharge produced by a longer rain may be little if any greater than that resulting from a rain lasting no longer than the period of rise. Therefore, for the sake of simplicity and at but little sacrifice in accuracy, it will be assumed that for small watersheds, the critical rainfall period may be taken equal to the period of rise.

TABLE 18

1	2	3	4	5	6	7	8
Watershed number	Area acres	Period of Rise min	Length of Base min	Peak Percentage Based on 10-min Intervals	Peak Expressed in cfs/sq mile/in.[1]	Peak Percentage Based on Intervals Equal to 0.1 of Base	Ratio of Length of Base to Period of Rise
183	74.2	27	95	32.7	1270	31.0	3.5
177	75.6	25	124	30.8	1195	38.2	5.0
10	122	27	130	25.1	974	33.4	3.7
196	303	32	200	18.4	715	36.8	6.2
20	373	42	210	17.3	671	36.4	5.0
92	920	72	300	10.5	407	31.5	3.6
95	2569	64	283	12.1	470	34.2	4.2
97	4581	102	563	6.7	250	37.7	5.5

[1] See pages 307 and 308.

Values of period of rise have been plotted against watershed area for two groups of basins in Fig. 105*b*. Again there is a noticeable trend among the points for each group. The relationship is by no means exact because all the watershed characteristics except size have been ignored.

Prediction of Runoff from Rainfall. If sufficient, continuous records are available on a stream so that unit hydrographs and distribution graphs can be determined, the runoff can be predicted with assurance by applying the distribution-graph percentages to units of rainfall excess. Furthermore, runoff from watersheds similar to those upon which records are available can be predicted with satisfactory accuracy by means of curves such as are plotted in Fig. 105. In either case the prediction of the volume of rainfall excess involves the selection of rainfall and infiltration rates by methods described in Chapters IV and VI.

As an example, suppose it is desired to predict the flood peak that will be equaled or exceeded once in 50 yr on a 1500-acre watershed located in central Ohio. It will be assumed that the topography, soil, vegetative cover, and other watershed charac-

teristics are similar to those of the North Appalachian Experimental Watershed for which curves are plotted in Fig. 105. From Fig. 105*b*, the period of rise is found to be approximately 64 min. The next step is to determine the maximum rainfall of this duration which will be equaled or exceeded once in 50 yr. This may best be done by analyzing local rainfall records (see Chapter IV, page 96). If there are no adequate records or if such accuracy is not desired, an approximate value may be obtained from the generalized curves in "Rainfall Intensity—Frequency Data."[1] From this publication it will be found that for central Ohio the maximum 50-yr rain of 60-min duration is 2.75 in. The approximate amount of precipitation for a 64-min duration might also be obtained from

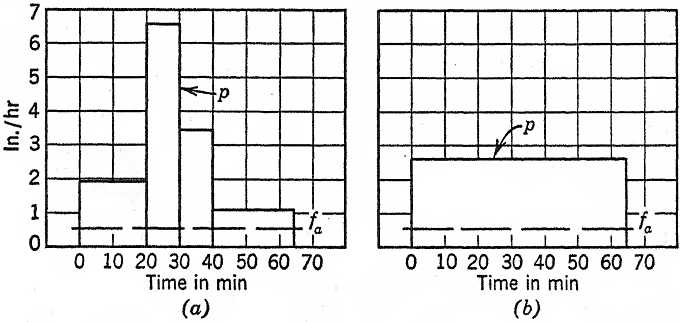

FIG. 106.

this same publication, but it is better to use the curves given in Fig. 39, page 128. From this figure, the location of a 2.75-in. per hr curve is found by interpolation and, for the selected values of duration shown in Column 1, Table 19, the values of rain intensity shown in Column 2 were determined. Column 6 shows the computed precipitation rates which are plotted in an arbitrary arrangement in Fig. 106*a*. In Fig. 106*b* is shown the same volume of precipitation with the intensity assumed to be uniform during the entire 64 min.

It is now necessary to determine the minimum value of f that may be expected at the time of this rain. This information may be gained from a study of multiple peaked hydrographs produced by rains of long duration. For the purpose of this problem, it will be assumed that f_c is 0.5 in. per hr. Also, for the sake of simplicity,

[1] David L. Yarnell, Rainfall Intensity—Frequency Data, *U. S. Department of Agriculture Misc. Pub.* 204, 1935.

it will be assumed that f is constant for the duration of the rain. In applying this value of f_c to the two intensity graphs of Fig. 106, it becomes apparent that the volume of infiltration is the same in each case and that, therefore, the volume of rainfall excess, P_e, would also be the same. The volume of rainfall excess from Fig. 106*b* is $(2.60 - 0.50)64/60 = 2.24$ in. The value of P_e from Fig. 106*a* is computed in Columns 8 and 9 of Table 19. If some portion

TABLE 19

1	2	3	4	5	6	7	8	9
Duration	Precipitation Intensity	Accumulated Precipitation	Time Increments	Precipitation during Time Increments			Precipitation Excess during Time Increments	
t	p	P	Δt	ΔP	p	f	p_e	ΔP_e
min	in./hr	in.	min	in.	in./hr	in./hr	in./hr	in.
			10	1.10	6.60	0.50	6.10	1.02
10	6.6	1.10						
			10	0.57	3.42	.50	2.92	0.49
20	5.0	1.67						
			20	0.66	1.98	.50	1.48	0.49
40	3.5	2.33						
			24	0.44	1.10	.50	0.60	0.24
64	2.6	2.77					$P_e =$	2.24

of the precipitation graph were below the f curve the value of P_e computed from a single block of rainfall such as Fig. 106*b* would be too small. Also, where runoff is being computed from a rain having a duration greater than the period of rise, the single block graph may not be substituted for a typical rainfall pattern.

The final step is to compute the maximum runoff rate. The precipitation excess is first converted to cubic feet as follows.

$$\frac{2.24}{12} \times 1500 \times 43{,}560 = 12{,}200{,}000 \text{ cu ft}$$

Because the peak percentages used in Fig. 105*a* are based on 10-min intervals, the volume of runoff must next be converted to cubic feet per second for 10 min by dividing by the number of seconds in 10 min as follows.

$$\frac{12{,}200{,}000}{10 \times 60} = 20{,}300 \text{ cfs for 10 min}$$

Finally the peak rate of runoff is obtained by multiplying by the peak percentage divided by 100. The peak percentage is found from Fig. 105*a* to be approximately 11 per cent. Therefore, the peak rate of runoff is $20{,}300 \times 11/100 = 2200$ cfs.

If it were desirable to design for the absolute maximum flood that would ever occur, in addition to using a correspondingly greater rainfall excess, it would be desirable to determine values of peak percentage from a curve similar to the one in Fig. 105*a*, but showing maximum peaks instead of average values.

Large Watersheds

The preceding paragraphs dealt with watersheds varying in area from a few acres to approximately 10 sq miles. The term "large watersheds" applies to basins having an area greater than 10 sq miles. However, the distinguishing feature of large watersheds is not that their area is greater than some arbitrary limit, but rather that they are of such size that, within the basin, there are likely to be major differences in rainfall duration and intensity and in soil permeability. On large watersheds, major floods are frequently the result of high rates of surface runoff from only a portion of the basin. Consequently, it is usually necessary to determine unit hydrographs for several different rainfall-distribution patterns. Also, in finding the average infiltration capacity for large basins, appropriate methods such as Horton's (see page 193) must be used. Finally, in correlating distribution graphs from different large watersheds, it is usually necessary to take into account other watershed characteristics in addition to size.

Determination of Unit Hydrographs. The selection of unit hydrographs requires judgment that may be gained only by experience. The procedure usually followed is to scan the runoff records for isolated hydrographs resulting from intense rains. These hydrographs are then plotted together with the rain intensity records from various portions of the basin as illustrated in Fig. 107. It is desirable to have an isohyetal map for each storm to show the rainfall distribution pattern. Those hydrographs that appear suitable may then be converted to distribution graphs in the manner described on page 291. For any basin, various distribution graphs obtained from similar rainfall distribution patterns should show good agreement among the larger ordinates, but considerable divergence among the low percentages at the beginning and end

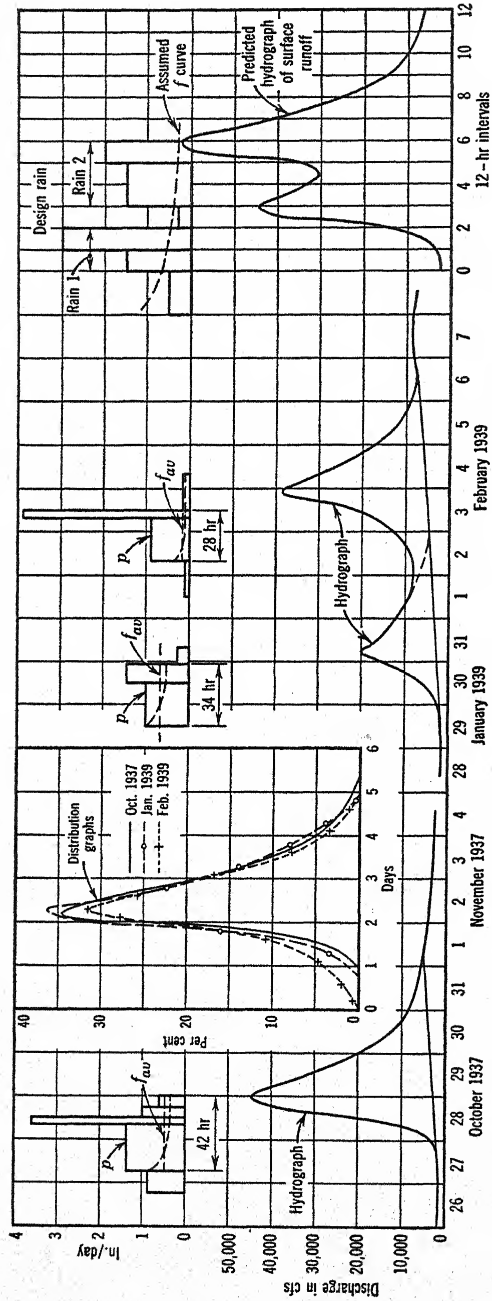
Design rain
Rain 1
Rain 2
Assumed f curve
Predicted hydrograph of surface runoff
12-hr intervals
February 1939
January 1939
November 1937
October 1937
Hydrograph
28 hr
34 hr
42 hr
p
f_{av}
Distribution graphs
Oct. 1937
Jan. 1939
Feb. 1939
Days
Per cent
In./day
Discharge in cfs
50,000
40,000
30,000
20,000
10,000

FIG. 107.

of the graph may be expected. If there are no great variations in rainfall characteristics, an agreement as good or better than that shown in Fig. 104 may be expected.

Various government agencies have determined distribution graphs for large numbers of basins. The peak values of some of these are plotted against area in Fig. 108. It will be seen that several large groups were derived from contiguous basins. Also

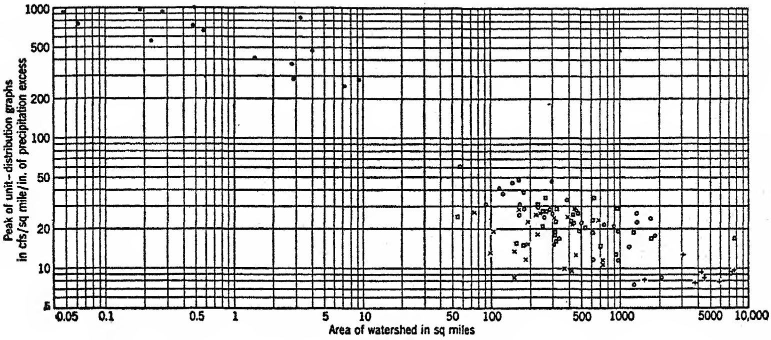

FIG. 108.

+ *U. S. G. S. Water-Supply Paper* 772, 1936.

× Gerald T. McCarthy, U. S. Engineer Office, Providence, R. I., *The Unit Hydrograph and Flood Routing,* March 1939. (All streams are within the basin of the Connecticut River.)

○ N. R. Laden, T. L. Reilly, and J. S. Minnotte, Synthetic Unit-Hydrographs, Distribution-Graphs and Flood Routing in the Upper Ohio River Basin, *Trans. Am. Geophys. Union,* 1940, Part II, p. 649.

□ Hydrological reports prepared by various district offices of U. S. Engineer Department. Points obtained from Hydrologic and Hydraulic Analyses, *Engineering Manual for Civil Works,* Part III, Chap. 5, 1946.

● Points transferred from Fig. 105*a*.

shown in Fig. 108 are many of the values for small watersheds that were plotted in Fig. 105*a*. It may be noted that within rather wide limits there is a definite trend among the values plotted.

The units of the peaks shown in Fig. 108 were taken as cubic feet per second per square mile per inch of rainfall excess, whereas in Fig. 105*a* the peak percentages were plotted. These two units are similar and readily convertible. The procedure is as follows. The application of 1 in. of rain on an area of A sq miles produces a volume equal to

$$\frac{1}{12} \cdot (5280)^2 A \text{ cu ft}$$

If the time interval into which the unit hydrograph was divided is Δt sec, then the volume may be converted to cubic feet per second for a time Δt by dividing by Δt. To get discharge per square mile, it is necessary to divide by A, and finally the peak is obtained by multiplying by the peak percentage and dividing by 100 as follows:

$$\left.\begin{array}{l}\text{Peak in cfs}\\ \text{per sq mile}\\ \text{per in. of}\\ \text{rainfall ex-}\\ \text{cess}\end{array}\right\} = \frac{A(5280)^2}{12 \times \Delta t \times A} \times \frac{\%}{100} = \frac{(5280)^2}{12 \times \Delta t} \times \frac{\%}{100}$$

An example of such a conversion is shown in Table 18, Columns 5, 6, and 7. In this case the time interval was 10 min so that Δt was 10×60 sec, whereas the distribution graphs presented in *Water-Supply Paper* 772 were divided into 24-hr intervals, thus making $\Delta t = 24 \times 60 \times 60$ sec. It is difficult to find a suitable name for a graph in which the ordinates have the units of cubic feet per second per square mile per inch of rainfall excess. It is more closely related to the distribution graph than the unit hydrograph. Perhaps a good term would be a unit-distribution graph. In this connection it is well to review the definition of a unit hydrograph. In his original presentation,[1] Sherman stated, "From the known hydrograph the 'unit' graph must be determined representing 1 in. of runoff from a 24-hour rainfall." In the accompanying example Sherman derived the "unit" graph with the ordinates expressed as cubic feet per second. However, with the introduction of the distribution graph by Bernard,[2] it is no longer necessary to reduce a hydrograph to the "unit" graph described above because the more useful distribution graph can be derived directly from the original hydrograph. Consequently in a later article,[3] Sherman has given the following definition for a unit hydrograph. "The unit hydrograph is the hydrograph of surface runoff (not including groundwater runoff) on a given basin, due to an effective rain falling for a

[1] L. K. Sherman, Streamflow from Rainfall by Unit-Graph Method, *Eng. News-Record*, 1932, p. 501.

[2] Merrill M. Bernard, An Approach to Determinate Stream Flow, *Trans. A.S.C.E.*, 1935, **100**, 347.

[3] O. E. Meinzer, Editor, Physics of the Earth IX, *Hydrology*, The Unit Hydrograph Method by L. K. Sherman, p. 514.

unit of time. The term 'effective rain' means rain producing surface runoff. The unit of time may be one day or preferably a fraction of a day. It must be less than the time of concentration." This is the definition adopted by the authors.

The authors have stated that the duration of a unit storm must not exceed the period of rise. This is substantially in agreement with Sherman's statement quoted above to the effect that the duration must be less than the time of concentration. The time of concentration is probably only a little longer than the period of rise. The authors believe that the period of rise is more easily defined and determined than the time of concentration. For large watersheds the duration of a unit storm may be less than the period of rise, possibly no more than half as long.

It must be emphasized that there is no relation between the duration of rainfall excess that produces a unit hydrograph and the time intervals into which the unit hydrograph is divided for the purpose of obtaining a distribution graph. The selection of the magnitude of these intervals depends entirely upon how accurate a reproduction of the hydrograph is desired and upon the time available for the work.

Predicting Runoff from Rainfall. When sufficient stream-flow records are available so that a representative distribution graph may be obtained, the construction of a hydrograph may be carried out with considerable assurance for any assumed rate of rainfall excess. No other procedure so far developed approaches in accuracy that which may be obtained by this method, even when only a few records are available. If the period of assumed rainfall excess is about the same as that which produced the unit hydrographs, the procedure is to multiply the volume of rainfall excess by successive distribution graph percentages to obtain the increment of volume to be expected at the outlet during each time interval. Each increment of volume must then be converted to cubic feet per second. If the duration of rainfall excess is greater than that which will produce a single unit hydrograph, the rainfall excess must be divided into portions of shorter duration and the above procedure applied to each portion. The final hydrograph may then be obtained by adding the various increments of runoff contributed to each time interval by the various portions of rainfall excess.

In reproducing or synthesizing hydrographs by this method, the

best results are obtained if the rainfall excess is divided at points of sudden change in intensity rather than at arbitrarily selected time intervals. The distribution graph is not a sufficiently precise tool to be sensitive to differences in the duration of rainfall excess that are small compared with the period of rise. For example, if the period of rise for a stream is 3 days, it is probable that periods of rainfall excess of 6, 12, or even 18 hr will produce nearly the same percentage graph. It will require further research before enough experimental evidence is available to establish the nature of the variation for such small changes in duration. Any refinements in the use of the distribution graph that may result from future research must include the effect of all the factors that influence the shape of the graph. (See page 32.) Such a refinement may be hoped for, but, while experimental evidence is accumulating, the distribution graph must be used with full cognizance of its limitations.

As an example of runoff prediction on a large basin, the Youghiogheny River at Connellsville, Pennsylvania, was selected. The drainage area above Connellsville is 1326 sq miles. Three hydrographs together with average rain intensity patterns are shown in Fig. 107. The surface runoff was separated from ground-water discharge as shown in the figure. The average surface runoff rate for each 12-hr period together with the instantaneous peak rate is given for each graph in Table 20, Column 3. Also shown in Column 3 is the total surface runoff for each graph. The average infiltration capacity for each storm was determined and plotted in Fig. 107. The procedure for determining the infiltration capacity of a large drainage basin has been fully explained (see page 193) and will not be repeated here. Also shown on each graph is an f curve sketched so that the volume of infiltration is the same as that beneath the f_a line.

The distribution graphs were derived by converting each of the values in Column 3, including the peak, to percentages of the total. These percentages are given in Column 4, Table 20, and plotted in Fig. 107.

This information will now be used to predict the runoff from the "design rain" shown in Fig. 107. For the purpose of this example an f curve was assumed and is shown superimposed on the design rain. The volume of rainfall excess for Rains 1 and 2 were then determined as 1.68 in. or 120,000 cfs for 12 hr and 2.02 in. or 143,000

TABLE 20

1 Date	2 12-Hr Time Intervals	3 Surface Runoff cfs	4 Distribution Graphs percentage
Oct. 1937	1	200	0.2
	2	4,500	3.7+
	3	35,000	29.1
	(pk)	(41,500)	(34.5)
	4	36,000	30.0
	5	22,000	18.3
	6	12,500	10.4
	7	6,000	5.0
	8	3,000	2.5
	9	1,000	0.8
		120,200 cfs for 12 hr or 1.68 in.	100.0
Jan. 1939	1	1,800	3.6
	2	8,000	16.1
	3	15,500	31.2
	(pk)	(17,500)	(36.3)
	4	11,000	22.2
	5	7,000	14.1
	6	4,000	8.1
	7	2,000	4.0
	8	300	0.6
		49,600 cfs for 12 hr or 0.70 in.	99.9
Feb. 1939	1	700	0.7
	2	2,000	2.0
	3	5,000	4.9
	4	11,000	10.8
	5	28,000	27.6
	(pk)	(32,000)	(31.5)
	6	26,000	25.6
	7	16,000	15.8
	8	8,000	7.9
	9	3,500	3.5+
	10	1,200	1.2
		101,400 cfs for 12 hr or 1.40 in.	100.1

cfs for 12 hr, respectively. It was deemed necessary to apply the distribution graph to the two parts of the design rain separately because the total duration of this rain is 72 hr, whereas the average period of rise of the three distribution graphs is only about half this much. It may be noted that the duration of rainfall excess for the three distribution graphs varied from 28 hr to 42 hr. Often an average-distribution graph is determined from a group such as is

TABLE 21

1	2	3	4	5	6
	Runoff			Ground-Water Flow	Total Stream Flow
12-Hr. Time Intervals	First Rain cfs	Second Rain cfs	Total cfs	cfs	cfs
1	200		200	2000	2,200
2	4,500		4,500	2300	6,800
3	35,000		35,000	2600	37,600
	(41,500)		(41,500)		(44,200)
4	36,000	300	36,300	2900	39,200
5	22,000	5,300	27,300	3200	30,500
6	12,500	41,600	54,100	3500	57,600
		(49,200)	(58,500)		(62,100)
7	6,000	42,800	48,800	3800	52,600
8	3,000	26,200	29,200	4100	33,300
9	1,000	14,900	15,900	4400	20,300
	Σ(120,200)				
10		7,200	7,200	4700	11,900
11		3,600	3,600	5000	8,600
12		1,100	1,100	5300	6,400
		Σ 143,000	Σ 263,200		

shown in Fig. 107. In this case the one for the October 1937 storm was selected to apply to the design rains. These percentages are applied to the corresponding volumes of rainfall excess to obtain the values shown in Columns 2 and 3 of Table 21. The values in Column 3 are started 36 hr after those in Column 2 to agree with the timing of the rain as shown in Fig. 107. Column 4 is the sum of Columns 2 and 3 and gives the predicted surface runoff. It is still necessary to superimpose these values upon some assumed ground-water flows such as are shown in Column 5. These are combined with the surface runoff to obtain the values of Column 6 which are plotted in Fig. 107.

The importance of carrying along the peak percentages cannot be overemphasized. All other percentage values are determined from average discharges during the selected time intervals. Only for extremely large basins would the average value for the maximum interval approach the value of the peak. In this example, an error of approximately 4500 cfs in the peak discharge would have resulted if the peak percentage had been ignored and as a result the predicted hydrograph would have had an incorrect shape.

Often it is necessary to estimate the hydrograph at a point where no discharge records are available. There are many variations in the conditions under which this problem arises. In some cases, for example, there may be gaging stations at other points on the stream from which unit hydrographs may be obtained. These may then be converted to the desired location by a flood-routing procedure (page 356). At other times there may be records on a number of other nearby basins having such similar watershed characteristics that a similarity between distribution graphs may be assumed.

In general, it is desirable to synthesize a hydrograph only on the basis of a thorough study of the graphs obtained at all available nearby stations. Because of the wide variety of rainfall and physical characteristics that are likely to exist on large watersheds, it is usually not possible to find a group so similar that peaks may be compared on the basis of size alone. This is demonstrated by the values plotted in Fig. 108 where even those from contiguous groups show a rather wide scattering. Some of the factors that cause the shape of the distribution graph to vary on any particular watershed are the rainfall-intensity pattern, the rainfall-distribution pattern, the direction of the storm path, the rainfall duration, the conditions of the soil at the beginning of the rain, the condition of the vegetation, the quantity of channel storage present at the beginning of the rain, the presence of ice in the stream, and the amount of vegetative growth in the stream. Some of the factors that cause variations in the distribution graphs for different basins are size, shape, slope of stream, slope of ground, stream density, roughness of stream channel, and the presence of natural or artificial channel storage.

The correlation of all these factors with the peak, period of rise, and base length of distribution graphs presents a difficult

problem. Obviously, it is necessary to select only the more important factors for study. From an intensive study of the Connecticut River system, McCarthy[1] found that the distribution graph characteristics could be determined with the following watershed characteristics as a basis: (1) area, (2) slope of area-elevation graph, and (3) the stream pattern expressed as the number of major stems of a watershed. The area-elevation graph as used by McCarthy is similar to the hypsometric chart shown in Fig. 14, page 47.

From a study of streams located in the Appalachian Highlands, Snyder[2] found that he needed only to consider size and shape of watersheds in order to correlate the properties of distribution graphs. He found that the distance, L, measured along the principal stream from the outlet to a point adjacent to the geographical center of the basin, could be used as a measure of the effect of shape on the properties of the hydrograph. He was able to determine the relation of both the "lag" and peak of the hydrograph to this distance, L. The term "lag" is defined by Snyder as "the time between center of mass of surface-runoff-producing rain of a specified type of storm and the occurrence of resulting peak discharge at the location being studied." If some stream-flow data are available from which to obtain certain coefficients, the relations developed by Snyder may be used to estimate the shape of a distribution graph for a watershed.

For the purpose of making a similar study in the upper Ohio basin it was reported[3] that neither of the methods mentioned above was entirely satisfactory but that a combination of the two was found useful. Perhaps no single universally applicable approach to this complex problem can be found. It must be expected that oftentimes where unit hydrographs are to be developed new problems may arise, the solution of which will require ingenuity and good judgment. Nevertheless, the unit hydrograph is by far the most dependable tool for predicting flood runoff that has so far been developed.

[1] Gerald T. McCarthy, U. S. Engineer Office, Providence, R. I., *The Unit Hydrograph and Flood Routing*, 1939.

[2] Franklin F. Snyder, Synthetic Unit-Graphs, *Trans. Am. Geophys. Union*, 1938, Part I, p. 447.

[3] N. R. Laden, T. L. Reilly, and J. S. Minnotte, Synthetic Unit Hydrographs, Distribution-Graphs and Flood Routing in the Upper Ohio River Basin, *Trans. Am. Geophys. Union*, 1940, Part II, p. 649.

MINIMUM FLOW

Except for streams that are fed by melting snows and those draining areas that contain a large amount of surface storage, low water flow is derived entirely from ground water. Horton has shown that the portion of the hydrograph that represents only ground-water flow is an exhaustion curve, and may be expressed by an equation of the form

$$Q = Q_0 e^{-cd^n}, \tag{16}$$

in which Q is discharge in cubic feet per second at the end of d days after termination of surface runoff; Q_0 discharge when d equals zero; e Napierian base, 2.718; and c and n are constants.

In order to evaluate Q_0, c, and n for any drainage basin, a composite ground-water-depletion curve should be constructed from the recession graphs resulting from a number of storms. The various segments of recession curves are shifted with respect to the time axis until they appear to match, and then an average or composite curve is drawn through them as shown in Fig. 3, page 24. In order to evaluate the constants of equation 16, the location of the time scale must be arbitrarily selected. The time when $d = 0$ may be taken at or before the time of occurrence of the highest point on any of the recession curves that is deemed to be free of any surface runoff. Such a point is illustrated by c in Fig. 3. The zero of the time scale occurs 8 days before c. For any other location of the time scale, the equation of the curve below c may also be determined, but the values of the constants will be different. Exactly the same extension of the recession curve to lower discharges would be obtained, however, by any other choice of origin.

Written in logarithmic form, equation 16 becomes

$$\log Q = \log Q_0 - cd^n \log e \tag{17}$$

Equation 17 represents a straight line in which $\log Q$ and d^n are the variables. In Fig. 109 it is shown that equation 17 becomes a straight line when $n = 0.5$. The value of Q_0 is the value of Q when $d = 0$ and is found to be 6000 in Fig. 109. The value of c, determined from the slope of the line, is 0.555. The equation for the line then becomes $Q = 6000e^{-0.555d^{0.5}}$.

By studying long-term records of rainfall in the region, the

longest period that is likely to elapse between rains of sufficient magnitude to produce ground-water accretion may be determined. If some point on the recession curve is selected as representing

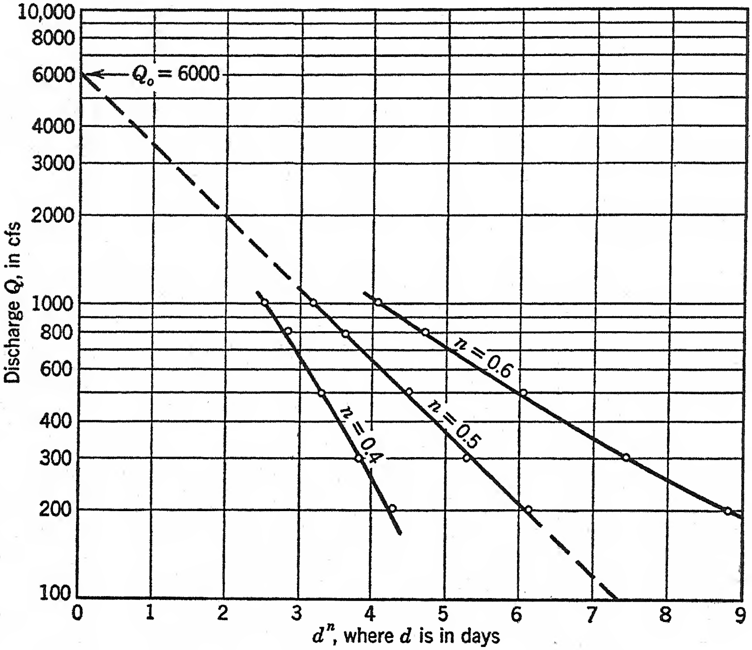

FIG. 109.

the conditions at the beginning of such a period, the daily discharge throughout the period may be determined from the derived equation or from an extension of the straight line of Fig. 109.

YIELD

It will be recalled that, in the determination of surface runoff, the method of procedure depended principally upon the size of the area drained. By way of contrast, the best procedure for determining yield depends not upon the size of basin but upon climate. For this purpose climate will be classified as *arid*, *semiarid*, and *humid*. Streams draining *arid* basins carry only surface runoff and, therefore, are intermittent and flow only during and immediately following periods of intense rainfall. The water table, if there is one in the basin, is always below the bed of the stream. Basins whose climate is here classified as *semiarid* have a rainy season and a dry

season. Nearly all the year's rainfall occurs in the rainy season during which stream flow is continuous. During the dry season the stream is dry most of the time, carrying water only intermittently after an occasional heavy storm. In *humid* regions, except for the smaller tributaries, the flow is continuous and the water table is always above the bed of the stream.

Arid Basins

Inasmuch as the entire yield of arid basins is derived from surface runoff, it follows that in order to determine their yield the same procedure may be followed as has already been described for determining surface runoff from any basin. (See page 305.) There is, however, this distinction, that in arid basins all infiltration may be included as a water loss because none of the water that once infiltrates into the soil ever reaches the stream draining that basin, nor does ground-water storage affect this problem. In this case, however, the problem is made easier because the infiltration capacity is nearly constant throughout the year.

From the records of a number of intense storms and their hydrographs of surface runoff the average infiltration capacity of the basin can be determined. This average value is then applied to each intense storm and the resulting runoff is found.

Semiarid Basins

As stated above, the flow of streams draining semiarid basins is usually continuous throughout the rainy season and intermittent during the dry season. The yield during the rainy season may be found by several methods depending upon whether a knowledge of total yield is sufficient or whether the manner in which that yield varies from month to month is required. In the first case a direct correlation may usually be found between total rainfall and total runoff for the rainy season. During the dry season, inasmuch as the entire flow comes from surface runoff, the yield is found in the same manner as described above for arid basins.

If, however, a knowledge is required of the manner in which yield varies from month to month, perhaps the best procedure is to plot a graph of mass runoff against mass rainfall for each rainy season for which records of runoff (or yield) are available. It will usually be found that if, for any rainy season, the mass runoff, expressed in any convenient units such as inches depth on the

basin, is computed month by month, and plotted against the mass rainfall, expressed in the same units, these graphs for different rainy seasons, plotted on the same coordinate axes, will practically coincide. When they fail to do so the reason can usually be traced to a difference in rainfall distribution or to incomplete records of rainfall on the basin.

If mass runoff is plotted on the horizontal axis and mass rainfall on the vertical, this curve will nearly always be concave downward because for each succeeding month throughout the rainy season the runoff is an ever-increasing percentage of the rainfall. Two reasons may account for this increase. At the beginning of the rainy season the water that infiltrates into the ground builds the water table up to a level above the bed of the stream. The portion above stream-bed level becomes stream flow later in the season. Also because of the increase in soil moisture, the inwash of fine materials, and other causes, the infiltration capacity may decrease as the rainy season progresses.

Having several such curves, each representing mass runoff for the season plotted against mass rainfall, a mean curve can be drawn in. This mean curve may be used for determining the monthly runoff for each rainy season for which rainfall records are available; it may also be used for predicting the probable monthly yield from any assumed seasonal rainfall pattern. If there have been diversions for irrigation or for other purposes, of course, such quantities must be added in computing these mass-runoff curves.

Attention should here be called to the fact that in the earlier years of records practically all precipitation stations were located in the valleys. With the knowledge that precipitation varies with altitude, during recent years an increasing number of stations have been established at higher altitudes. Therefore, the relation between rainfall and runoff or yield may appear from the records to have changed, whereas in reality the only change has been as a result of the increase in the number of rainfall stations located at higher altitudes.

Humid Basins

In humid regions such as the eastern part of the United States, except for streams that drain only small areas, the flow is continuous. The water table never drops below the bed of the stream.

As a result, the yield of such a stream for any year or for any month is dependent upon the elevation of the water table at the beginning of the period. The change in ground-water storage, ΔS, now becomes important, whereas in arid regions it had no significance, and in semiarid regions it had only the effect of reducing the monthly yield during the early months of the rainy season and increasing it correspondingly during the later months; it had no effect upon the total seasonal or annual yield.

For humid basins the *average annual yield* usually can be satisfactorily determined from a comparatively short period of records. For instance, having available 10 yr of records of rainfall and runoff, the value of ΔS would be small compared with the total yield. Also the average annual water loss as determined from 10 yr of records should not depart very much from the long-term average. Therefore, if the long-term mean annual rainfall and the corresponding average annual water loss are known, the average yield may be taken as the difference between the two. However, the determination of yield, month by month or even year by year, cannot be made with the same degree of accuracy because of the increasing importance of ΔS as the time interval decreases and because of the relatively greater influence of the various factors that affect evaporation opportunity. As an illustration, referring to *U. S. Geological Survey Water-Supply Paper* 846, page 57, it will be observed that the water losses on the Tittabawassee basin above Freeland, Michigan, for 1916 were 13.0 in., rainfall 28.0 in., and temperature 45°. In 1914 the rainfall was 32.2 in., the temperature 45.6°, but the water loss was 24.4 in. As a result the yield in 1916 was 15 in. as compared with 7.8 in. in 1914 although the rainfall was 4.2 in. more in 1914. Also on the West River at Newfane, Vermont, in 1920 the precipitation was 36.9 in. and the water loss was 7.5 in., whereas in 1930 the rainfall was 38.7 in. and the water loss was 17.5 in., with only a small difference in temperature. Many other similar illustrations can be given which show a lack of correlation between water loss, rainfall, and temperature for short periods. (Also see Fig. 55.)

Because of the influence of soil, vegetative cover, land use, depth to water table, and other factors that often affect the water losses of adjacent basins in varying degrees, the average water losses cannot be reliably determined for a basin on which no stream-flow records are available.

Use of Records on Adjacent Basins. Yield varies more or less regionally. As a result, if long-term records are available on one or more adjacent basins, those records may often be used advantageously. Two different conditions will be considered. In the first, no discharge records are available on the stream in question,

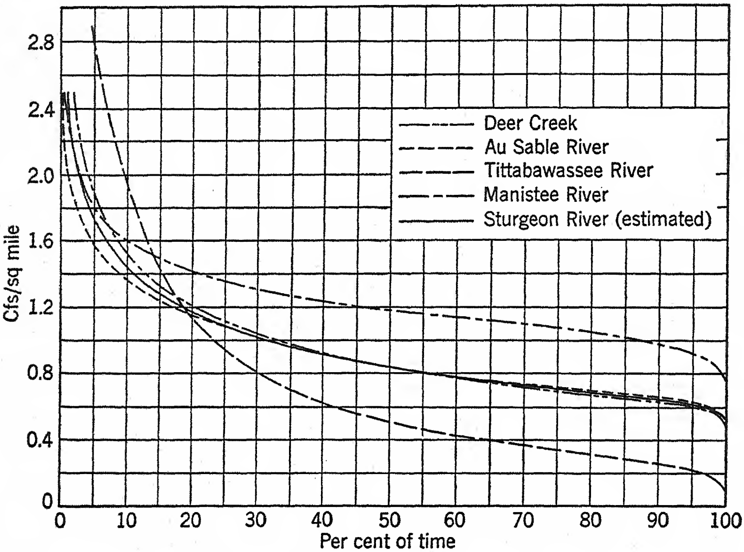

FIG. 110.

whereas in the second, records are available for a period of at least a few years that are also covered by those on the adjacent stream.

The solution of a problem of the first type is illustrated in Fig. 110 in which the duration of flow curve for the Sturgeon River in Michigan is estimated by comparison of the duration of flow curves for Deer Creek, Au Sable River, Tittabawassee River, and Manistee River. Of these streams, the Sturgeon River basin lies nearer to and is perhaps more similar to the Deer Creek basin on the west and to the Au Sable River basin on the southeast than it is to the other two. The Manistee is farther away to the southwest, and the Tittabawassee is still farther away to the southeast. Consequently the estimated duration-of-flow curve for the Sturgeon River is drawn in between the duration curves for the Au Sable River and Deer Creek, being influenced only slightly by the curves for the Manistee and the Tittabawassee Rivers.

This study was made for the City of Petoskey in 1927 at a time when no discharge records on the Sturgeon River were available. At that time the average yield of the Sturgeon River at Wolverine was estimated to be 202 cfs. In 1941, the U. S. Geological Survey established a gaging station at this same site and the actual average measured discharge since that time has been 208 cfs. Such a

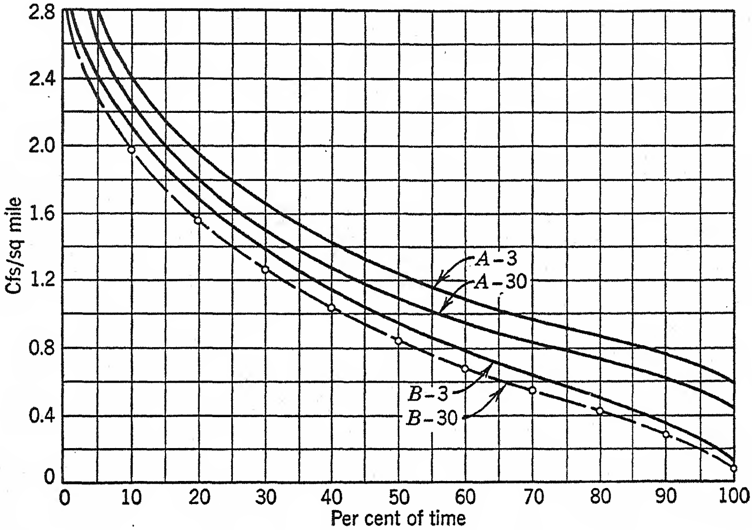

FIG. 111.

high degree of accuracy should not always be expected, but, in general, this method will be found to lead to more dependable results than will be obtained from a theoretical determination of the probable water losses.

In Fig. 111 is illustrated a method in which duration of flow curves can be used in estimating the yield of a stream for which short-term records are available and for which simultaneous and long-term records are available on at least one adjacent stream. In this case it is assumed that there are records covering a period of 3 yr on Stream *B*, and on Stream *A* there are records covering a period of 30 yr, 3 yr of which are simultaneous with the records on *B*. Duration curves are then plotted for both streams for the 3 yr for which the records are simultaneous. Also the duration curve is plotted for Stream *A* for the entire period covered by the records, which in this case is assumed to be 30 yr. Then, in order

to obtain the 30-yr duration curve for Stream B, the discharge values obtained from the 3-yr curves for B are multiplied by the ratio of the 30-yr yield to the 3-yr yield on A for a number of points on the curves. For instance, during 60 per cent of the 30 yr of records on A, the flow exceeded 0.96 cfs per sq mile, whereas for the same percentage of time during the 3 yr of records it exceeded 1.10 cfs per sq mile. For the same percentage of time the flow of Stream B, during the 3 yr of records, exceeded 0.78 cfs per sq mile. Therefore, for 60 per cent of the time the flow of Stream B equaled or exceeded

$$\frac{0.96}{1.10} \times 0.78 = 0.68 \text{ cfs per sq mile}$$

The estimated yield of Stream B for other percentages of time are obtained in the same manner.

If similar records are available for other adjacent basins, an estimated duration-of-flow curve should be derived from each. After plotting all such curves on the same sheet and after a careful comparison of the physical characteristics of each of these basins with those of Stream B, a final estimated curve can be drawn in.

If, for storage or other purposes, it is desired to obtain an estimated hydrograph of Stream B, this can be done in the following manner. Suppose that, at a certain time, the recorded flow of Stream A was 0.84 cfs per sq mile. From the 30-yr duration curve for Stream A it is found that this flow corresponds to 70 per cent of the time. For this same percentage of time, the 30-yr duration curve for Stream B shows a flow of 0.55 cfs per sq mile. The estimated flow for each successive time interval is found in a similar manner. In this way a continuous hydrograph, based upon either daily, weekly, or monthly average discharges, may be obtained for the entire period of records on the adjacent stream. However, the time interval used in deriving the hydrograph must be the same as was used in constructing the original duration curves.

Because the rainfall pattern for a long period is not the same on any two basins, it should not be expected that every minor variation in the hydrograph of any given stream can be accurately reproduced by this method. Nevertheless the general behavior of the stream, the number of peaks of each different magnitude, and the number of periods of low flow should all be correctly shown. Consequently a hydrograph derived in this manner should be just

as useful in making storage, water-supply, or power studies as though the fluctuations were chronologically represented perfectly. The value of the results is dependent only upon whether or not the various factors that affect the rainfall-runoff relation during the period of simultaneous records were normal. Therefore, the longer the period of overlapping records the more reliable will be the results.

Other Methods of Estimating Yield. From time to time various other procedures have been suggested for estimating yield. One of the earliest of these was proposed by Vermeule[1] in 1894. He was probably the first to recognize the importance of ground-water storage and to point out that there is a limit to the volume of this storage on any basin. This fact has also been discussed by Horton.[2] A number of articles describing this or related methods of predicting long-term yield are listed below;[3] many others could be added.

[1] C. C. Vermeule, *New Jersey Geological Survey*, **3**, 11.

[2] R. E. Horton, Maximum Ground-Water Levels, *Trans. Am. Geophys. Union*, 1936, Part II, p. 344.

[3] F. F. Snyder, A Conception of Surface Runoff, *Trans. Am. Geophys. Union*, 1939, Part IV, p. 725; R. K. Linsley, Jr., and W. C. Ackermann, A Method of Predicting the Runoff from Rainfall, *Trans. A.S.C.E.*, 1942, **107**, 825; C. R. Hursh, M. D. Hoover, and P. W. Fletcher, Studies in the Balanced Water-Economy of Experimental Drainage-Areas, *Trans. Am. Geophys. Union*, 1942, Part II, p. 509; C. K. Cooperrider, H. O. Cassidy, and C. H. Niederhof, Forecasting Stream-Flow of the Salt River, Arizona, *Trans. Am. Geophys. Union*, October 1945, p. 275; F. Paget, A New Forecasting Curve for the Kaweah, *Trans. Am. Geophys. Union*, June 1946, p. 389.

CHAPTER IX

FLOODS

Importance of Flood Studies

Floods in the United States cause a property damage that has been estimated to average over $35,000,000 a year in addition to a toll of human lives on which no monetary value can be placed. A full realization of the magnitude of the stakes that are involved in the solution of the flood problems existing on the various streams throughout the country should impress those in charge of these studies with a deep sense of responsibility and obligation and should imbue them with a determination to spare no efforts to obtain the most reliable results possible. On the other hand, except in the more important hydraulic structures, it is not uncommon for those in charge to devote an incredibly short time to the determination of the magnitude of the flood for which the structure should be designed. Far too frequently all that is done is to apply a few convenient formulas, or perhaps determine from the records what the maximum flood has been in the past, and then add 25 or 30 per cent as a factor of safety. They then spend months on its structural design. This fact perhaps explains why dams, bridges, and culverts rarely fail because of structural defects; for it is a matter of record that by far the greater number of such failures are the direct result of faulty determinations of the magnitude of the floods for which these structures should be designed.

In this problem as in all others the proper solution lies in making the best possible use of all available data. What are the data that are pertinent? In the first place there are, of course, the records of floods in the past, if any records exist; but just as important are the records pertaining to *the factors that affected and determined the magnitudes of those floods.* This latter type of data is too frequently ignored. It is difficult to understand why these additional data are not more frequently used, for they are practically always available whenever flood records are available. Methods of using the above-mentioned data will be outlined in this chapter following

a general discussion of floods and of methods of estimating their magnitude and frequency.

Definition of Flood

Normally a flood is considered an unusually high stage of the river. It is perhaps better described as that stage at which the stream channel becomes filled and above which it overflows its banks. The valley then becomes "flooded." In *Webster's New International Dictionary*, a "flood" is defined as "a great flow of water; . . . especially, a body of water, rising, swelling, and overflowing land not usually thus covered; a deluge; a freshet; an inundation."

Inasmuch as the banks of a stream vary in height throughout its course, there is no definite stage above which a river can be said to be in flood and below which it is not in flood. Frequently, however, especially on the more important rivers an arbitrary elevation has often been established, either by the U. S. Army Engineers or by others in authority, that is called "flood stage."

Causes of Floods

All floods are primarily due to surface runoff. Any drainage basin with soil so pervious that its infiltration capacity is never exceeded is rarely subject to floods. As an illustration, the Manistee River in Michigan is fed primarily by ground water, and it experiences no serious floods. (See Fig. 9.)

Floods may result from (1) an intense rainfall, (2) the melting of accumulated snow, or (3) the melting of snow combined with rain. In the southern states the melting of snow is not a factor in flood production, whereas in many parts of Canada it is the most common cause. In the latitude of southern New York and Michigan, floods result from all three of these causes. It is not uncommon in this latitude that the greatest floods on large drainage basins result from an early warm spring rain falling on melting snow, whereas on small basins in the same area the largest floods result from intense summer thunderstorms.

A well-defined technique has been developed for the determination of the probable maximum flood resulting from rainfall alone that may be expected from any size of drainage basin with a given frequency of occurrence.

In any flood study the first problem is the determination of the

factors that combine to produce floods on that particular basin. It may be found that some floods on the basin being studied result from each of the above three causes. If the problem involves the determination of the maximum flood to which the basin would ever be subjected, attention should be directed toward learning the cause or causes that produce the major floods. It may so happen that the majority of the floods on this stream are caused by summer storms but that all major floods result from the melting of snows. If such is the case, all summer floods may be disregarded and the investigation confined to floods produced by the melting of snow.

Furthermore, if it is found that all major floods are the result of rainfall, attention should next be directed toward determining the character and origin of these storms. As an illustration, Ruff,[1] in a study of the nature and origin of the storms that cause floods in Pennsylvania, examined the paths of the low-pressure areas for several days preceding a number of the largest floods that have occurred on all the principal streams and their tributaries in the state. As a result he found that winter floods occurring throughout the state were caused by storms that moved up the Ohio Valley in a northeasterly direction. Summer floods on streams west of the Appalachian Mountains were found to result from storms having the same origin and direction of travel. However, for all streams east of the Appalachians, the summer flood-producing rains came, in general, from the south. Such information is of great value especially in comparing the magnitude and frequency of storms and floods in two or more nearby drainage basins.

Seasonal Distribution of Floods

In most regions floods occur more frequently in certain seasons of the year than in others. In *Water-Supply Paper* 771 the U. S. Geological Survey presents data on the principal floods that have been recorded on many of the more important streams in the United States. In order to show the seasonal occurrence of floods and the manner of its variation throughout the country, ten typical streams whose records are listed in this publication have been selected and the percentages of the total number of recorded floods that have occurred during each month of the year have been computed. The results are shown in Fig. 112. Also shown for each

[1] Charles F. Ruff, Maximum Probable Floods on Pennsylvania Streams, *Proc. A.S.C.E.*, September 1940, vol. 66, No. 7.

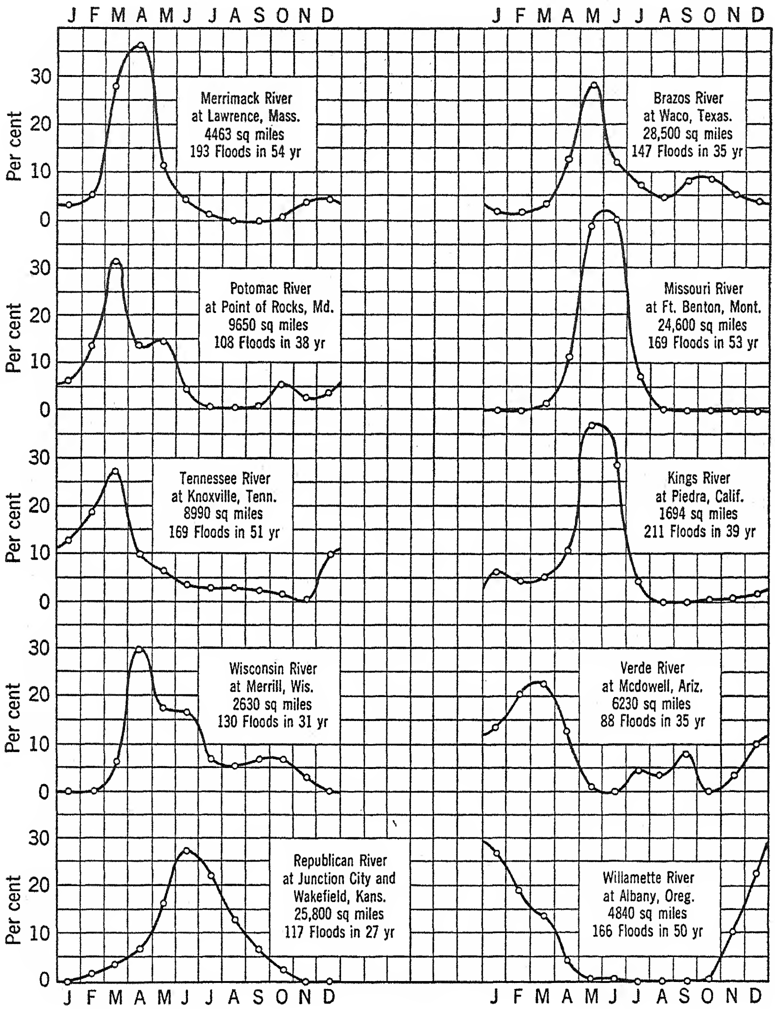

FIG. 112.

stream are the area of basin, the number of floods, and the length of record.

A study of this figure shows that in every basin there are a few months each year during which a major portion of the floods occur. For instance, in New England the Merrimack River is much more subject to floods during the months of March, April, and May than

at other times, with the greatest number occurring in April. On the Potomac and on the Tennessee, the season advances slightly, with the greatest number occurring in March. On midwestern streams it is delayed somewhat, reaching the peak in April, May, or June. In the far West the flood period varies greatly. For instance, on the King's River in California and on the Missouri River in Montana, most of the floods occur in May and June, whereas on the Willamette River in Oregon nearly all occur in the winter with only a rare occurrence during the summer months from May to October inclusive.

Although usually the major floods occur during the period of greatest frequency, they perhaps do so only in conformance to the theory of probability. As a result, one of the greatest floods on a stream may occur at a time when least expected. As an illustration, of 153 floods in 54 yr of records on the Merrimack River, only six occurred in November. Nevertheless one of those six was the second largest ever recorded on that stream.[1]

Is the Flood Hazard Increasing?

During recent years disastrous record-breaking floods have occurred on so many streams throughout the United States that there is a widespread belief among the general public that floods are increasing in magnitude and in frequency. A hasty survey might easily lead to such a conclusion. For instance, in over 100 yr of records at Sewickley, Pennsylvania, the maximum known discharge prior to 1936 was 413,000 cfs on March 15, 1907. On March 18, 1936, a peak flow of 574,000 cfs was recorded. On the Potomac River at Point of Rocks, Maryland, in nearly 90 yr of records the maximum flood prior to 1936 was 320,000 cfs on June 2, 1889. In March 1936, the discharge was 480,000 cfs. On the Connecticut River at Springfield, Massachusetts, in 140 yr of records, the greatest flood recorded was 281,000 cfs on March 19, 1936. The second highest flood in this period was 188,000 cfs on November 6, 1927. Many similar cases could be cited tending to show that the greatest floods have occurred in the most recent portion of the period of record.

Caution should be used, however, in coming to the conclusion

[1] A table showing records of 997 unusual flood peaks in the United States and foreign countries, with the dates of their occurronces, is given in Creager, Justin, and Hinds, *Engineering for Dams*, Vol. I, John Wiley.

that flood discharges are increasing in magnitude. It is well to remember that flood records are constantly growing longer. If there are available 50 yr of discharge records on a given stream and then another 50 yr of records are added, the chances are even that the second 50-yr period will contain the record of a flood that is greater than any that was recorded during the first 50 yr. If one were to study the records of all the streams in the United States and find that in all cases, or even in a goodly majority of the cases, the more recent half of those records contained flood flows that are higher than any that were recorded in the earlier years, he would then have good evidence that the magnitude of floods is increasing. It is doubtful, however, if he would obtain this result.

Although it is questionable whether flood *discharges* are increasing on the majority of streams, it is only reasonable to believe that flood *stages* are increasing on most rivers, especially on those flowing through populous areas. Every levee and flood wall that is built along rivers to protect adjacent property from overflow raises the stage upstream and by reducing the volume of channel storage tends to increase both stage and discharge at all points downstream. Every bridge and every dam that is built across a stream and every building or other structure that is built upon the banks or on the flood plain constitutes an encroachment upon the stream that tends to raise the stage of the water at all points upstream. In populous centers where these encroachments are numerous the summation of all the resultant rises oftentimes becomes surprisingly large.

The final answer to the question of whether or not flood *discharges* are increasing is directly dependent upon the answers to two other questions: (1) are storms increasing in intensity? and (2) are the physical characteristics of the drainage basin changing, or have they been changed by man, so that for any given storm the rate of surface runoff has been increased? The answer to the first of these questions is most certainly in the negative; at least, if there are any climatic changes taking place that tend to affect the intensity of storms occurring in any given area, those changes are so slow that they go unobserved in the brief space of a century. Furthermore, the forces that govern our climate are so vast and widespread that any changes effected on the earth's surface by man have such a feeble influence thereon as to be utterly negligible.

The answer to the second of the above questions is not so simple.

That changes in land use do affect infiltration capacity and, therefore, the rate of surface runoff seems only reasonable and is so obvious that it can hardly be denied by anyone. However, the nature and the magnitude of the effect of those changes are difficult to determine. On this point there is a wide difference of opinion among scientists. In the opinion of the authors the conversion of undrained forested areas into drained agricultural lands may have the net effect either of increasing or of decreasing flood flows. It is their belief that in the majority of cases the tendency is to increase the magnitude of floods, but the percentage of increase is far greater in small drainage basins than in large ones.

The Design Flood

No structure of any importance, either in or adjacent to a river, should ever be planned or built without due consideration to the damage it may cause or to which it may be subjected in time of flood. The destructive powers of a raging torrent are enough to fill the bravest heart with awe and respect. History reveals much evidence of the tremendous power for destruction that is possessed by a flood. The transportation of huge boulders and masses of concrete and the excavation of deep holes at the foot of dams, all attest to the need for careful design of any structure that is likely to be subjected to these tremendous forces.

To avoid destruction, dams must have sufficient spillway capacity and adequate protection against scour at the toe; bridges must have the needed waterway opening; flood walls and embankments must be high enough so that they will not be overtopped; reservoirs must have the required capacity; and so on. The maximum flood that any such structure can safely pass is called the *design flood.*

If a flood of a given magnitude occurs with an average frequency of once in 100 yr there is a 1-per-cent chance, or 1 chance in 100, that such a flood will occur during any one year; a flood whose magnitude is likely to be exceeded on an average of once every 25 yr is a 4-per-cent-chance flood, etc. This method of designating flood frequency as suggested by Hazen,[1] is to be preferred for the reason that the average person considers a flood having "a frequency of once in 100 yr" as carrying no present threat, but likely to occur only after a lapse of 100 yr. On the other hand, a "1-per-

[1] Allen Hazen, *Flood Flows*, John Wiley, 1930, p. 10.

cent-chance flood" at once conveys the impression that there is 1 chance in 100 that such a flood will occur within a year; furthermore, that it is just as likely to occur this year as any other year, and that is the exact impression that should prevail.

The first characteristic to be determined for any design flood is not its magnitude as perhaps might be expected but the chance or probability of its occurrence. In other words, should the structure be safe against the 2-per-cent-, or the 1-per-cent-, or the 0.1-per-cent-chance flood or against the maximum flood that may ever be anticipated? After this question has been answered, the problem is to determine the magnitude of the flood that may be expected to occur with that average frequency.

Contrary to common opinion there is but little relationship between the frequency of the design flood and the normal life of the structure. If the structure has an anticipated life of 50 yr, it would plainly be folly to design it so that it would be safe only against floods that occur with an average frequency of once in 50 yr or less. If this were done, the chances would be even that it would be destroyed by flood before it had served its purpose through half of its normal life. Especially would this be folly if, as is often the case, by a slight additional cost it could have been made practically immune to such destruction.

How far one is justified in going in order to make a structure or flood-prevention system safe against damage or destruction by floods depends upon the following considerations.

1. *The extent to which human life would be endangered* by the occurrence of a flood of greater magnitude. It is impossible to place a monetary value upon human life. In the preparation of the plans for flood protection in the Miami Conservancy District, the engineers decided that the flood of 1913 was one of the great floods of centuries. Nevertheless, because of the great danger to human life in case of failure of the system, they designed protective works that would take care of a flood nearly 40 per cent greater than the flood of 1913. In a similar manner any structure whose failure would seriously endanger human lives should be designed to pass safely the greatest flood that will probably ever occur at that point.

2. *The value of property that would be destroyed* as a result of floods of greater size. This consideration is purely economic. By determining the total losses that would result from greater floods

during a long period of time, reduced to an annual basis, and comparing this figure with the increased annual cost of the protective system that would be required to prevent those additional losses, one can determine the limiting size of flood against which it is profitable to protect.

3. *The inconvenience resulting from failure* of the structure due to greater floods. If, for instance, a bridge on a trunk-line highway should be destroyed because of insufficient waterway opening, all traffic might have to be detoured for a considerable distance over poor roads perhaps for several months until a new bridge is designed and built.

One other factor should be mentioned in connection with the design of any flood-control system, viz., the sense of security against floods that will be provided by that system. Although this is another intangible value, it is, nevertheless, important. Some cities in protected basins use the security provided by their flood-protective system as an inducement to attract manufacturing enterprises to locate there. In general, property values within the areas protected are enhanced, often to a considerable extent.

If due consideration is given to these factors, the proper frequency of the design flood can be determined although, of course, good judgment must always play a major part in connection with the first and third items.

Frequency of Storm Producing Design Flood

Let us assume that a dam and its appurtenances are to be designed that can safely pass the maximum 1-per-cent-chance flood. Although such a flood occurs only once in 100 yr, storms of a magnitude capable of producing such a flood will occur with a much greater frequency because a great many such storms will occur at times when the infiltration capacity and storage are unfavorable to the production of high surface runoff. If, for instance, favorable flood-producing conditions exist during only 10 per cent of the time when such storms are likely to occur, the storm that will produce the maximum 1-per-cent-chance flood will occur with an average frequency of once every 10 yr. Therefore, in order to determine the average frequency of the storm that is capable of producing the design flood, the frequency interval of that flood should be reduced by the probability of the conditions being unfavorable to high surface runoff.

Critical Storm Period

For any drainage basin there is a certain definite duration of storm that will cause the maximum flood that can be expected to occur with any given frequency. The reason for this fact is that ordinarily the intensity of storms varies inversely with their duration. The greatest flood will, therefore, result from that storm that lasts just long enough to bring the hydrograph to its peak; if it lasted any longer the width of the graph would be increased, but because of the reduced intensity of storm the magnitude of the flood would be no greater. The duration of that storm that causes the greatest peak runoff is called the critical period for that basin.

No definite rule can be given whereby the critical period can be exactly determined for any given basin. Perhaps the best method is to select a number of short, intense storms and determine from the resulting flood hydrographs the length of the period from the time that rainfall excess begins until the peak runoff is reached. Normally this interval does not vary greatly for different storms and may usually be safely taken as the critical period.

Classification of Methods

Although scores of different procedures have been proposed for the determination of the size of flood that may be expected to occur on any stream with a given average frequency, all these methods may be grouped into three general classes:

1. By means of empirical formulas.
2. By statistical or probability methods.
3. By use of the infiltration capacity and unit-hydrograph principles.

The advantages and disadvantages of these methods will now be discussed. Before doing so, however, it may be well to review the factors affecting surface runoff, for floods are almost entirely the result of surface runoff. It therefore follows that every factor that in any way affects surface runoff necessarily affects flood flows.

Factors Affecting Flood Flow

These factors naturally fall into two classes, viz., those that determine the intensity of the storms that are likely to visit any

drainage basin and the physical characteristics of the basin as they affect and determine the disposal of that rainfall. They are as follows.

A. Storm Factors

1. Distance from ocean.
2. Direction of prevailing winds with respect to ocean.
3. Mean annual temperature.
4. Elevation.
5. Topography.

B. Drainage Basin Factors

1. Size of basin.
2. Shape of basin.
3. Orientation of basin.
4. Location with respect to storm paths.
5. Kind of soil.
6. Condition of vegetation.
7. Condition of ground surface.
8. Soil-moisture deficiency.
9. Type and extent of artificial drainage.
10. Extent of surface storage in lakes and swamps.
11. Condition of stream channels.
12. Slope of stream channel.
13. Mean slope of basin.
14. Character of drainage net.

The influence exerted by these various factors upon surface runoff has been discussed in earlier chapters and need not be expanded here. It is well enough, however, to keep this multiplicity of influences in mind in connection with the discussion that follows.

Determination by Use of Empirical Formulas

Perhaps the earliest method proposed for the determination of the probable flood flow of a stream was the use of empirical formulas. These usually expressed the relation between flood flow and one or two of the above-listed variables. Seldom were definite specifications given for the frequency with which the computed flood flows might be expected to occur. Sometimes they were described as frequent or rare, terms that are comparative and, therefore, practically meaningless. When we consider that the magnitude of a flood of any given frequency depends upon all the above variables, the futility of attempting to correlate that magnitude with only one or two of those variables and obtain dependable

results at once becomes apparent. A quick and very rough approximation is all that should be expected. Nevertheless a few typical formulas of this type will be presented here although the student is urgently advised never to use them unless he has first investigated their origin, has become familiar with the data upon which they were based, knows the conditions under which they are intended to be used, knows the areas to which they are adapted, and understands the restrictions placed upon their use by their authors. Unless these precautions are taken, the blind use of any one of these formulas is likely to give results grossly in error—not by 20 or 30 per cent but by several hundred per cent and in some cases by a thousand per cent or more. In all these formulas Q is flood flow in cubic feet per second, and A is area of drainage basin in square miles.

One of the oldest and also one of the best known of these formulas that has been used extensively for storm sewer design is the so-called rational formula[1]

$$Q = 640KI_tA$$

where K is the percentage of the rainfall that becomes surface runoff, and I_t is the intensity of rainfall in inches per hour for the critical period, t.

The relationship expressed by this formula is at once apparent when one recalls that 1 in. of rain falling in 1 hr on 1 acre is equivalent to 3630 cu ft of water which, running off at a uniform rate in 1 hr, is equal to approximately 1 cfs. Perhaps, therefore, this formula should not be classed as an empirical formula as are the others that follow.

Although this formula has been called "rational," the results of hydrological research indicate that this term is misleading except possibly for very small runoff areas of the type discussed on pages 274 and 290. Only for such small areas can the relation between rainfall intensity and runoff intensity that is implied by the formula become a reality. Even then it would seem desirable to determine rate of runoff on the basis of sound principles, i.e., as the difference between rainfall rate and infiltration capacity rather than by the use of the coefficient f. For areas of 5 acres or more it need only be recalled that two storms of entirely different intensities but producing equal amounts of rainfall excess within the period of rise

[1] Philip A. Morley Parker, *Control of Water*, D. Van Nostrand, pp. 277–282.

may produce the same rates of runoff (page 129) to know that there is no direct relation between rainfall rates and runoff rates. In other words this formula ignores the fact that for any watershed, runoff occurs in the form of a typical hydrograph as illustrated by Fig. 103.

Inasmuch as area is an important and also one of the most apparent factors affecting the magnitude of the flood flow of a stream, it is only natural that a great many formulas have been proposed in which the flood flow is made directly dependent upon the area of the basin. Formulas of this type take the form

$$Q = KA^n$$

where K is a coefficient depending upon the rainfall and runoff characteristics of the basin and n is a constant whose value usually lies between 0.5 and 1.0.

Of this general type is the modified Myers formula,[1]

$$Q = 10{,}000p\sqrt{A}$$

in which p has a value of unity for the stream that has the greatest flood flow of that area. For any other stream, p is the percentage that the flood flow of that stream is of the above maximum. For different streams the value of p varies all the way from about 0.2 per cent to 100 per cent.[2]

Although scores of other formulas have been proposed by various investigators for the quick determination of the flood flow of streams, the above suffice to show their general character. When one considers the enormous range in the values of the coefficients and exponents in most of these formulas, a condition resulting directly from the fact that those terms must represent the combined effect of a dozen or more characteristics, such as porosity of soil, character of vegetation, slope of basin, storage, drainage, and many other items, the extreme difficulty encountered in making satisfactory and intelligent use of these formulas is apparent.

Determination by Use of Statistical or Probability Methods

A very logical procedure to follow in the prediction of the occurrence of natural phenomena is to base that prediction upon the

[1] C. S. Jarvis and others, Floods in the United States, *U. S. Geological Survey Water-Supply Paper* 771.

[2] C. S. Jarvis, Flood Flow Characteristics, *Trans. A.S.C.E.*, 1926, **89**, 994.

records of the past. Such a procedure possesses fundamental merits that cannot be denied. There can be no question but that this method applied to the determination of the maximum flood that may be expected to occur in any stream with a given frequency will yield correct results, *provided that there are sufficient records available upon which to base that determination and also provided that there have been no important changes in the regimen of the stream during or subsequent to the period of record.*

Especially prior to Sherman's discovery of the unit-hydrograph principle, statistical or probability methods were often used in making flood studies. On this subject the discussion that follows is by no means complete but is intended only to demonstrate the general nature of the methods employed. These methods of estimating flood magnitudes and frequencies may be divided into two classes:

1. By the use of duration curves.
2. By the use of probability curves.

These two types of curves are closely related, as will presently appear.

Although a great many methods have been suggested for determining flood frequency by the use of duration or probability curves, only two of the simpler procedures will be explained. These methods can perhaps best be explained by use of an example. For this purpose the 54-yr record of floods on the Merrimack River at Lawrence, Massachusetts, as presented in *U. S. Geological Survey Water-Supply Paper* 771, has been selected.

In Column 2, Table 22, is shown the number of floods in this period during which the maximum 24-hr flow occurred between the limits as shown in Column 1. In Column 3 is given a summation of these values starting with the maximum. In Column 4 are shown the percentages of the total number of floods when each of the lower values in Column 1 was equaled or exceeded. For instance, 19 per cent of the 153 floods that occurred in this 54-yr period equaled or exceeded 40,000 cfs. They all exceeded 20,900 cfs, and so on. In Fig. 113 the percentages shown in Column 4 are plotted against the corresponding lower values in Column 1. In Fig. 114 these same data are plotted on logarithmic probability paper. The principal advantage in the latter method of plotting lies in the fact that especially throughout the lower percentages

TABLE 22

1 Flood Peak Limits 1000 cfs	2 Number of Occurrences	3 Mass Totals	4 Percentage of Total Occurrences
20.9–22.9	1	153	100
23.0–24.9	6	152	99.3
25.0–27.4	31	146	95.4
27.5–29.9	23	115	75.1
30.0–32.4	20	92	60.1
32.5–34.9	19	72	47.0
35.0–37.4	13	53	34.6
37.5–39.9	11	40	26.1
40.0–44.9	13	29	19.0
45.0–49.9	7	16	10.5
50.0–59.9	3	9	5.9
60.0–69.9	5	6	3.9
70.0–88.2	1	1	0.65

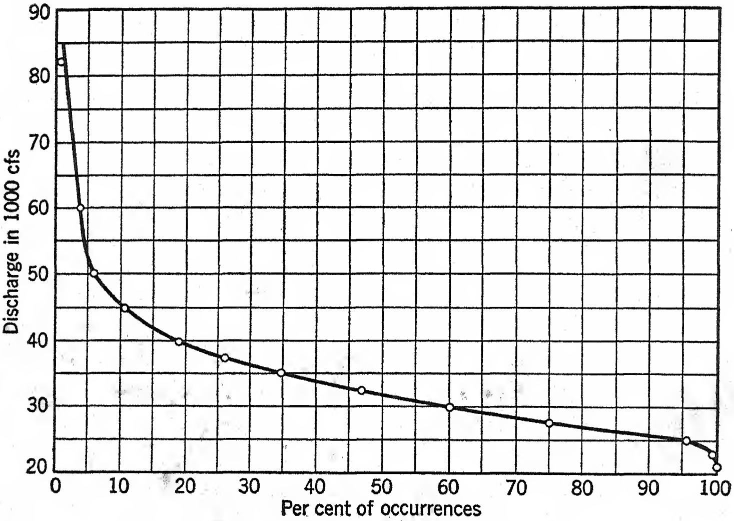

FIG. 113.

this curve is much flatter than the one shown in Fig. 113. It can therefore be extended more readily to cover the very low percentages.

It has been suggested by those who advocate the use of probability methods in making flood studies, that curves such as these

may be used in the following manner. Suppose that we wish to determine the maximum flood that may be expected to occur on the Merrimack River with a frequency of once in 1000 yr. We must not lose sight of the fact that both these curves show the percentages of the *total number* of floods that occur in any given period that equal or exceed the magnitudes as shown on the left. In the 54 yr of records there were 153 floods or 2.83 floods per year. Therefore, in 1000 yr there should be 2830 floods, and the greatest one to be expected in that period would be of the magnitude shown at 100/2830 per cent or about 0.035 per cent of the time. In Fig. 114, we find that for this percentage the magnitude of flood

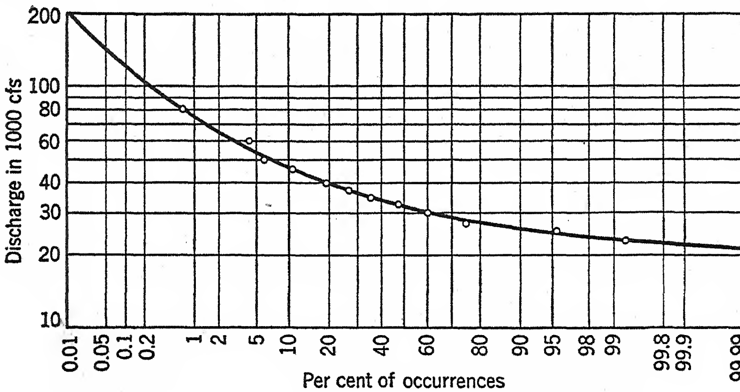

Fig. 114.

is about 154,000 cfs. In Fig. 113 the scale is such that the corresponding value falls off the drawing, but the two curves should give the same result.

Many refinements of the above methods as well as a number of quite different procedures have been suggested[1] for flood-frequency determinations by probability methods but they all possess the same inherent defects. The discussion that follows applies equally to all these methods.

[1] Allen Hazen, *Flood Flows*, John Wiley, 1930.

H. Alden Foster, Duration Curves, *Trans. A.S.C.E.*, 1934, **99**, 1213.

J. J. Slade, Jr., An Asymmetric Probability Function, *Proc. A.S.C.E.*, October 1934, p. 1007.

R. D. Goodrich, Straight Line Plotting of Skew Frequency Data, *Trans. A.S.C.E.*, 1927, **91**, 1.

F. G. Switzer, Probability of Flood Flows, *Proc. A.S.C.E.*, April 1927, p. 563.

L. Standish Hall, The Probable Variations in Annual Runoff as Determined from a Study of California Streams, *Trans. A.S.C.E.*, 1921, **84**, 191.

Accuracy of Probability Methods

In determining the value of any particular variable, the greater the number of samples upon which that determination is based the more accurate will be the result. If, for instance, we wish to determine for a given stream the maximum flood that may be expected to occur with a frequency of once in 10 yr and only 10 yr of records are available, the error in the result is likely to be great because the answer is based upon only one sample. If the same determination were to be based upon 100 yr of records, the probable error would be greatly reduced, for we would now have ten samples upon which to base our judgment instead of only one. Or again, if we had a 1000 yr of records or 100 samples, the probable error would then be so small as to be negligible. If on the other hand we wish to determine the maximum flood that is likely to occur with an average frequency of once in 100 yr and only 50 yr of records are available, the probable error is then very high, perhaps several hundred per cent or more, because in this case not even one whole sample, or period of observation, is available upon which to base our judgment. It should be emphasized that the probable error depends upon the number of independent samples available and that no amount of juggling or manipulation of data can possibly reduce that error.

About ten independent samples should provide a satisfactory determination of the size of flood that may be expected to occur with any given frequency although a greater number will increase the accuracy. This means that the statistical or probability method may be used to determine the magnitude of floods that will occur with a frequency of once a year, once every 2 yr, or even once every 5 yr, if 50 yr of records are available; but, for the determination of the greatest flood that may be expected once in 100 yr or once in 1000 yr with no more than 50 or 100 yr of records, these methods are entirely inadequate and likely to result in enormous errors.

Apparent justification for the above conclusions is to be found in the results of some studies by Hazen[1] in the determination of the probable maximum floods to be expected on the Hudson River at Mechanicsville, New York, and on the Arkansas River at Pueblo, Colorado. Prior to 1913 the maximum flood on the Hudson River

[1] Allen Hazen, *Flood Flows*, John Wiley, 1930, p. 88.

for which there is any record occurred in 1869 during which the peak flow reached about 67,000 cfs. The records prior to 1888 however were considered none too reliable, and therefore Hazen used only the records subsequent to that date. The greatest floods that occurred each year throughout the 23 yr of record from 1888 to 1912, arranged in the order of their magnitudes, were plotted on logarithmic probability paper. By extrapolating on this curve, he found that the 1-per-cent-chance flood which is the maximum flood that can be expected to occur once in 100 yr, is 64,600 cfs. The maximum 1000-yr flood would be, by this same method, about 10 per cent greater. In 1913 a flood occurred at Mechanicsville that reached a peak of 118,000 cfs.

In a similar manner throughout the 29 yr of record prior to 1921, the maximum flood on the Arkansas River at Pueblo, Colorado, was about 10,000 cfs. By the above method, using only the records for this 29-yr period, Hazen estimated the maximum 100-yr flood to be about 11,650 cfs. In 1921 a flood occurred on this stream that was believed to exceed 100,000 cfs. This is many times greater than the maximum that would ever be expected on this stream as determined by the probability method. Nor are these two cases the only ones that could be cited of a stream suddenly going on a rampage in a flood. The Republican River in Kansas, the Miami River in Ohio, and scores of others afford similar examples.

Cause of Extraordinary Floods

This rather strange phenomenon wherein an occasional flood occurs that is at least several times greater than any previously recorded may be explained as follows. A major flood on any particular basin depends primarily upon the following three considerations:

1. Upon the occurrence on that basin of a storm of high intensity or the sudden melting of a heavy accumulation of snow.
2. Upon the condition that the storage in lakes, swamps, ponds, depressions, and channels is filled.
3. Upon a low infiltration capacity of the basin at the time of the storm or the melting of the snow.

It so happens that the latter two conditions are very likely to occur simultaneously, and as a result it is safer to combine them as a single condition rather than to consider them as independent variables.

To simplify our problem let us assume that the drainage basin under consideration is located in one of the southern states where floods do not result from the melting of snows but only from intense rainfall. Using the area-year method previously described (page 102), suppose that it has been found that a storm of a given intensity will occur on the basin with an average frequency of, for instance, once in 50 yr. Furthermore, assume that such a storm is just as likely to occur immediately following another storm as at any other time since the records show for example that during only 5 per cent of the time is the surface storage filled and the infiltration capacity at the minimum. The chances are, therefore, only one in twenty that this condition will exist simultaneously with the occurrence of the storm. As a result only once in twenty times will a storm of the given intensity occur when these other conditions are favorable for the production of a maximum flood. Consequently, under these conditions the most intense storm that occurs with an average frequency of once in 50 yr will produce the maximum flood only once in a thousand years. It is, therefore, not likely that these two conditions will occur simultaneously within a short period of records. If they do not, a record of 50 or even 100 yr will reveal no unusually high flood, but when they do happen to occur at the same time a flood many times greater than anything that has ever before been recorded is likely to follow with disastrous results.

In this country but few streams have reliable discharge records covering a period of more than 50 yr. Therefore, if one should wish to determine the magnitude of the greatest flood that is likely ever to occur on a given basin, as would be the case if the failure of the proposed structure would cause the loss of human life and the destruction of valuable property, it is clear that such a determination cannot be based upon the application of purely statistical methods to past records. It is easily conceivable that such a peak flood might either have been recorded in a 50-yr period or might exceed by 1000 per cent or more the maximum recorded in such a comparatively short time.

Nor is it possible to combine the records of a number of different streams in a manner similar to that followed in the station-year method of determining rainfall frequencies. This fact is self-evident when we consider the remainder of the many factors listed above, which influence and determine the magnitude and frequency of

occurrences of floods on different drainage basins. Because of the large number of these factors, it is manifestly improbable that any two basins would ever have the same flood-producing potentialities, without which their records cannot properly be combined.

Unit Hydrograph—Infiltration-Capacity Method

Although a number of methods have been suggested whereby the maximum flood that may be expected to occur on any particular drainage basin with a given frequency may be determined, the best method that has yet been proposed is based upon Horton's fundamental concept of infiltration capacity and Sherman's theory of the unit hydrograph. This method requires the collection and preparation of the following basic data.

The first step is to select from the records a number of floods to be studied in detail. Each of these floods should be the result of a single isolated storm or of a number of storms separated by time intervals of sufficient length that each produces a separate peak, even though portions of the resulting hydrographs overlap. After first deducting the ground-water flow from the recorded discharges the resulting hydrographs of surface runoff are separately plotted for each flood. The rainfall causing each storm is plotted on the same time scale with convenient time intervals. The infiltration capacity should be determined for each flood.

The period that elapses from the beginning of each rise until the crest is reached is observed in each case. These intervals should not differ a great deal, and one half of the average period may be taken as the critical storm period for that basin. Suppose in the present instance that interval is found to be 4 days, the area of drainage basin is 5000 sq miles and the recurrence interval of the design flood is 100 yr. For basins on which there are not sufficient records available to permit the use of the area-year method (page 102), the alternative method of finding the intensity of storm that will produce the design flood (see page 105) must be used. By applying the station-year method to all the rainfall records of over 20 yr in length that are available at meteorologically homogeneous stations in and near this basin, it is found, let us assume, that the maximum 2-day rainfall occurring at any station with an average frequency equal to that of the design flood is 12 in. From the curves in Fig. 35*b* it is found that a 2-day storm having a maximum station intensity of 12 in. will have an average intensity on an area of 5000 sq miles

equal to 65 per cent of the maximum or 7.8 in. A synthetic storm pattern should be made up (see page 303) having this total amount falling in 2 days. With these basic data we are now ready to construct the flood hydrograph that should result from this storm.

In this procedure perhaps the most difficult and uncertain step is in the determination of the character of the infiltration-capacity curve that should be used. That the method of determining this curve depends upon the size of the basin and upon the available data has already been discussed in Chapter VI. However, regardless of the method employed the infiltration capacity of the basin should be determined for a number of typical unit storms that have occurred during various seasons of the year and under various conditions of antecedent rainfall. Then, if the problem is one involving loss of life and great destruction of property in case of failure of the structure, the minimum value of infiltration capacity should be used, or perhaps even a lower value should be used if it appears that the conditions prevailing during the floods of record were not conducive to a low infiltration capacity. In each individual case judgment alone can determine the proper value of the infiltration capacity that should be used.

Having determined the infiltration capacity and with the distribution graphs for each of the storms for which the infiltration capacity was determined, we obtain the average-distribution graph. Using the synthetic storm pattern above referred to, together with the infiltration capacity as determined from the available hydrographs, we can apply the distribution graph to the rainfall excess occurring in the design storm, and the resulting flood flow is determined, as illustrated on page 310.

Relief from Flood Hazard

Damages from floods may be avoided in three different ways.

1. By the construction of protective works.

2. Through the reduction of flood stage without appreciable change in the peak discharge.

3. Through the reduction of flood flows by storage, change in land use, or similar methods.

It should be noted that inasmuch as the first two of these methods do not reduce the rate of runoff, they afford protection only from flooding and do not prevent soil erosion. Which of these

three methods or what combination of methods should be used for flood relief in any particular drainage basin or locality is a question that can be answered only after careful study and consideration of all the various possibilities. A method that is admirably adapted to provide the maximum possible relief and at the lowest cost in one case might prove wholly inadequate and tremendously expensive in another.

In this field to a greater extent perhaps than in any other there is an overwhelming tendency on the part of the public never to lock the barn until the horse has been stolen. It is doubtful if any flood protection or flood reduction measures have ever been taken on any river until after a disastrous flood has been experienced. If prior to 1913 all the engineers in America had shouted from every house top in the Miami valley giving warning of the impending disaster that awaited that area, they would never have succeeded in arousing public sentiment to organize a Miami Conservancy District and construct the admirable flood-prevention system that was subsequently built—only a flood that took a toll of 360 lives and destroyed more than $100,000,000 worth of property could do that. Nor is this intended as the slightest reflection on the intelligence of the people living in that area. Human nature is the same everywhere. This is proved by the many danger centers still existing throughout the country where the inhabitants are peacefully lulled by a sense of false security, blissfully ignorant of the terrible havoc that can and some day will be wrought by the innocent-looking stream that flows by their doors and that throughout the memory of the oldest inhabitant has always been comparatively docile and well behaved.

Flood Protection

Flood protection is provided mainly by means of levees and flood walls that are built along the banks and afford protection only locally to people and property within reach of the flood waters. Their purpose is to confine those waters within the natural stream channel. In so doing they actually increase the stage of the river at points upstream because of backwater and at points downstream because of the increased discharge resulting from the reduction in storage.

Although this method oftentimes provides satisfactory protec-

tion against the ordinary flood it carries with it a serious danger. Because of the presence of a protecting levee or flood wall, the adjacent property owners acquire a new sense of security against floods and as a result new homes are built at lower levels where before they did not dare to build. A levee will protect only until it is overtopped; after that it is utterly useless. Consequently, when the exceptional flood comes along and overtops the levee, the resulting devastation and loss of life are likely to be far worse than if it had never been built.

Stage Reduction

Without affecting the rate of discharge, the flood hazard can oftentimes be greatly lessened by stage reduction. This may be accomplished in any instance by either or both of the following methods.

1. By straightening and deepening the river channel. Inasmuch as $Q=AV$, it follows that for any given Q, by increasing V, A is correspondingly reduced, and hence the stage is reduced. The extent of the benefits that can be obtained in this manner depends upon the initial conditions of the channel. Even though it is extremely sinuous, as some of the reaches in the lower Mississippi, if the fall in the river is slight, the amount of stage reduction that can be accomplished by this method is usually quite limited. Furthermore, nature objects to man's interference when he attempts to show her how to improve the course of a mighty river, and, as a result, unless he paves the channel or at least the side slopes, he must expect an almost constant maintenance expense. If there is sufficient fall available, by paving the channel through populous areas the depth of water will be reduced in practically the same proportion as the velocity is increased.

2. By providing a by-pass or additional flood channel past danger centers. Oftentimes large cities are located on rivers or on other bodies of water. It is here that because of encroachments by bridges, buildings, and filled-in areas, a bottleneck is created. Appreciable widening of the river channel is usually out of the question because of the cost. Not uncommonly, however, a flood channel can be constructed around the city at reasonable cost. The flood peaks on the Mississippi River at New Orleans and for a distance of more than a hundred miles upstream are materially

reduced by the diversion of a portion of those flood waters into the Atchafalaya River which thus becomes a by-pass channel direct to the Gulf.

Reduction in Peak Discharge

Peak discharges can be reduced by either (1) temporarily storing a portion of the surface runoff until after the crest of the flood has passed or (2) reducing the amount of surface runoff through a change in land use which increases the infiltration capacity. Either of these methods, if properly carried out, will produce beneficial results at all downstream points.

Reduction by storage is accomplished in two different ways, (1) by a large number of small, individual farm reservoirs located in the headwaters of the main stream or of its many tributaries, or by the use of terraces that detain the surface runoff long enough to permit it to infiltrate into the soil; and (2) by large reservoirs located in the valleys farther downstream.

Regardless of the size of the reservoir, however, there are two types of storage, controlled and uncontrolled. In controlled storage, gates in the impounding structure may regulate the outflow in any manner that is thought desirable. Only in unusual cases will such a reservoir have sufficient capacity to completely eliminate the peak of a major flood. As a result, the regulation of the outflow must be carefully planned.[1] The operators must estimate how much of the early portion of a flood may be safely impounded, taking into consideration the danger of having the reservoir filled before the peak of the flood has been reached. Where reservoirs exist on several tributaries, the additional problem of the timing of the release of the stored waters becomes a matter of very great importance. It might be possible to so release these waters that the peak flows would combine at a downstream point with disastrous results.

In uncontrolled storage, there is no regulation of the outflow capacity of the impounding dam. Such structures usually contain overflow spillways, and the only flood benefits obtained from them result from the modifying and delaying effects of the storage above the spillway crest.

[1] Robert M. Morris and Thomas L. Reilly, Operation Experiences, Tygart Reservoir, *Trans. A.S.C.E.*, 1942, **107**, 1349.

The effectiveness of a large number of small reservoirs in the headwaters of a stream has never been demonstrated on any large scale. However, it has been given considerable publicity by its advocates who urge its use for soil-conservation purposes as well as for flood prevention. The value of such reservoirs in the prevention of floods is questionable because of the probability of their being either filled or partly filled at the time of a flood-producing rain.

On the other hand, terraces have been used, and if properly designed they are effective in flood reduction and as a soil-conservation measure. They follow the contour of the ground and have a base width of 5 or 6 ft and a usual height of 6 or 8 in. They are spaced close enough together to impound the surface runoff without overtopping. Actually they form small detention reservoirs which hold the water back long enough to give it time to infiltrate into the soil. This requires but a short time so that they are soon empty and ready for the next rain. They thus serve to reduce flood flow, prevent soil erosion, and increase ground-water supply.

Much has been written of the advantages arising from the multiple use of storage reservoirs. It is sometimes claimed that the same storage capacity can be utilized for flood control, storing the flood waters to be used later (1) as an aid to navigation by increasing the low water flow, (2) for the production of power, (3) for irrigation, or (4) for water supply and other purposes. Large storage reservoirs provide the most generally effective and satisfactory method of flood prevention that is in common use, but, unless a reservoir has a capacity far greater than is needed for storing the excess waters of a major flood, its value for other purposes will be very limited if it is to be fully effective for flood control. This necessarily follows from the fact that if a reservoir has only sufficient capacity to hold the waters from one major flood it must be emptied as quickly as possible after each flood in order to have its storage capacity available for the next flood when it comes along. By always keeping sufficient storage capacity available for any probable flood, any additional storage can be utilized for other purposes. The reservoir capacity necessary to completely store the waters of any flood, reducing the outflow to some safe rate, can easily be found by means of a mass diagram. This capacity is given

by the maximum ordinate between the mass curve of inflow and the mass curve of outflow.

Protective works such as levees and flood walls are always built at those places along the river that require protection. On the other hand, storage reservoirs are almost invariably located at some considerable distance upstream from the cities or areas that are to be protected. They must be located where the topography is suitable and cost is not excessive. Unless a topographical map of the basin is available, a survey is necessary to determine the existence of suitable reservoir sites on the main stream and on each of the several tributaries. The cost of development of each of the possible sites must be determined, and, after reducing this cost to an annual basis, it must be compared with the annual benefits that would accrue from its development. Only then can one determine which sites, if any, are economically feasible to develop.

Determination of Benefits from Flood Reduction

To determine the annual benefits that would result from any flood-control program[1] it is first necessary to establish a number of flood profiles throughout those reaches of the river, at least, where considerable damages occur. Also a design-flood profile should be determined as well as the minimum profile at which there are appreciable damages. A careful field survey is then necessary to determine the amount of the damages that result from floods corresponding to the established profiles throughout the damage zone. These values can be plotted in the form of a curve, and for any intermediate stages desired the damages can be found.

Next the frequency with which floods of each of these various magnitudes occur must be determined. With these data at hand, the manner in which they are used will be illustrated.

Suppose that at the downstream end of the damage zone the range of the flood stage for which damages occur is 10 ft. Let us assume that the amount of damages has been determined throughout the entire length of the damage zone for profiles corresponding to flood stages at the lower end of 2, 4, 6, 8, and 10 ft respectively. Also the number of times in 100 yr when each of these stages has

[1] For a more complete discussion of this subject see Edgar E. Foster, Evaluation of Flood Losses and Benefits, *Trans. A.S.C.E.*, 1942, **107**, 871–924.

been reached or exceeded has been determined. A table will then be prepared as follows:

TABLE 23

Flood Stage, feet	Number of Times in 100 Years that Stage Is Exceeded	Number of Floods of this Stage but No Higher	Damages per Flood	Total Damages
10	1	1	$1,200,000	$1,200,000
8	2	1	800,000	800,000
6	5	3	500,000	1,500,000
4	9	4	250,000	1,000,000
2	20	11	100,000	1,100,000
			Total Damages per 100 yr	$5,600,000

If methods can be found whereby these flood stages can be reduced by 4 ft, a revised table similar to Table 23 would be as follows:

TABLE 24

Flood Stage, feet	Number of Times in 100 Years that Stage Is Exceeded	Number of Floods of this Stage but No Higher	Damages per Flood	Total Damages
10	0	0	0	0
8	0	0	0	0
6	1	1	$500,000	$500,000
4	2	1	250,000	250,000
2	5	3	100,000	300,000
			Total Damages per 100 yr	$1,050,000

The annual benefits would therefore amount to $45,500. It should perhaps be explained that it would have been impossible to determine the amount of the damages occurring within any 2-ft interval and then consider this figure as being constant for all floods that reach or exceed this stage because these damages vary for the different magnitudes of floods. For instance, a flood of a 10-ft stage will uproot trees, carry away houses, and destroy bridges that are located at the lower levels and would not be seriously damaged by a 2-ft or 4-ft stage of flood.

Flood Routing

It was assumed above, for purposes of illustration, that methods had been found whereby flood stages at the lower end of the

danger zone could be reduced 4 ft. It remains to be explained how the *amount* of this reduction in stage can be computed. For this purpose it is necessary to learn the character of the contribution to the flood hydrograph at the lower station from each tributary area whose contribution to that hydrograph we propose to change. To do this we must first take the hydrograph of a recorded flood from the upper area and determine how and when that water reaches the lower stations, for this hydrograph will change as it moves downstream. Although the volume of water will ordinarily remain practically constant, the base of the hydrograph will broaden, the peak will be reduced, and, of course, the time will be delayed. Then after determining to what extent this upper-station hydrograph can be reduced by storage or other means, we can route the revised graph down to the lower station and find how much the proposed upstream improvement will affect flood peaks at the downstream station. The process whereby the hydrograph of a flood as it occurred at an upstream station is transferred to some point downstream is called flood routing through river channels.

Without this procedure no intelligent planning of flood relief is possible. In fact, it could happen that a reservoir located on a certain tributary and constructed at considerable cost might actually prove detrimental by delaying the flood waters a sufficient time so that they would arrive at the downstream damage point at the crest of the flood instead of passing harmlessly by at an earlier period.

However, the flood hydrograph at any downstream station will be affected and determined not only by changes in land use, by storage in small upstream reservoirs, and by channel storage, but also by storage in large downstream reservoirs. Before taking up the method of routing floods through river channels, the much simpler plan of routing through an uncontrolled reservoir will be considered. This latter procedure will be presented by means of a numerical example.

Flood Routing through Reservoirs

In Fig. 115, Curve 1 represents two successive stream rises on the Monongahela River at Greensboro, Pennsylvania. It was assumed that a dam having a spillway length, L, of 600 ft, would be built across this stream and that the relation between the discharge, Q, and the head, H, on the spillway is expressed by

the equation

$$Q = 3.5LH^{3/2} \tag{1}$$

This relationship is shown graphically as Curve 1, Fig. 116. The relation between H and reservoir-surface area is shown by Curve 2, Fig. 116. This curve permits the computation of the storage capacity, S, for various values of H by starting with zero storage when $H = 0$ and adding successive volumes for larger values of H.

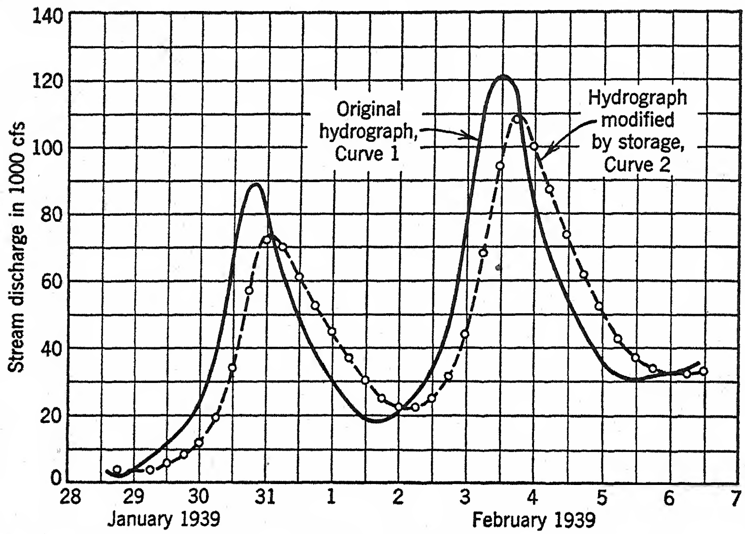

FIG. 115.

Since storage is a function of H, equation 1 might be written in terms of storage rather than H as follows:

$$Q = CS^n \tag{2}$$

The storage equation may now be written for some convenient time interval.

Let T = number of seconds in the time interval.
I = average rate of inflow in cfs during interval.
Q_1 = rate of discharge over the spillway in cfs at the beginning of a time interval.
Q_2 = the corresponding rate at the end of a time interval.

S_1 = storage above the spillway crest in cu ft at the beginning of a time interval.

S_2 = corresponding storage at the end of a time interval.

Then

$$S_1 + IT - \left(\frac{Q_1 + Q_2}{2}\right) T = S_2 \tag{3}$$

The unknowns in this equation for any time interval are S_2 and Q_2. These may be determined by simultaneous solution of equa-

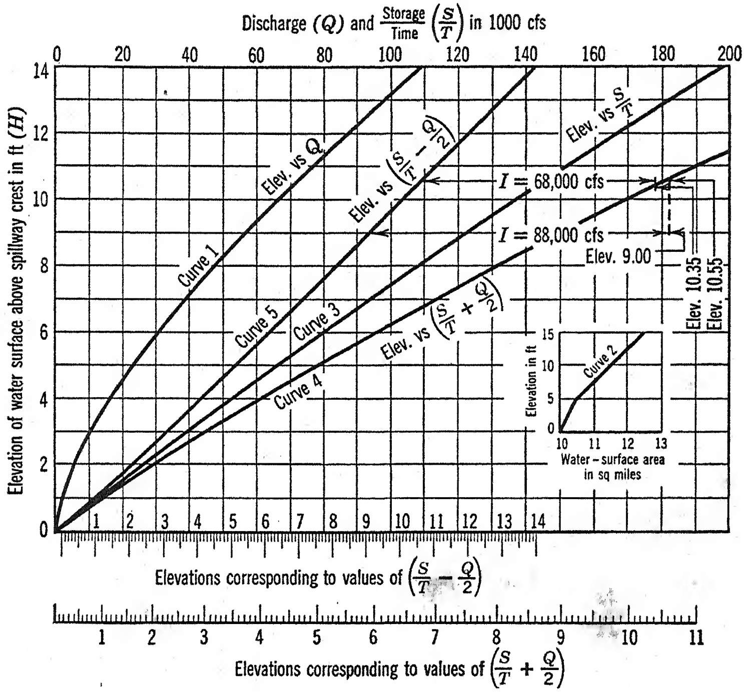

FIG. 116.

tions 2 and 3. Because the values of S_2 and Q_2 thus determined are the values of S_1 and Q_1 respectively, for the next interval the procedure may be repeated for successive intervals until the entire hydrograph of outflow is determined. The above equations can be solved only by successive approximations, which make the process very tedious.

The following graphical procedure greatly facilitates the solution.[1,2]

Equation 3 may be rewritten as follows:

$$\left(\frac{S_1}{T} - \frac{Q_1}{2}\right) + I = \left(\frac{S_2}{T} + \frac{Q_2}{2}\right) \tag{4}$$

In this example a time interval, T, of 6 hr was selected. Values of S/T were determined from Curve 2, Fig. 116, for various elevations and plotted as Curve 3, Fig. 116. Values of $Q/2$ were then added to and subtracted from the abscissas of Curve 3 to obtain Curves 4 and 5 respectively. For any time interval, equation 4 requires that the abscissa of Curve 5, corresponding to the water-surface elevation at the beginning of an interval, plus I, be equal to the abscissa of Curve 4 corresponding to the elevation of the water surface at the end of the interval. Values of I are obtained from Column 2, Table 25. If it is assumed that the outflow is equal to the inflow during the first interval, then, since there is no change in storage throughout this period, the elevation at the beginning of the second interval will be the one corresponding to a Q of 3500 cfs, or 1.45 ft. Beginning at this elevation on Curve 5, Fig. 116, the value of I of 2800 cfs, is added graphically by measuring to the right and the corresponding elevation at the end of the interval determined from Curve 4. This is also the water-surface elevation at the beginning of the third interval thus permitting the process to be repeated. The steps involved in two intervals beginning at 6 AM, January 31, are shown on Fig. 116. The elevation at 6 AM is shown in Table 25 to be 9.00 ft. The value of I, 88,000 cfs, is added to Curve 5 at this elevation and the elevation at the end of the interval, 12 noon, is found to be 10.55. The next I, 68,000 cfs, is added to Curve 5 at this elevation and by dropping down to Curve 4 the elevation at 6 PM is 10.35.

This procedure may be simplified by projecting values of elevations taken from Curves 4 and 5 on to horizontal scales as shown

[1] An outline of this graphical procedure was presented by L. G. Puls, Flood Regulation of the Tennessee River, *House Document* 185.70, Congress 1st Session, 1928, pp. 43–55. The method was also derived independently at about the same time by E. R. Gustafson, of the U. S. Engineer Office, Duluth, Minn.

[2] Other graphical procedures for solving reservoir storage problems have been presented. See for example: R. D. Goodrich, Rapid Calculation for Reservoir Discharge, *Civil Eng.*, February 1931, pp. 417–419; I. H. Steinberg, A Method of Flood Routing, *Civil Eng.*, July 1938, pp. 476–477.

TABLE 25

1 Date 1939	2 I Average during Interval	3 Elevation at End of Interval	4 Q at End of Interval
Jan. 29	3,500	1.45	3,500
	2,800	1.40	3,500
	5,500	1.50	3,500
	10,000	1.90	5,500
Jan. 30	14,500	2.45	8,000
	19,000	3.15	11,500
	32,000	4.35	19,000
	55,000	6.40	34,000
Jan. 31	82,000	9.00	57,000
	88,000	10.55	72,000
	68,000	10.35	70,000
	53,000	9.50	61,500
Feb. 1	43,000	8.55	52,500
	36,000	7.70	45,000
	29,000	6.80	37,000
	21,000	5.85	30,000
Feb. 2	18,000	5.15	24,500
	19,000	4.80	22,000
	23,000	4.80	22,000
	29,000	5.15	24,500
Feb. 3	40,000	6.00	31,000
	60,000	7.60	44,000
	93,000	10.15	68,000
	118,000	12.60	94,000
Feb. 4	120,000	13.85	108,000
	93,000	13.15	100,000
	75,000	11.95	87,000
	61,500	10.70	73,500
Feb. 5	50,000	9.50	61,500
	41,000	8.45	52,000
	33,000	7.45	42,500
	30,500	6.75	37,000
Feb. 6	30,000	6.35	33,500
	31,500	6.20	32,500
	32,000	6.15	32,000
	34,000	6.25	33,000

in Fig. 116. Then, a scale of discharge values may be cut to slide between these horizontal scales, thus permitting values of I to be added rapidly to values on the upper scales of elevations at the beginning of intervals to get elevations at the end of intervals

directly from the lower scale. Having determined the water-surface elevation at the end of each interval, the corresponding discharge values may be read from Curve 1, Fig. 116. These values, shown in Column 4, Table 25, are plotted as Curve 2, Fig. 115. The modification of the hydrographs resulting from reservoir storage may be determined by a comparison of this curve with the original hydrograph (Curve 1).

Flood Routing through River Channels

The method of flood routing described in the previous pages provides a means for the determination of the modified flood hydrograph that enters a river from a flood-control reservoir. Because flood-protection reservoirs are likely to be located many miles upstream from the cities where most of the flood damage occurs, it is necessary to route the flood hydrographs to these downstream localities. The problem is similar to routing through a reservoir in that the stream channel itself is an elongated reservoir, and the solution is again effected by successive applications of the storage equation in conjunction with the relationship between storage and discharge. However, a number of difficulties arise in river routing that make the problem more complex. In an extensive study of this problem, Thomas[1] states: "In a large river with many tributaries, the movement of a flood wave is a phenomenon of such utter complexity as to defy complete and exact analysis by human beings." In this same paper Thomas suggests a graphical "trial-and-error" solution which satisfies the fundamental equations of wave movement. This method, presented in more detail in a later paper,[2] is the only one of the several proposed by various writers that might be termed an exact method, all others being properly called approximate methods. Owing to the laboriousness of the exact method or because inaccuracies in basic data do not justify its use, most routing has been done by the approximate methods.

A complete description of the routing procedure by any one of the approximate methods will not be given here because any particular method would usually apply only to a limited number of

[1] Harold A. Thomas, The Hydraulics of Flood Movements in Rivers, *Engineering Bul.*, Carnegie Institute of Technology, Pittsburgh, Pa., 1937.

[2] H. A. Thomas, Graphical Integration of the Flood-Wave Equations, *Trans. Am. Geophys. Union*, 1940, pp. 596–602.

cases. Generally it is necessary to select or devise a procedure to meet the needs of any particular situation. A brief résumé of some of the published material on the subject will be given, followed by a general discussion of the methods used.

In 1939 an article was published[1] giving a rather complete description of methods used on the Tennessee River. The operations presented in this article applied to cases where complete and accurate information concerning storage capacity of the river was available. In 1942 the authors presented a method[2] that was devised for the case where no storage data were available. Another method that is used extensively by the U. S. Engineers Corps has been described by a number of writers.[3,4,5,6] This has been called the "coefficient method"[5,7] or the Muskingum method,[6] and its development has been credited to[6] T. S. Burns, F. B. Harkness, and G. T. McCarthy.

Four difficulties encountered in river routing that did not appear in reservoir routing will now be discussed.

1. *Determination of Storage.* The storage capacity of the river at various stages must be known to provide the relationship between discharge and storage (equation 2, page 352). Accurate topographical data of the type used to determine reservoir capacity are frequently not available for long river reaches. The storage may be determined by two other methods. If hydrographs of a flood are available at the lower and upper ends of the reach, the method used by the authors[2] may be employed. In this method the fact that the lower portion of the recession side of the hydrograph represents outflow from storage is utilized. This idea has also been

[1] Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder, Flood Routing, *Trans. A.S.C.E.*, 1939, pp. 275–313.

[2] C. O. Wisler and E. F. Brater, A Direct Method of Flood Routing, *Trans. A.S.C.E.*, 1942, pp. 1519–1562.

[3] Gerald T. McCarthy, U. S. Engineer Office, Providence, R. I., The Unit Hydrograph and Flood Routing. A paper presented at the conference of the North Atlantic Division, U. S. Engineer Department at New London, Conn., June 24, 1938; revised, March 21, 1939.

[4] W. B. Langbein, Channel-Storage and Unit Hydrograph Studies, *Trans. Am. Geophys. Union*, 1940, pp. 620–627.

[5] B. R. Bilcrest and L. E. Marsh, Channel-Storage and Discharge Relations in the Lower Ohio River Valley, *Trans. Am. Geophys. Union*, 1941, pp. 637–649.

[6] C. O. Clark, U. S. Engr. Office, Winchester, Va., Storage and the Unit Hydrograph, *Trans. A.S.C.E.*, 1945, pp. 1419–1488.

[7] N. R. Laden, T. L. Reilly, and J. S. Minnotte, Synthetic Unit-Hydrographs, Distribution Graphs and Flood Routing in the Upper Ohio River Basin, *Trans. Am. Geophys. Union*, 1940, pp. 649–659.

presented by Horton.[1] A second method requires, in addition to the hydrographs at the upper and lower ends of the reach, a hydrograph of inflow from the intervening area, i.e., the drainage area contributing flow to the river between the upper and lower ends of the reach. With this information available, all quantities in the storage equation (equation 3, page 353) except $S_2 - S_1$ become known, and increments or decrements of storage may be determined for all intervals throughout the flood period. If some assumption is made as to the value of storage at the beginning of the flood the increments may be added cumulatively to determine actual values of storage.

2. *Inflow from the Intervening Area.* Usually in reservoirs, the increase in drainage area between the upper and lower ends is so small that the inflow from this area may be neglected. For river reaches, however, this area may be of considerable magnitude, and the inflow from it cannot be neglected. If hydrographs for a flood at the upper and lower ends of the reach are available and if the relation between discharge and storage is determined, the inflow from the intervening area can be found by successive applications of the storage equation.[2] In this case the storage equation would be written with inflow at the upper station, Q, and inflow from the intervening area, I, as separate terms as follows:

$$S_1 + \left(\frac{Q_1 + Q_2}{2}\right) T + \left(\frac{I_1 + I_2}{2}\right) T - \left(\frac{O_1 + O_2}{2}\right) T = S_2 \quad (5)$$

Values of Q and O being known and with values of S determined from the storage-discharge relation, values of I are the only unknowns.

If either the upper or lower hydrograph or the storage is unknown, the inflow from the intervening area must be estimated from rainfall, utilizing an assumed distribution graph to synthesize the hydrograph. This procedure is likely to give uncertain results unless hydrographs from a very similar watershed are available as a guide in estimating the shape of the distribution graph. A number of writers have discussed this problem.[3,4,5]

[1] R. E. Horton, Natural Stream Channel Storage, *Trans. Am. Geophys. Union*, 1936, pp. 406–415 and 1937, pp. 440–456.

[2] Footnote 2, page 357.

[3] Footnote 6, page 357.

[4] Footnote 7, page 357.

[5] Franklin F. Snyder, Synthetic Unit-Graphs, *Trans. Am. Geophys. Union*, 1938, p. 447.

3. *Storage-Discharge Relationship.* In very large reservoirs, the water surface is nearly level at all times so that a change in storage must be accompanied by a corresponding change in water-surface elevation at all points in the reservoir. Therefore, since the rate of outflow, O, is directly related to water-surface elevation, it may also be directly related to the storage as shown by equation 2, page 352. In a long river reach, however, the storage begins to increase as soon as the flood wave arrives at the upper end of the reach. It continues to increase until the wave front reaches the lower station, which may be hours later. During all this period of increasing storage the outflow, O, may have been constant. It follows, therefore, that in this case O is not directly related to storage. Based on the assumption that the storage in any reach is related to the average of water-surface elevation at the two ends and therefore to the average of the discharges at the two ends, the authors have plotted storage against $(Q + O)$ with satisfactory results.[1]

A somewhat similar procedure is used in the Muskingum method. In this method storage is related to the weighted average of the total inflow $(Q + I)$ and O as follows: $S = K(xI_t + (1 - x)O)$, where $I_t = (Q + I)$. The value of x is selected to give as nearly as possible the same storage-discharge relation during rising and falling river stages. When $x = 0$ the relation applies to reservoir storage (compare with equation 2, page 352). The author's method[1] described above also provides for a variable value of x, because I was not used in their plotting. Consequently the weight given to I_t depends upon the relative magnitudes of Q and I. If, for example, I is approximately equal to Q, S would, in effect, be plotted against $\left(\frac{I_t}{2} + O\right)$, which would correspond to using $x = \frac{1}{3}$ in the Muskingum formula. When I is zero the weighting would correspond to $x = 0.5$. In the Tennessee River work[2] previously mentioned, a graphical method is used to accomplish this same purpose.

4. *Variable Stage-Discharge Relations.* Unlike outflow from a reservoir, discharge at a river-gaging station may vary with the slope of the energy gradient as well as with the stage. At such stations the relation between gage height and discharge are somewhat different for rising and falling stages (see Chapter X, page 394).

[1] Footnote 2, page 357.
[2] Footnote 1, page 357.

It follows that a relation between channel storage and discharge would also depend to some extent upon whether the discharge was increasing or decreasing. Several methods of correcting for such effects have been described.[1,2] Each of these was developed for a particular situation and is not sufficiently general to warrant a detailed description here.

[1] Footnote 1, page 357.
[2] Footnote 2, page 357.

CHAPTER X

STREAM-FLOW RECORDS

If the exact relationships between rainfall, evaporation, transpiration, infiltration, and runoff were fully understood, there would be little need for the establishment of gaging stations and the collection of discharge records. Inasmuch as long-term rainfall records are available practically everywhere the necessary stream-flow data could then be computed more quickly and at less expense than they can be collected in the field. However, these relationships are not and perhaps never will be sufficiently well understood to enable the engineer to make a satisfactory determination of the yield of any stream without having actual discharge records covering a more or less extended period. Regardless of the advances that will unquestionably be made in this science it is, therefore, doubtful if discharge records will ever cease to play an important role in the solution of all problems involving a knowledge of stream flow.

A brief treatment of the technique involved in obtaining stream-flow records is given here. For a more complete discussion the reader is referred to the various treatises on the subject.[1]

Methods of Obtaining Discharge Records

The principal types of installations used to obtain continuous records of stream discharge are classified as follows:

1. Weir stations.
2. Control meter stations.
3. Power plants.
4. Velocity area stations.

No one of these methods is the best suited to all conditions, but in every case one is to be preferred to the others. It is the engineer's

[1] Grover and Harrington, *Stream Flow*, John Wiley, 1943.
W. A. Liddel, *Stream Gaging*, McGraw-Hill, 1927.
D. M. Corbett and others, Stream-Gaging Procedure, *U. S. Geological Survey Water-Supply Paper* 888.

duty to study carefully each situation and determine the best procedure.

Weir Stations

Weirs are overflow structures for which there is a mathematical relationship between head, or height of water above the crest, and the discharge. A record of head may, therefore, be translated into a record of discharge. Weirs are often installed on small streams for the purpose of measuring discharge. Large installations are seldom made because the cost is prohibitive. However, spillways of dams are often used as weirs to measure the discharge of large streams. There are two types of weirs, sharp-crested and broad-crested. Sharp-crested weirs are characterized by a knife-edge overflow section from which the jet springs free. Broad-crested weirs include all other types of cross sections, such as flat-topped weirs and ogee spillway sections. Either type requires that the elevation of the tail water be low enough to avoid backwater at the crest.

Sharp-crested weirs, when built in accordance with standard practice, have the advantage of not requiring the establishment of a rating curve from field measurements. V-notch weirs are used to measure small discharges. A 90° V-notch is suitable for accurate determination of flows as low as 0.01 cfs. Some of the formulas derived for these and other types of weirs are found in standard books on hydraulics. Rectangular sharp-crested weirs are used for larger flows. Sharp-crested weirs are not adaptable to streams carrying loads of silt and debris. Such material collects in the stilling pool above the weir thus raising the velocity of approach higher than under calibration conditions. Floating debris is likely to injure the crest of the weirs or to become lodged on the sharp crest thus affecting the weir reading.

Broad-crested weirs may also be precalibrated, but for accurate results they are usually calibrated in the field. When such weirs have a horizontal crest they follow the general law that

$$Q = CLH^{3/2} \tag{1}$$

where Q is the discharge in cfs, L is the length of the crest in feet, H is the head in feet, and C is an empirical coefficient that must be determined from discharge measurements. King[1] gives a large number of values of C for various types of broad-crested weirs. In

[1] H. W. King, *Handbook of Hydraulics*, McGraw-Hill, 1939.

order to provide a greater sensitivity to low discharges, broad-crested weirs having a parabolic notch have been utilized with good results. One such type developed by the U. S. Geological Survey is called the Columbus Deep Notch. It has the additional advantage of permitting the passage of most floating material and of being only slightly sensitive to changes in velocity of approach such as result from sedimentation.

Control Meters

A control meter is a structure built in the channel of a stream, whereby critical depth is produced by raising the bottom of the channel or by decreasing the width, or both. The throat of the section is usually made rectangular or trapezoidal in cross section. Such a device permits the application of the well-known relationships between discharge, minimum energy, and critical depth[1] for determining the discharge. The floor of the throat is made level while the floor of the expanding outlet is given a sufficiently steep slope to cause the water to leave the throat at supercritical velocities and thus insure the presence of critical depth at some point in the throat. The expanding side walls of the outlet have the same effect as the steep bottom slope. A properly designed control meter is characterized by the presence of a hydraulic jump. The jump is below the constriction when the original depth in the channel was greater than critical depth and above the control when the original depth was less than critical.

It is not possible to measure critical depth directly since its exact position in the throat varies with the discharge and is not easily found even for a particular discharge. The discharge is determined by measuring the head at a point above the throat and applying the relationship between discharge and specific energy at critical depth. For a rectangular throat, this relationship is $Q = 3.087bH^{3/2}$, in which b is the width of the throat and H is the specific energy at critical depth as shown in Fig. 117. An application of Bernoulli's equation between Point 1 where the gage is placed and the throat, and solved for H, results in the following relationship,

$$H = d_1 - x + \frac{V_1^2}{2g} - h_e \tag{2}$$

[1] H. W. King, *Handbook of Hydraulics*, McGraw-Hill, 1939.

in which only the two minor quantities V_1 and h_e are unknown. V_1 may be found by trial, thus leaving only the energy loss unknown. Studies made by the U. S. Bureau of Reclamation[1] indicate that, when all surfaces are connected by tangent curves, the energy loss is so low that it may be neglected. A meter with one angular change in direction at the beginning of a constriction was found to have a coefficient of discharge of 0.95.[2] The length of the throat must be kept as short as possible to avoid energy losses and at the same time long enough to insure that the critical depth will fall

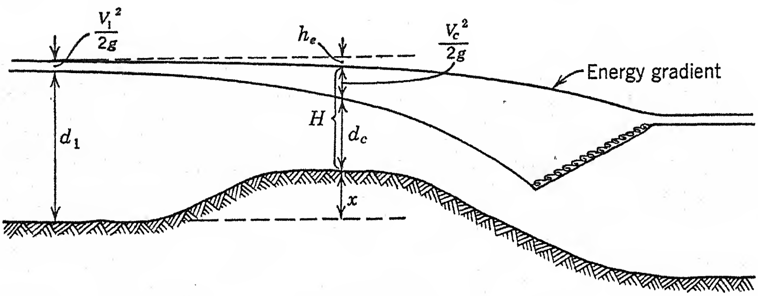

FIG. 117.

within the throat. A throat length of about three times critical depth has been found to give good results.

The chief advantage of the control meter over the weir is its ability to measure discharge in debris- and silt-laden streams.[3] Since the flow immediately above the control is accelerated rather than retarded as in the case of the stilling pool of a weir, silt and debris tend to be swept through the control. Its rugged construction permits the passage of logs and debris that would seriously damage a sharp-crested weir. This type of control is especially suited to intermittent streams on which there is little opportunity to find a natural control. The capacity may be made as large as desirable by simply making the throat wider. The lower limit for accurate discharge measurements increases with the capacity of

[1] Julian Hinds, discussion of The Improved Venturi Flume, *Trans. A.S.C.E.*, 1926, **86**, 859.

[2] Dr. F. V. A. E. Engel, The Venturi Flume, *Engineer*, August 3 and 10, 1934.

[3] H. G. Wilm, J. S. Cotton, and H. C. Storey, Measurement of Debris-Laden Stream Flow with Critical-Depth Flumes, *Trans. A.S.C.E.*, 1938, **103**, 1237.

the flume. A typical ratio of maximum to minimum discharge is 125 to 1. Since it is usually necessary to measure discharges below the lower limit of these large meters, a second smaller control meter or a weir is often set in parallel with the large one.

Power Plant Records

At most modern power plants, operating records are kept that provide an excellent basis for the determination of the mean daily discharge past those plants. On many streams the head is so fully developed for power purposes that no suitable places are left for velocity-area stations because of backwater. In addition, there are many other streams where the climate is rigorous and it is impossible to get discharge records without ice interference, and where this occurs good records are both difficult to obtain and expensive.

The total flow that passes a power plant is equal to the sum of that (1) through the turbines, (2) over the spillway, (3) through the various gates, sluices, fish ladders, etc., and (4) the leakage through, underneath, and around the dam, powerhouse, and embankments.

The hydraulic turbine is an excellent water meter. The discharge capacity at various gate openings of most modern turbines has been accurately determined. If it is unknown, it can be found by means of tests. If it is a low-head plant and the turbines are set in an open wheel pit, the discharge for the various gate openings can be measured by current meters in the head race, by the salt-solution method, or possibly by means of a weir in the tail race. If it is a high-head plant and the water is conducted to the turbines through a closed penstock or pipe line, it may be more convenient to measure the discharge by the salt-velocity method, the Gibson method, or possibly by means of a Pitot tube. After a turbine has once been rated for the various gate openings, its rating is affected only slightly by years of service.

The coefficients (equation 1) of discharge for nearly all types of spillways as they are built at the present time have been quite accurately determined.[1] For other types, the coefficient can be

[1] Robert E. Horton, Weir Experiments, Coefficients and Formulas, *U. S. Geological Survey Water-Supply Paper* 200, Government Printing Office, Washington, D. C.

H. W. King, *Handbook of Hydraulics*, McGraw-Hill, third edition, 1939.

determined by current meter measurements made at a time when all turbines, gates, sluices, etc., are closed. Practically all modern power plants keep a complete log of head-water and tail-water elevations. If the elevations of the crest of the dam and of the zero of the head-water gage are known, the head over the spillway is found, and, with the length of spillway known, the discharge is computed.

A similar procedure is followed for finding the discharge through the various gates, sluices, etc. If they are of such construction that the proper coefficients cannot be found in engineering literature, they can be calibrated by the use of current meters or by other means. Horton[1] has provided a good method for determining the proper coefficients to be used for partially open Tainter gates.

The leakage past the dam may be determined by closing the turbines and other openings for a short time during low water with the head drawn down enough to prevent overflow during the test and then measuring with a current meter the total flow downstream from the plant. The quantity is sometimes so small that it may be neglected.

By adding these quantities the total daily discharges are determined. With regard to accuracy, the results obtained by this method are the very opposite in character to those obtained at weir stations. In this instance, during periods of low flow practically all the water passes through the turbines and that flow should be accurately determined. In rigorous climates the winter flows are usually low and therefore difficult to determine accurately at either weir or velocity-area stations. Again during flood periods the peak discharges can ordinarily be determined more accurately from power-plant records than by either of the other methods.

Velocity-Area Stations

At velocity-area stations discharge measurements are made by dividing the stream cross section into a number of parts for each of which the area, velocity, and discharge are determined separately. By adding these partial discharges the total is obtained for the stream. After a sufficient number of such measurements have been made, they are plotted against their corresponding gage heights to produce a station-rating curve or a discharge curve as

[1] Robert E. Horton, Discharge Coefficients for Tainter Gates, *Eng. News-Record*, January 24, 1934.

it is called. Such a curve is shown in Fig. 118. After having been once established, gage readings that have been taken at regular intervals, such as daily or oftener, may be applied to the curve and the corresponding discharges thereby determined.

Every velocity-area station has three essential features: (1) a control, (2) a gage, and (3) a metering section. The characteristics

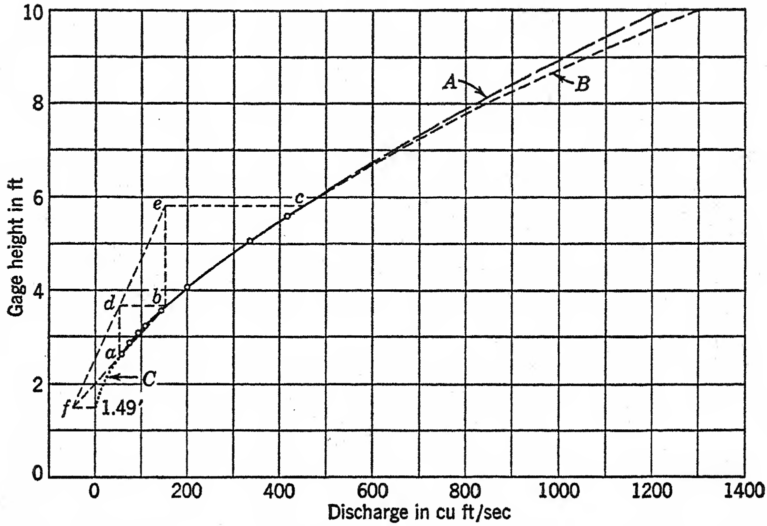

FIG. 118. Discharge curve for Huron River near Whitmore Lake, Mich. Extended by: A, $A\sqrt{D}$ method; B, logarithmic method; C, running method.

and relative locations of these component parts may vary considerably at different stations.

The control is a cross section or reach of river channel that determines the relationship between stage and discharge at that section and for some distance upstream. The gage is an instrument that is installed upstream from but within the range of influence of the control for the purpose of determining the fluctuations in stage with respect to time. The metering section is the cross section of the stream where the discharge is measured.

Inasmuch as the discharge, measured at the metering section, is plotted against stage, measured at the gage, it follows that there should be no appreciable inflow to or outflow from the stream between these two points. Beyond that, there is no restriction as to their relative locations. In other words, although the gage must be

located as above noted, with respect to the control, the metering section may be either upstream or downstream from either the gage or the control, as long as the quantity of water passing it per second is substantially the same as that passing the gage.

The Control. The control may well be considered the most important of the three component parts of the station. Upon its permanence very largely depend both the cost and value of the discharge records obtained. In establishing a station the control should, therefore, receive first consideration; the gage should never be installed until after the location of the control has been determined.

In Fig. 119 are shown the longitudinal profiles of the bed of a stream and the water surface for a given reach. *ABCDEF* represents the profile of the water surface during a low stage of the stream. At this stage the section at *C* serves as a control for the

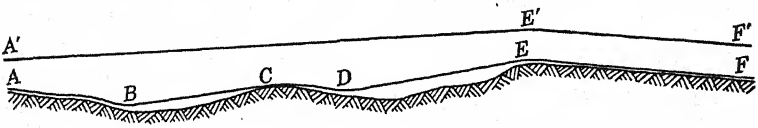

FIG. 119.

reach extending up to *B*, and the section at *E* is the control for the reach from *D* to *E*. The reaches *AB*, *CD*, and *EF* act as their own controls throughout their respective lengths at this low stage. A reasonable amount of erosion or sedimentation at any point throughout these reaches would not appreciably affect the elevation of water surface for any considerable distance upstream. Nor would any similar change occurring anywhere between *B* and *C* or between *D* and *E* measurably affect the water surface anywhere upstream. On the other hand, any change occurring either at *C* or at *E* will produce a corresponding effect upon the water surface throughout the reach *BC* or *DE*, respectively, during the low stages of the river. However, when the water surface rises approximately to the stage represented by the profile *A'E'F'* the section at *E* becomes the control for the entire length of channel from *A* to *E*. For this stage or for higher stages, any ordinary changes in the cross section of the channel, even though they occur at *C* or between *A* and *B*, have but slight effect upon the elevation of water surface although a similar change at *E* will produce a marked effect throughout the entire reach from *A* to *E*. Thus many sections

of the channel which act as controls during low stages of the river become completely drowned out in the high stages and then have practically no effect on the water surface at points upstream.

Except in cases where a rock outcrop creates either a waterfall or a rapids with considerable drop, a longitudinal river profile is a practical necessity in determining the location of the various controls in any given length of stream channel. The best control is the one that is the most nearly permanent and also functions as a control throughout all stages of the river.

No control is absolutely permanent. The continuous flow of water combined with the effects of temperature changes will cause even the hardest granites to slowly disintegrate. Nevertheless a control that is formed by an outcropping of good, hard, resistant rock may be considered permanent at least for several decades. Most other controls are subject to a slow and gradual change through long periods or to a more rapid change during floods. After a gaging station has been established and sufficient discharge measurements have been made to determine the relationship between stage and discharge, it becomes the engineer's duty and responsibility to make additional discharge measurements at various stages from time to time to learn just how rapidly or how slowly the control is changing. Especially should such measurements be made soon after the occurrence of major floods for ordinarily the most marked changes occur then. Never should he assume that the control is permanent until it has been proved so by a number of measurements made after the passage of such floods.

In some locations, especially on small streams, artificial controls are installed. The use of control meters and weirs has already been discussed. When properly installed, such devices require no special calibration. More generally, however, an artificial control is a low overflow dam or lined section of stream bed built for the purpose of stabilizing the stage-discharge relationship. The low dam type serves the additional purpose of providing sufficient depth for a current meter in shallow streams. An artificial control should have adequate cutoff walls and an apron on the downstream side as protection against erosion. The shape of the cross section should be designed to facilitate the passage of floating debris and bed material, whereas the curvature of the crest should be designed to provide the desired sensitivity at low discharges. The U. S. Geo-

logical Survey has developed a number of standard types of low dam controls.[1]

The Gage and Its Location. All gages may be classified as either recording or nonrecording. Recording gages draw a continuous graph of the fluctuations in stage. Nonrecording gages require an observer who reads the gage and records the readings at regular time intervals.

Regardless of the type of gage employed, it should be so located as to conform to the following specifications.

1. It must be upstream from, but within the range of influence of the control.

2. Its support should be rigid and immovable so that the elevation of the datum will be unlikely to change.

3. It should be as sensitive as possible; in other words, it should be located where the greatest range of fluctuations in stage occurs. In Fig. 119 the proper location would be at or below D.

4. It should be in a protected spot so that destruction by ice or other floating debris would be improbable.

5. It should be easily accessible.

6. If the gage is nonrecording and on a northern stream, it should be located at a point in the stream where the velocity of the water is great enough to prevent ice formation. Oftentimes a good location is on the side of a bridge pier near the downstream end. At this point the water usually flows smoothly, does not freeze or pile up on the gage, and the danger of destruction and probability of change in datum is slight.

7. The gage should never be located upstream from the junction with another stream so near as to be affected by backwater from that stream. Nor should it ever be located within the influence of backwater from a hydroelectric plant. At neither of these locations is the relation between stage and discharge constant.

Recording Gages. Automatic recording gages have so many advantages over the nonrecording type that they are being used more and more. Their principal advantages are:

1. The personal equation is almost entirely eliminated. In disagreeable weather or at times when the observer of a nonrecording gage is busy with other matters, there is a strong temptation not to read the gage for considerable periods at a time and then to interpolate the missing values. Authenticity of records may therefore be questionable at times.

[1] Grover and Harrington, *Stream Flow*, John Wiley, 1943.

2. A nonrecording gage is usually read by the observer once or twice a day. Especially on the smaller streams, large fluctuations may occur between readings, and as a result the hydrograph obtained by plotting the discharges from the daily gage readings

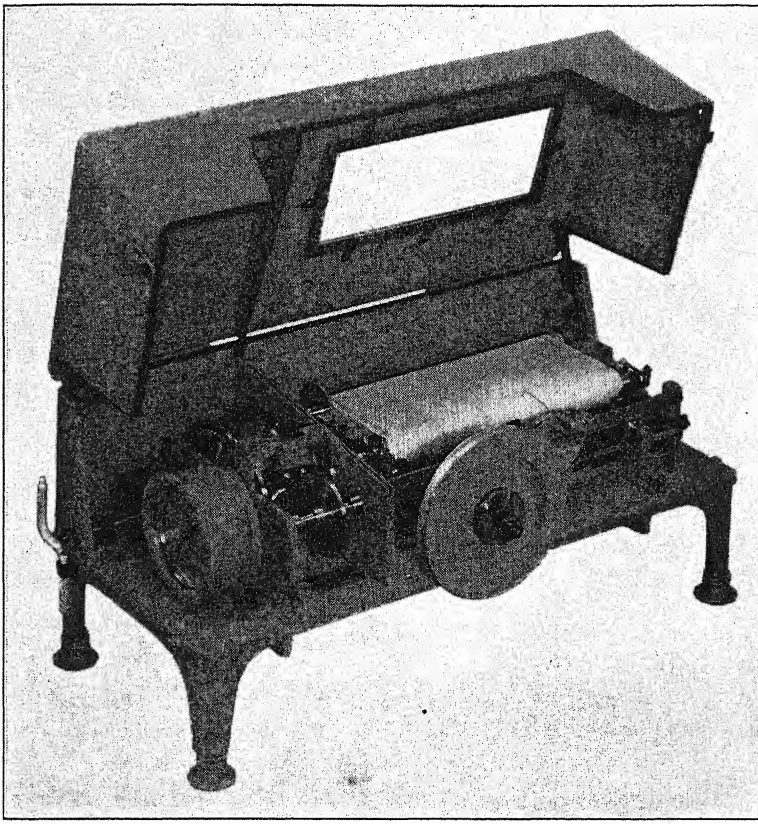

Fig. 120. Stevens Water-stage Recorder Type A35. Courtesy Leupold & Stevens Instr.

does not present a true picture of the actual behavior of the stream as is illustrated in Fig. 2, page 17. This matter becomes doubly important when one or more power plants are located upstream from the gage and also when diversions for irrigation, municipal, or industrial uses are made at upstream points, changing the natural flow of the stream.

In Fig. 120 is shown a continuous-recording river gage. In this

instrument the cylinder is driven by the clock at a constant speed represented on the record sheet by scales varying from 12 in. per day to 864 in. per day. The pen carriage, actuated by a float as the water surface in the well rises and falls, travels back and forth. The rate of travel varies from 0.2 in. to 10 in. for each foot of fluctuation in stage. To permit the recording of unlimited fluctuations in gage height the pen carriage automatically reverses direction at the top and bottom of the chart. In Fig. 121 is shown a record sheet covering such a reversal period. This type of gage is so designed that the chart must be changed as often as each day or as infrequently as every 2 mo, depending on the time scale selected.

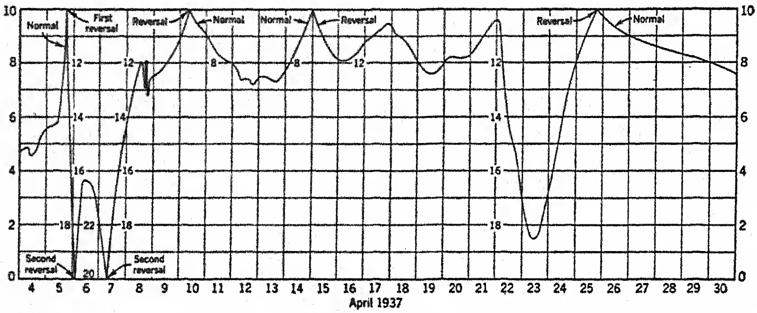

FIG. 121.

In the gage illustrated in Fig. 122, the clock drives the pen carriage at a constant speed across the record sheet while the float turns the cylinder as the stage rises and falls, which is the reverse of the arrangement in the gage shown in Fig. 120. The operating period for this type of gage ranges from 1 day to 10 days.

The recording gages described above are entirely satisfactory when installed at hydroelectric power plants or at other points where an attendant is present who, in addition to his other duties, changes the record sheet, winds the clock at regular intervals, and sees to it that the gage operation is continuous.

When these gages are installed in isolated places with no attendant present, there is always the danger of a break in the continuity of the records because of the clock stopping or because of some other instrumental failure. The longer the period of operation without attention the more delicate is the mechanism and therefore the greater is the probability of breakdown. Gages operating

no longer than a week or 10 days are relatively sturdy in construction and free from failures. Even when such gages stop, the interruption is for such a short period that the missing records can be interpolated quite satisfactorily. On the other hand, it is practically impossible to interpolate records that are missing for a

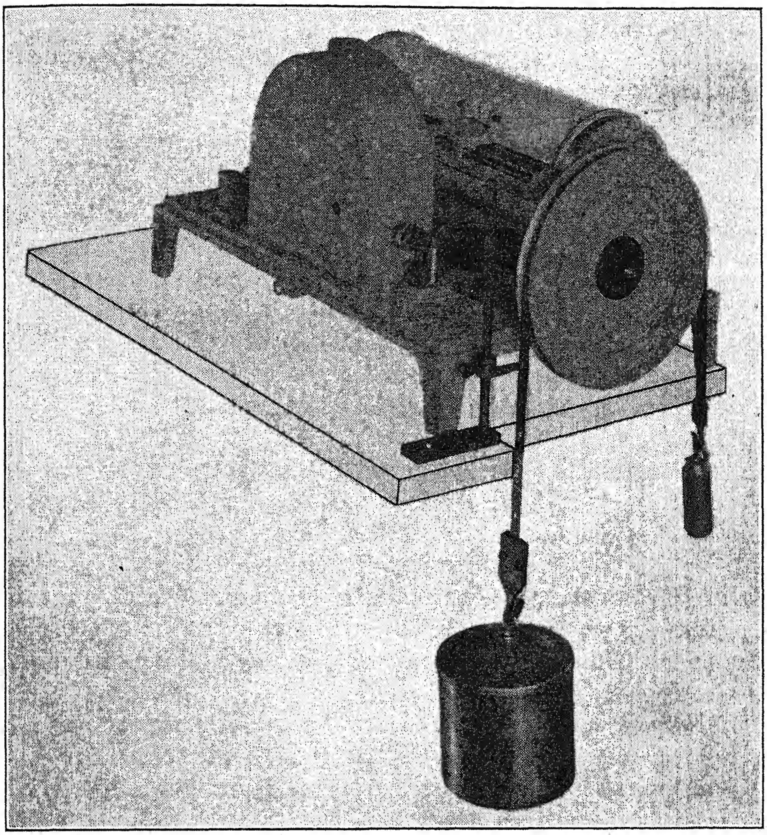

FIG. 122. Stevens Water-stage Recorder Type F. Courtesy Leupold & Stevens Instr.

period of a month or longer. The value of discharge records increases tremendously with their continuity. Frequent interruptions render them of little value for many purposes.

In the long-distance transmitting type of gage, the transmitter is installed at the point on the stream where the records are to be

obtained. By means of an electric line, such as a telephone circuit, the gage heights are automatically transmitted to any distant point where receiving instruments are installed. The receivers are usually of the continuous-recording type, although they may also be of the indicating type in which the gage height is only indicated on a dial and is not recorded. Although this type of gage is expensive to install, especially where the records are to be obtained at a remote point and a long transmission line must be built, it has several great advantages over other types of gages. The receiver may be installed in a power plant or office where someone is always present to observe a break in operation and to dispatch a repair man to find and correct the trouble thereby insuring continuity of records. No additional employee in the field or long trip from the office or power plant is necessary to change the record sheet at regular intervals. Radio-operated remote-recording systems are also available. They have the advantage of not requiring a wire between the gage-height transmitter and the recorder.

All recording gages that are not in power plants or in pumping stations should be housed in permanent shelters preferably of concrete and located on the stream bank. The floor of the gage house should be above the extreme high-water level. The float well should be excavated to a depth below the extreme low-water stage so that the float can rise and fall freely with the fluctuations of the stream. A pipe having a diameter of about 4 in. should connect the float well with the deepest portion of the stream. A bank of earth or other material around the gage house may prevent freezing in the winter. Otherwise an electric light in the well or some other device must be employed for this purpose.

Nonrecording Gages. Nonrecording gages are of four general types, viz., (1) staff, (2) weight, (3) float, and (4) hook.

Staff gages are either vertical or inclined and are usually located at or near the water's edge to be readily accessible and easily readable by the observer. The zero of the gage should be below the stage of extreme low water to prevent the occurrence of negative readings. If the stream is subject to large fluctuations in stage and the banks slope gently to the water's edge, either two or more sections of vertical staff or an inclined staff may be used. If the vertical staff is adopted special care should be taken to insure that the several sections are so installed that the gage readings on all

sections refer to exactly the same datum. If an inclined staff is used, it should be securely anchored to concrete piers that extend well below the frost line to prevent a change in datum through frost action.

Vertical-staff gages may be either metal or wood although the former are much to be preferred. Alternate subjection of wood-staff gages to air and water causes rapid deterioration and painted graduations soon peel off. Enameled metal staffs have a much longer life and can be obtained in various lengths of section and graduated to tenths, half tenths, and hundredths of feet if desired. Inclined staffs, on the other hand, are usually made of wood. The graduations are put on after the gage has been installed, the spacing depending upon the slope at which the gage is set.

Weight gages are so designed that a weight, attached to the end of a tape, chain, or wire can be lowered to the water surface and the gage is then read, thus determining the elevation of the water with respect to some fixed datum. A gage of this type is illustrated in Fig. 123. In this instrument the weight, W, is lowered by means of a narrow metal tape, T, having a low coefficient of expansion. The tape, which is graduated in feet only, is wound on the reel, R, passing over two pulleys, P, and with a small clearance over the scale, S, which is 1 ft long and is graduated in tenths and hundredths of feet. When the reel is released and the weight is lowered to the water surface, only one foot mark can be over the scale. The gage reading is represented by that foot mark plus the tenth and hundredth appearing on the scale directly beneath the foot mark. Only the back and base of the gage box are shown in the figure, the remainder with hinges at H having been removed.

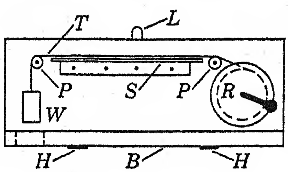

FIG. 123.

A float gage consists of a float and a counterweight connected by a tape, wire cable, or chain that passes over a pulley located near the upper end of a vertically graduated scale board. The gage reading is indicated on the scale by an index marker on the tape, cable, or chain. To prevent its being carried away by the current, the float must be enclosed in a stilling well which consists of a vertical box usually made of timber or concrete and having holes

in the sides or bottom to permit the water to stand at the same level inside and outside. The principal use of such gages is at power plants and at pumping stations.

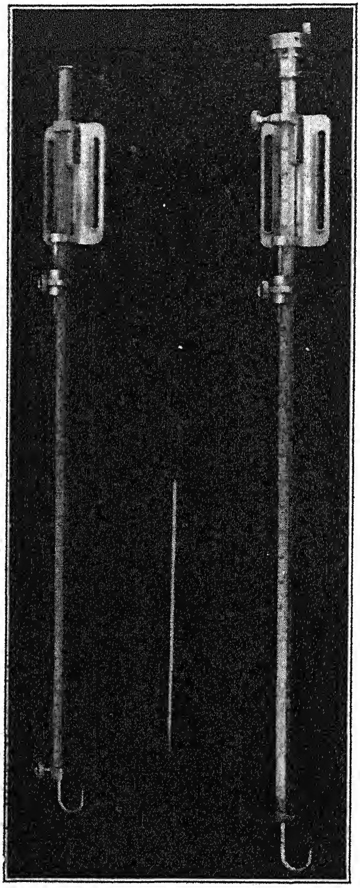

FIG. 124. Hook gages. Also straight point that can be substituted for hook. Courtesy W. & L. E. Gurley.

Hook gages, examples of which are shown in Fig. 124, are the most accurate of all types of gages. A hook gage is usually installed in a stilling well in order to provide a calm water surface. The hook, which is attached to the lower end of a vertical rod, is lowered until the point is beneath the water surface where it is clamped in position. By means of a slow-motion screw it is then raised until

the point causes a slight rise to appear on the water surface. By means of a vernier the stage can be read to a thousandth of a foot. Because the accuracy attained is not justified in view of the difficulty in making the readings, this type of gage is seldom used in stream-gaging work except at some weir stations where slight fluctuations in stage cause appreciable variations in discharge.

Gage Datum. Regardless of the type of gage that is used, it is of utmost importance that the elevation of its zero be determined with respect to at least two permanent bench marks in the near vicinity. After a gage has once been installed, its datum should if possible be kept unchanged.

The maintenance of a permanent datum is of such fundamental importance that the elevation of the zero of the gage should be checked annually and the gage should be reset whenever any change is found to have occurred. Unless this precaution is taken, long periods of records are likely to have doubtful value because of a change in the elevation of the gage. Inasmuch as the time when this change occurred is unknown, corrections are impossible. With an annual check, however, this difficulty is greatly reduced.

Measurement of Velocity

For the determination of the velocity of a stream a great many different kinds of instruments have been developed. These instruments fall into three general classes depending upon the principles on which they operate: (1) floats, (2) pressure instruments, and (3) current meters.

Floats, inasmuch as they move with the same velocity as the adjacent water, provide a direct means of measuring that velocity. There are three kinds of floats: surface, subsurface, and rod. For surface floats almost anything that floats may be used such as apples, oranges, bottles partly filled with water, cakes of ice, driftwood, pieces of paper, and so on. The surface velocity is found by timing the travel of these floats at various points across the stream through measured distances that usually range between 100 ft and 500 ft. The mean velocity in the vertical is taken as the product of the observed surface velocity and a coefficient. This coefficient should be determined experimentally (see page 390). Rod floats are usually of wood, about 2 in. square, and of a length slightly greater than the depth of the water. They should be well coated with waterproof paint or varnish to prevent them from becoming

water logged and should be so weighted at their lower end that they float vertically and almost submerged. Subsurface floats are no longer used. The use of all kinds of floats should be restricted to straight stretches of river channel having a practically uniform cross section throughout. Because natural river channels seldom if ever fulfill this requirement, floats are not extensively used in stream-gaging work.

From time to time various instruments have been devised for the purpose of measuring the velocity at a point in the stream based upon the principle of converting the kinetic energy of the water into pressure energy and then measuring the pressure head. The Pitot tube, which is the most noteworthy of these instruments, can be used successfully in pipes and in experimental channels but is not adapted for use in natural rivers. Another instrument of this type utilizes a flat plate, the pressure on whose face is transmitted through a hollow tube and indicated on a dial. Numerous other devices employing this principle have been developed, but none have proved satisfactory for measuring velocities in natural streams.

At practically all velocity-area stations the velocities are determined by current meters in which a wheel is made to rotate about its axis by the force of the current, the speed of rotation depending upon the velocity of the water. Inasmuch as this relationship is affected by many factors such as bearing friction and the shape and surface condition of the moving parts, it is necessary to rate each meter even though it may appear to be an exact replica of another rated meter. The rating should be checked after severe usage or when it has been accidentally damaged and about once a year under ordinary usage.

Rating of Current Meter

The usual method of rating a current meter is to draw it through still water observing the time of travel and the number of revolutions made as the meter travels a given distance. The number of revolutions per second and the corresponding velocity in feet per second are then computed. When these quantities are plotted, one against the other, on ordinary cross-section paper, a straight line is usually found to fit the points closely enough for all practical purposes. Ordinarily, however, there is a change in the slope of this line at some certain velocity that seems to vary for different meters

so that two equations for this relationship must be derived, one for the higher velocities and the other for the lower. These equations are then solved for all different velocities and a rating table

TABLE 26

RATING TABLE FOR PRICE METER #9295

Rated in University of Michigan Naval Tank, October 19, 1931

Sec/rev	10	20	30	40	50	60	70	80	90	100
30	0.87	1.64	2.42	3.20	3.97	4.75	5.53	6.30	7.08	7.86
31	.84	1.59	2.34	3.10	3.85	4.60	5.35	6.10	6.85	7.61
32	.82	1.55	2.27	3.00	3.73	4.46	5.19	5.91	6.64	7.37
33	.80	1.50	2.21	2.91	3.62	4.32	5.03	5.74	6.45	7.15
34	.78	1.46	2.14	2.83	3.52	4.20	4.89	5.57	6.26	6.94
35	.76	1.42	2.09	2.75	3.42	4.09	4.75	5.41	6.08	6.75
36	.74	1.38	2.03	2.68	3.33	3.97	4.62	5.27	5.91	6.56
37	.72	1.35	1.98	2.61	3.24	3.87	4.50	5.13	5.76	6.39
38	.70	1.32	1.93	2.54	3.16	3.77	4.38	5.00	5.61	6.22
39	.69	1.29	1.88	2.48	3.08	3.67	4.27	4.87	5.47	6.06
40	.67	1.26	1.84	2.42	3.00	3.58	4.17	4.75	5.33	5.91
41	.66	1.23	1.80	2.36	2.93	3.50	4.07	4.64	5.20	5.77
42	.64	1.20	1.75	2.31	2.86	3.42	3.97	4.53	5.08	5.64
43	.63	1.17	1.71	2.26	2.80	3.34	3.88	4.42	4.96	5.51
44	.62	1.15	1.68	2.21	2.74	3.27	3.79	4.32	4.85	5.38
45	.61	1.13	1.64	2.16	2.68	3.20	3.71	4.23	4.75	5.27
46	.60	1.10	1.61	2.12	2.62	3.13	3.64	4.14	4.65	5.15
47	.59	1.08	1.58	2.07	2.57	3.07	3.56	4.06	4.55	5.05
48	.58	1.06	1.55	2.03	2.52	3.00	3.49	3.97	4.46	4.94
49	.57	1.04	1.52	1.98	2.47	2.94	3.42	3.89	4.37	4.84
50	.56	1.02	1.49	1.95	2.42	2.89	3.35	3.82	4.29	4.75
51	.55	1.00	1.46	1.92	2.37	2.83	3.29	3.74	4.20	4.66
52	.54	0.99	1.43	1.88	2.33	2.78	3.23	3.67	4.12	4.57
53	.53	0.97	1.41	1.85	2.29	2.73	3.17	3.61	4.04	4.49
54	.52	0.95	1.38	1.82	2.25	2.68	3.11	3.54	3.97	4.40
55	.51	0.94	1.36	1.78	2.21	2.63	3.06	3.48	3.90	4.33
56	.51	0.92	1.34	1.75	2.17	2.59	3.00	3.42	3.83	4.25
57	.50	0.91	1.32	1.72	2.13	2.54	2.95	3.36	3.77	4.18
58	.49	0.89	1.30	1.70	2.10	2.50	2.90	3.30	3.71	4.11
59	.48	0.88	1.28	1.67	2.06	2.46	2.85	3.25	3.64	4.04
60	.48	0.87	1.26	1.64	2.03	2.42	2.81	3.20	3.59	3.97

is made up as shown in Table 26. For an observation of any number of revolutions in any given time, the corresponding velocity is found from the table, interpolations being made if necessary. If an observation shows 40 revolutions in 65 sec, the velocity is the same as for 20 revolutions in 32.5 sec or, from Table 26, 1.52 ft per sec.

Among the current meter rating stations in the United States are those at the Bureau of Standards, Washington, D. C., University of Michigan, Ann Arbor, Michigan, Cornell University, Ithaca, New York, and Rensselaer Polytechnic Institute, Troy, New York. At all these stations the meter is suspended from a car that is driven by a motor at uniform speed, running on a track over a tank or channel filled with water. Time, distance, and number of revolutions are automatically recorded on a revolving drum.

It has been found that for any given velocity of the car a meter supported by a rod will revolve faster than if it were supported by

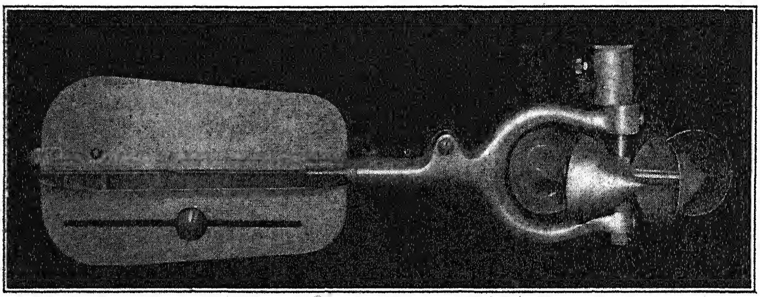

FIG. 125. Price current meter. Courtesy W. & L. E. Gurley.

a cable. Hence, when a meter is being rated it should be supported in the same manner in which it will be used in making measurements. If at times it will be supported by rod and at other times by cable, it should then be rated both ways.

Types of Current Meters

All current meters may be divided into two general classes, differential and direct-acting, depending entirely upon the type of wheel employed. In the differential meter, of which the Price (Fig. 125) is an example, the wheel consists of a series of cups that rotate on a vertical axis. It will be observed that half of the cups are convex and half are concave to the current. The difference in pressure produces the rotation. In the direct-acting meter, such as shown in Fig. 126, the wheel resembles a propeller, carrying a number of vanes on a horizontal axis whose direction coincides with the direction of flow. The force of the current acts directly and uniformly on all the vanes, tending to cause rotation.

Current meters may also be classified in accordance with the

manner in which the observer determines the number of revolutions that the wheel makes during the period of observation. Under this method of classification, the different types are: (1) mechanical, (2) acoustic, and (3) electric.

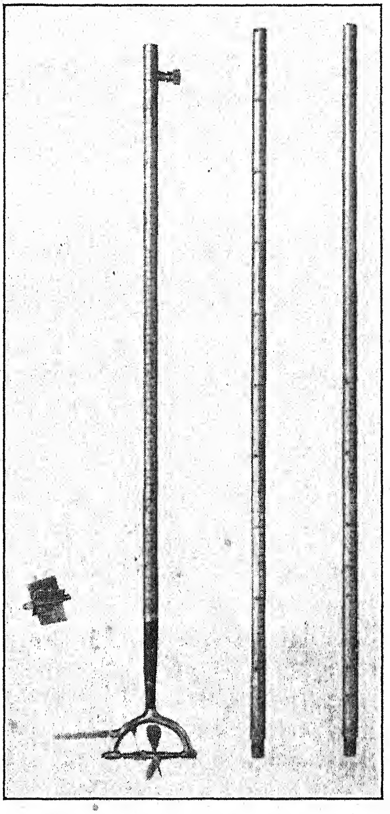

FIG. 126. Direct-acting meter. Courtesy Leupold & Stevens Instr.

The mechanical meter, through a worm gear on the shaft of the meter wheel, operates a set of gears that indicate the number of revolutions the meter wheel has made during the observation. A cord or light wire-cable connection enables the operator to throw the gears in and out of mesh at the beginning and end of each observation. On account of trouble from floating debris and the inconvenience of taking the meter out of the water for each reading, this meter has never gained extensive use.

The acoustic meter and also the electric meter are so designed that a signal is produced at each revolution or at every second, fifth, tenth, or other number of revolutions. In the acoustic meter the signal is conveyed directly to the ear of the observer through a hollow tube. As these meters must be held in position by the observer, their use is restricted to shallow streams. In the electric meter a telephone circuit, energized by a battery, carries the signal to a telephone receiver at the ear of the observer. Because of its general adaptability to the wide variety of conditions encountered in natural rivers, this type of meter is almost universally used.

In making an observation with either an acoustic or an electric meter, the meter is submerged to the desired depth whereupon the observer with the aid of a stop watch determines the time required

for a certain number of revolutions. As the velocity at any given point in a river is not usually constant but is subject to pulsations, a sufficient number of signals should be timed so that the true mean velocity at that point is closely approximated. For this purpose the time of observation is usually about 50 or 60 sec. With the time required for a given number of revolutions known, the velocity is determined from the rating table (see page 379).

The vulnerable parts of current meters are the cups or vanes and the bearings. After every measurement the meter should be dismantled and carefully dried, and the bearings should not only be dried but oiled, using only high-grade watch oil. One of the advantages of differential meters is that they have vertical shafts and therefore operate on point bearings with a minimum amount of friction. To protect the bearings from injury, they should be released from contact when not in use. The spinning time of every meter should be known. Just before starting the gaging, the meter should be given the spinning test in still air to make certain the bearings are properly adjusted. The operation of the meter wheel should be watched constantly. If it appears to be sluggish, repeat the spinning test. Cup meters are slowed down when algae become wound around the axis near the bearings either above or below the wheel. When algae or other foreign matter is found in the bearings upon the completion of the gaging, all measurements that were made after the bearings were last inspected or tested should be repeated. In operating the meter near bridge piers or rocks great care should be taken to prevent injury to the meter wheel. If the cups or propeller are dented, bent, or chipped in any way, its rotating speed for any velocity of water will most likely be changed, and the meter should be rerated before being used again. When the contact signals are received irregularly, the observer should determine the cause; such irregularity may result from pulsations in the currents or from missing contacts or from breaks in the electric circuit.

Stop watches also are delicate instruments and should be frequently checked to determine whether or not they properly record the time.

Methods of Making Discharge Measurements

Current meter measurements may be made (1) from bridges, (2) from cableways, (3) from boats, and (4) by wading. Bridges

provide the most convenient method when they are available, and the cross section and velocity of the stream are favorable to accurate measurement. However, the best cross section is rarely found at a bridge. If there are piers in the river, they usually create turbulence and scour. Often old cofferdams and piling are left from the construction of the bridge, and these impair the value of the section for current meter work. It then becomes a question whether or not the convenience provided by the bridge more than offsets the inaccuracy resulting from its use.

Cableways are frequently used by the U. S. Geological Survey. Although rather expensive to install, they have the advantage that they can be built at sites that are the most favorable for gaging. From the cableway a car is suspended by means of which the observer propels himself from point to point across the stream. The cable, which is usually ⅝ in. to ¾ in. stranded steel wire, may be suspended between trees or between specially constructed and well-anchored supports.

Sometimes discharge measurements can be made advantageously from a boat, especially if the velocity of the stream does not exceed 3 or 4 ft per sec. For higher velocities and for rough water it becomes difficult to hold the boat steady enough to permit accurate measurements. For large, wide rivers the boat must be anchored and its position determined by means of sextant or by triangulation. For narrower streams it can be held in position by a cable stretched across the river, and tagged with the stationing usually at 10-ft intervals. The meter should be suspended from a boom projecting several feet from the boat so that the measured velocities are unaffected by the presence of the boat.

Measurements by wading are restricted to streams that are relatively shallow and of moderate velocity. A tagged measuring line or tape must be stretched across the river near the water's surface. The meter must be held far enough away from the observer that his presence does not affect the velocities being measured.

Measurement of Area

In making discharge measurements with current meters the biggest errors usually result from measurements of area rather than of velocity. For this reason the seemingly simple operation of measuring the depth of the stream will receive considerable attention. When the velocities are low, less than about 4 ft per

sec, no serious difficulties are encountered; but, when the velocities are around 5 or 6 ft per sec or more, and when the river is deep, accurate measurements are not easily made.

Occasionally the stream bed is practically permanent, being subject to neither erosion nor sedimentation. In such cases a standard cross section should be taken in time of low water. With a level and rod, elevations of the river bed are obtained at such intervals as are necessary to define its contour properly. This cross section is plotted and the depths below zero gage height are indicated at all points where velocity measurements are made. The depths existing at the time of any measurement are then found by adding these values to the gage height. When such standard cross sections are used they should be checked at frequent intervals to make certain of their permanence.

If the stream bed at the metering section shifts too much to permit the use of a standard cross section, it is necessary to determine the depth by sounding at each measuring point every time a gaging is made. For this there are two general methods of procedure, depending somewhat upon the depth, velocity, and equipment at hand.

With ordinary velocities, the depths are easily measured. However, when the velocities are high, a heavy lead weight suspended from a small but strong wire cable can be swung upstream and dropped so that it will reach the bed upstream from the metering section. By pulling up the cable until it becomes taut the weight can be lifted just enough to allow it to slide along the bed until it reaches the measuring section. Then with the cable as near a vertical line as possible it is gripped or marked at some reference point, as for instance, the corner of the bridge rail if the measurement is being made from a bridge. The weight is then lifted until the bottom just grazes the water surface. The depth is found by measuring the amount the cable was raised. Such soundings are made successively at all points where velocity measurements are to be made later. Frequent gage readings should be taken to determine whether or not the stage is changing. If the regular gage is not near at hand, a temporary gage or reference mark should be established. If the stage does change, the proper corrections must be made to the measured depths when the velocity readings are taken. In this manner the depths are found at all the stations before any velocities are measured.

In the other method, and this is perhaps the more common procedure, the meter, weight, and cable are first used for sounding the depth and then for measuring the velocity at that same station before moving up to the next station. When measurements are made by this method in swift, deep water, especially if they are made from a bridge or cableway, corrections will have to be made

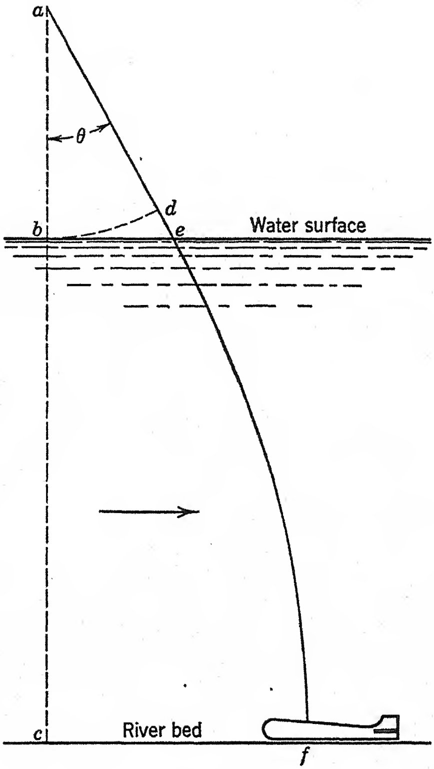

FIG. 127.

in order to determine the vertical depth of stream and be able to submerge the meter at the proper depth below the surface.

In the Annual Report of the Chief of Engineers, U. S. Army, 1900, F. C. Shenehon describes a method of correcting soundings and meter placements in deep, swift rivers. In Fig. 127 is shown the position assumed by the sounding line as the weight, just off the bed of the stream, is supported entirely by the line. From this figure it is seen that if the weight is slowly lowered to the bottom

of the stream, then from the length of cable af must be deducted the distance ae and the difference between the lengths of ef and bc in order to determine the depth bc, assuming the bed cf is horizontal. Both these corrections are functions of the angle θ. In the figure, $ae = ab \sec \theta$. The difference between the length of ef and the depth bc is determined empirically and is equal to $K \times ef$. In Table 27 are given values of $\sec \theta$, and also values of K as determined by Shenehon.

TABLE 27

θ	$\sec \theta$	K	θ	$\sec \theta$	K
4	1.0024	0.0006	22	1.0785	0.0248
6	1.0055	.0016	24	1.0946	.0296
8	1.0098	.0032	26	1.1126	.0350
10	1.0154	.0050	28	1.1326	.0408
12	1.0223	.0072	30	1.1547	.0472
14	1.0306	.0098	32	1.1792	.0544
16	1.0403	.0128	34	1.2062	.0620
18	1.0515	.0164	36	1.2361	.0698
20	1.0642	.0204			

In making measurements by this method the procedure is as follows.

1. Measure the distance ab.
2. Lower the weight until it rests on the bed of the stream, and measure af.
3. Find ae from $ab \sec \theta$, interpolating for $\sec \theta$ if necessary.
4. Subtract ae from af, obtaining ef which is then multiplied by K and the result is subtracted from ef to get bc.
5. To make a velocity measurement at eight tenths of the depth of the stream, the meter is raised from the position shown in Fig. 127 a distance equal to two tenths of the depth, bc, minus the distance from the bottom of the weight to the center of the meter wheel.
6. To make a measurement at two tenths of the depth the meter is lowered until the center of the wheel is distant from a an amount equal to ae plus two tenths of ef.

As an illustration of the above procedure, suppose that ab = 16.5 ft, af = 29.7 ft, and $\theta = 28°$.

$$ae = 16.5 \times 1.1326 = 18.7$$

$$ef = 29.7 - 18.7 = 11.0$$

$$bc = 11.0 - 0.0408 \times 11.0 = 10.55$$

If the distance from the bottom of the weight to the center of the meter wheel is 0.5 ft, then with the bottom of the weight just touching the stream bed at f, in order to make a velocity measurement at eight tenths of the depth, the cable should be raised an amount equal to $(0.2 \times 10.55) - 0.50 = 1.6$ ft. Also if a velocity measurement is to be made at two tenths of the depth below the surface, the meter should be let down a distance of $18.7 + 0.2 \times 11.0 = 20.9$ ft below a.

Although the values in Table 27 are given to the fourth decimal place, this is done mainly for purposes of interpolating, and depths should be recorded to the nearest tenth of a foot only. Corbett[1] warns that this method is based upon the following assumptions.

1. That the weight and wire are such that the weight will go to the bottom despite the force of the current.
2. That the sounding is made with the weight at the bottom but entirely supported by the wire.
3. That the horizontal pressure on the weight, when in the sounding position, is neglected.
4. That the table of coefficients is applicable for any wire or sounding weight provided the wire or weight is designed so as to present as little resistance as possible to the current.

Warning is also given that the method yields correct results only when the direction of the current is approximately normal to the metering section. If it deviates by more than 10 degrees, an additional correction should be made to the vertical angle. In such cases, however, the results are perhaps af questionable accuracy because of the probable difference in depth at f from that at c.

Mean Velocity in Vertical

At any point in the cross section of a stream the velocity varies from the surface down to the bottom. If velocities are plotted as abscissas and depths as ordinates, the resulting vertical velocity curve will normally resemble a parabola with its horizontal axis located at about 20 or 25 per cent of the depth below the surface. The exact shape of the curve varies greatly, however, for different depths of stream and for different velocities. For any given mean velocity, the deeper the stream the less will be the maximum

[1] Don M. Corbett, Stream Gaging Procedure, *U. S. Geological Survey Water-Supply Paper* 888, Government Printing Office, Washington, D. C.

velocity and the more nearly will the velocity be uniform throughout.

The more commonly used methods of determining the mean velocity in the vertical with a current meter are the following.

1. Vertical velocity curve.
2. Two-point method.
3. Six-tenths depth method.
4. Two-tenths depth method.
5. Three-point method.
6. Surface method.
7. Integration.

In the vertical velocity-curve method measurements are made at regular intervals from a point just beneath the surface down to the bottom of the stream. For each curve the velocity should be measured at no less than six points and preferably at ten or twelve. The points are plotted on cross-section paper, the curves are drawn in and extended up to the water surface and down to the bottom of the stream. The area within each curve is then planimetered and divided by the total depth to find the mean velocity. At least one vertical velocity measurement should be made at every station at a high stage of the river, another at a medium stage, and another at a low stage. The results of these measurements are used to determine the coefficients that must be applied to the results obtained by each of the short-cut methods of measurement. For each of these methods, for which coefficients are found to be needed, a curve can be obtained by plotting the coefficients, as determined by the measurements, against gage height. By extending this curve to higher stages the coefficient can be found for any gage height.

In the two-point method an observation is made with the meter at two tenths of the depth and again at eight tenths of the depth below the surface. The average of these two results is taken as the mean velocity in the vertical. Seldom is a coefficient necessary in this method if the channel is straight, uniform, and unobstructed for some distance upstream from the section. This method is the most widely used and perhaps most generally satisfactory of all. It cannot be used, however, at points where the depth is less than five times the distance from the bottom of the meter weight to the

center of the meter wheel because in such cases the meter wheel cannot be submerged to eight tenths of the depth.

In the six-tenths method a single observation is made at six tenths of the depth below the surface, and the result is taken as the mean velocity in the vertical except where the vertical velocity measurements indicate the need of a corrective coefficient. Although in most cases this method gives fairly accurate results, it is not as dependable as the two-point method and is not often used except where the two-point method cannot be used because of insufficient depth or because of interference by grass, weeds, or rocks at eight tenths of the depth.

Before the 0.2-depth method can be used at a station, it is necessary to make a number of discharge measurements at various stages using either the vertical velocity-curve method or the two-point method. For each such measurement a hypothetical discharge is computed by considering the mean velocity in each vertical as being the velocity that was found at two tenths of the depth. Each of these hypothetical discharges is then plotted against the corresponding values of the true measured discharge. The results will usually be found to produce a straight, or nearly straight, line passing through the origin. Sometimes a more nearly straight line may be obtained by plotting the weighted mean 0.2-depth velocities against the true mean velocities in the cross section. The weighted mean 0.2-depth velocities are obtained by dividing the hypothetical discharges by their corresponding cross-sectional areas. Whichever method produces the more nearly straight line is the one to be used in any instance.

One of the advantages of the two-tenths method arises from the fact that at two tenths of the depth the velocity is either the maximum or very nearly so, and therefore in that vicinity the velocity curve is practically vertical, and the velocity varies but slightly with considerable variations in depth. For this reason it is not as important that the meter be placed at exactly the proper depth as is the case in a six-tenths measurement. Also this relationship is more nearly constant than the relation between the surface velocity and the true mean velocity. This method possesses considerable merit.

The three-point method is a combination of the two-point and the six-tenths methods. Observations are made at two tenths, six tenths, and eight tenths of the depth, and the mean of the three

is taken as the mean velocity in the vertical. Sometimes the mean of the velocities obtained at the two-tenths and eight-tenths depths is averaged with the velocity at six-tenths depth. This procedure does not seem to be logical, for it is admitted that the six-tenths method does not give as accurate results as the two-point method; it follows, therefore, that the average of these two methods is not as accurate as the two-point method. It may be advantageous, however, to use the three-point method in gaging deep streams, giving equal weight to all three measurements.

Surface measurements are sometimes made in which the meter is submerged either a constant depth below the surface for measurements made at all stages or the meter is submerged a certain percentage of the total depth. Whichever method is used, a coefficient must be applied to each surface velocity to get the mean velocity in the vertical. These coefficients are obtained from vertical velocity measurements by dividing the mean velocity in each vertical by the velocity obtained from the curve at the depth at which the surface measurement is made. By plotting these coefficients against gage height and, if necessary, extending the curves to higher stages, the proper coefficients can be found for any gaging.

In the integration method the meter is lowered from the surface to the bottom and returned to the surface at a constant rate, at each vertical, the revolutions being counted and timed for each observation. Because of the facts that (1) it is difficult to raise and lower the meter at a uniform speed, (2) the observations are affected by the vertical movement of the meter, and (3) the lowest velocities occurring near the stream bed are not included in the integration, this method is not often used nor is it to be recommended.

Characteristics of a Good Metering Section

Regardless of the method employed, the accuracy of current meter measurements will depend in a large measure upon the characteristics of the metering section. If those characteristics are ideal or even favorable, an amateur at this work should obtain satisfactory results without much difficulty. On the other hand, if these conditions are adverse, it may tax the ingenuity of the most skillful and experienced hydrographer to make a satisfactory discharge measurement. The conditions that favor good results are as follows.

1. The section should be straight and uniform for a distance upstream equal at least to five times the width of the stream and for a distance downstream equal to twice the width of stream.

2. The bed of the stream should be smooth. It should be free from vegetal growth, boulders, or other obstructions. Bridge piers are particularly objectionable.

3. The bed and banks of the stream should be firm and stable.

4. The current should be normal to the metering section.

5. Velocities should be greater than 1 ft per sec and less than 4 ft per sec.

6. There should be no large overflow section at flood stage.

7. The section should be accessible.

Spacing of Verticals

The number of sections into which the stream should be subdivided depends largely upon the character of the cross section. If the channel is straight, smooth, and uniform for some distance above and below the metering section so that the velocity variations are uniform, perhaps no more than ten sections are necessary, except for very wide streams. Normally, however, there should be more. At two adjacent verticals, neither the depth nor the velocity should differ excessively. The reason for this is that the method of computing the discharge is only approximate. The mean velocity in the section is really not the same as the average of the velocities in the verticals at the ends of the section. If the two depths were the same, the velocities could differ or, if the two velocities were the same, the depths could differ without causing an error; but, if one depth is, for example, 1 ft and the mean velocity is 1 ft per sec and the other depth is 5 ft and the corresponding velocity is 5 ft per sec, the mean velocity for the section is not 3 ft per sec because the same weight should not be given to the velocity occurring at the 1-ft depth as to that at the 5-ft depth. If the velocity varies as a straight line between these two verticals, true mean velocity occurs at the center of gravity of the section. At the vertical through that point the mean velocity is 3.44 ft per sec instead of 3 ft per sec, an error of about 15 per cent. Had an intermediate vertical been taken where the depth was 3 ft and the mean velocity was 3 ft per sec, this error would have been reduced to about 4 per cent. Errors of this kind are nearly always cumulative for the reason that almost invariably the higher velocity occurs at the

same vertical with the greater depth. Because of this, the computed discharge is too low.

Effect of Swaying of Meter

Especially if the meter is suspended on a cable from a point that is some distance above the water surface there is a tendency for it to be carried not only downstream but also back and forth across the current. All meters are affected by such motion although in a different manner; differential meters are speeded up by the full amount of the movement, whereas direct-acting meters are retarded to a certain extent. This lateral motion can be prevented, if the stream is not too wide, by means of a light stay line fastened to the meter cable near the water surface and held taut from each bank. Even when a meter is supported on a rod, swift water often causes a short but rapid vibration that affects the velocity observations. The proper solution of difficulties of this sort can usually be best determined by the observer in the field, as the problem varies considerably with the conditions attending each case.

Effect of Angle of Current

The discharge of a stream is the product of the area of cross section and the velocity component normal to that section. Seldom is the direction of the current normal to the cross section, especially at bridges. A meter suspended from a cable swings into line with the current, and therefore any velocity so measured should be multiplied by the cosine of the deflection angle in order to get the velocity component normal to the cross section. Where this angle is no more than 8 degrees the correction is less than 1 per cent and may ordinarily be ignored. For great angles, however, it should be measured and correction made. If no protractor is at hand, the angle can be measured by holding the notebook parallel with the metering section and, with a ruler or straightedge lined up with the direction of the current, drawing a line whose angle with the normal is later measured with a protractor.

Effect of Turbulence

On page 378 it was explained that in the usual rating of current meters, the meter is drawn at known velocities through still water. The effect is the same as though the meter were held still and the

water moved past it—*but with streamline motion.* All meters are affected in one way or another by turbulence. Differential meters overregister in turbulent water by an amount that depends upon the degree of turbulence. Direct-acting meters underregister. An investigation by Yarnell and Nagler[1] indicated that for both kinds of meters the percentage of error increased quite rapidly with the degree of turbulence and was roughly about the same for each type. Their tests, however, were all conducted with the meter suspended on a rod, and it is known that a meter behaves differently for different types of support. By far the greater number of the gagings of natural streams are made with meters suspended on cables rather than on rods.

Groat[2] found that differential meters overregister from three to six times as much as direct-acting meters underregister. In view of the difference in the results obtained in these two investigations, it seems that further study should be given to this subject. From the information at present available, it appears that for streams that are only slightly turbulent a measurement made by either type of meter should give satisfactory results. In gaging very turbulent streams, however, if a high degree of accuracy is desired, neither type of meter used alone should be trusted. Either duplicate gagings should be made using both types of meters simultaneously, taking perhaps the average of the results obtained by the two as being correct, or some other method of measurement should be used.

Measurements under Ice Cover

When a stream freezes over, the wetted perimeter is increased by the width of the stream. The hydraulic radius and, therefore, the velocity and discharge are correspondingly reduced for any given stage. Because of the friction with the ice cover, the velocity of the water near the ice is retarded and the location of the thread of maximum velocity is lowered. Despite this redistribution of velocities in the vertical, however, it has been found that the average of the measurements made at two tenths and at eight tenths of the depth gives the mean in the vertical accurately

[1] David L. Yarnell and Floyd A. Nagler, Effect of Turbulence on the Registration of Current Meters, *Trans. A.S.C.E.*, 1931, **95**, 766–860.

[2] B. F. Groat, Characteristics of Cup and Screw Current Meters, *Trans. A.S.C.E.*, 1913, **76**, 819–870.

enough for practical purposes. The technique of gaging is changed slightly, however.

Holes are cut in the ice preparatory to making measurements at the same points in the cross section as those used during open-water conditions. The gage height of the water surface is recorded. The distance from the water surface to the bottom of the ice is measured at each hole and deducted from the total depth. Both these measurements are recorded in the notes. Two tenths and eight tenths of the difference are computed and to these values is added the distance from the water surface to the bottom of the ice. After placing the center of the wheel on the water surface, these distances are measured off on the cable, the meter is lowered, and the observations made. Where the depth under the ice is less than five times the distance from the bottom of the weight to the center of the meter wheel, a single measurement should be made at mid-depth below the ice cover, and a coefficient of about 0.85 should be applied to this result to obtain the mean velocity in the vertical. In each case a number of vertical velocity curves should be derived to check whatever short-cut method is used.

Effect of Rising or Falling Stage upon the Discharge Curve

During a rising stage of river the velocity and discharge are greater than they are for the same stage when the discharge is constant. Likewise, during a falling stage the discharge is less for any given gage height than it is when the flow is steady. This means that the result of discharge measurements made during rising or falling stages, when plotted, will not fall on the true discharge curve but will fall to the right or left thereof respectively. The amount of departure from the true curve does not depend upon the total rise or fall during the measurement but only upon the rate at which that change occurred. It is, therefore, important that whenever discharge measurements are made for the purpose of determining or of checking the discharge curve, the gage height and time should be recorded at the beginning and at the end of the gaging in order that the rate of change may be computed. In Fig. 128 is shown the discharge curve for the Ohio River at Wheeling, West Virginia. On this curve are also shown the results of measurements made during the flood of March 1905, together with the rates of rise and fall expressed in feet per hour.

The engineers of the U. S. Geological Survey have developed

several methods of correcting discharge measurements and gage heights obtained during changing stages. Most of these methods depend upon the relationship between discharge and slope. The simplest method of making these corrections is to derive curves of relation between percentage of correction in discharge and rate of change in stage expressed in any convenient units such as feet per hour or feet per day. Such curves can be easily obtained after a number of discharge measurements have been made during both rising and falling stages and also at constant stage. A smooth dis-

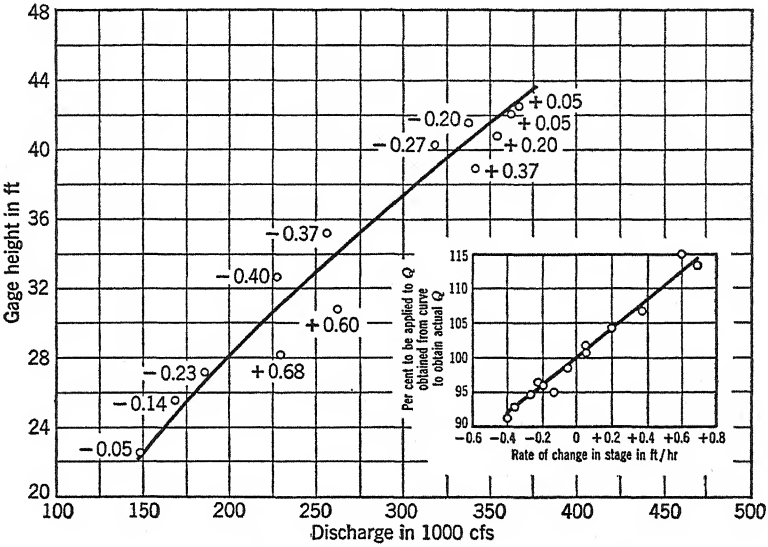

FIG. 128.

charge curve is then drawn in, passing through or near those points that were obtained at constant stage, to the left of those obtained at a rising stage, and to the right of those for which the stage was falling. Then for each measurement during a changing stage the percentage is computed by which the discharge obtained from the curve would have to be changed to make it agree with the measured discharge for the same gage height. These percentages are plotted against the respective rates of change in stage as shown in Fig. 128. A different relationship will be found to exist during a rising stage from that of a falling stage. Usually this relation is expressed as a straight line or a very flat curve.

With such a set of curves available for any gaging station, it is

easy to determine the rate of rise and fall during periods of sudden change, read off the percentages of correction, and apply these corrections to the discharge obtained from the rating table.

The Effect of Ice on the Stage-Discharge Relation

Three different kinds of ice occur in natural rivers: surface ice, anchor ice, and slush ice or frazil. Everyone is familiar with surface ice or sheet ice that forms on the surface of still or slowly moving water when its temperature falls below 32° F. Anchor ice in the form of long, slender needles freezes on the surface of rocks in the beds of rivers and on steel bars, beams and other hard objects beneath the water surface. It forms usually at night and only in very cold weather. If during the day even for only a few minutes the rays of the sun strike objects on which anchor ice has formed, the temperature of those objects will be raised enough to cause the ice to release its grip and rise to the surface. Frazil or slush ice occurs in the form of scales or flakes and gives the water a muddy appearance when it occurs in quantity. It usually forms at rapids, on clear nights when the air temperature is well below zero. At such times enough cold air becomes mixed with the water to reduce its temperature the small fraction of a degree below freezing that is required for the formation of ice. When this frazil ice comes in contact with steel, rocks, or the under side of surface ice, it is likely to catch hold and collect there, forming large masses. This ice is often very troublesome at power plants and at other intakes, collecting on the trash racks and completely choking off the water supply.

Ice affects the stage-discharge relation in three different ways.

1. Anchor ice may form on the control. If the control consists of rocks extending practically continuously across the stream and anchor ice forms thereon, the gage height may be raised by approximately the thickness of the ice formation. Inasmuch as this thickness does not usually exceed a few inches and the ice leaves with the first sunshine, the errors in the discharge records resulting from this cause are not usually serious. Besides, if the station is equipped with a recording gage the occurrence of anchor ice at the control is easily detected and corrected for. In such cases the hydrograph will show a sudden unnatural rise. The open-water-discharge curve can usually be applied by deducting the amount of this rise from the ordinates of the hydrograph.

2. Ice jams may form below the control, backing the water up, drowning out the control, and completely destroying the stage-discharge relation during their existence. These jams may result from floating cakes of surface ice wedging between bridge piers or at other constricted sections or at the upstream margin of unbroken ice cover. They may also result from frazil ice being carried under surface ice where it catches on the under side and collects in the form of huge masses, almost completely choking the channel, forcing the water to flow over the top of the surface ice.

3. Surface ice may form at the control, entirely changing the relation between gage height and discharge. The magnitude of this change varies, however, throughout the period that the control remains frozen over. It appears to be roughly proportional to the cumulative below-freezing air-temperature deficiency beginning with the initial ice formation.

Obtaining Discharge Records with Control Frozen Over

Perhaps the best procedure for obtaining discharge records at stations where the control freezes over is as follows: (1) make discharge measurements at the regular metering section at frequent intervals, the necessary frequency being learned only from experience; (2) determine from the open-water-rating curve or table the gage height at which the measured discharge would occur during open water and find the amount of backwater by deducting this gage height from the gage height existing at the time of the measurement; (3) plot these amounts of backwater as gage-height corrections with the time scale as abscissas; (4) now compute the cumulative temperature deficiency below freezing, starting with the date of the initial freezing over, and plot these values to the same time scale and on the same drawing with the gage-height corrections; (5) draw a gage-height-correction graph through the points obtained from the discharge measurements and as nearly as possible parallel to the mass-temperature-deficiency curve; (6) from the recorded gage heights subtract the corrections, day by day, as obtained from this correction graph and apply these corrected readings to the open-water-rating curve or table to obtain the daily discharges.

As long as the control section remains free from ice and no ice jams form downstream to drown out the control, the open-water stage-discharge relationship will not be appreciably affected by the

formation of ice anywhere upstream from the control; under those conditions the only effect on the gage is the result of the increased friction with the ice cover between the gage section and the control, and this is usually negligible. As soon as the control section freezes over, however, the above statement no longer holds true, for then the constricted section at the control is entirely changed, and as a result the stage-discharge relation at the gage may be radically altered.

Obtaining Records on Stream with Shifting Control

Only in exceptional cases is it definitely known at the time when a gaging station is established, that the control is going to be really permanent; practically always the determination of the degree of permanency should claim first attention. At first, discharge measurements should be made at as many different stages as possible, but after a lapse of some time they should be repeated at the same stage in order to determine whether the stage-discharge relation has remained constant in the interim. Especially should measurements be repeated after periods of high water, for it is usually during such times that the most pronounced changes in the control occur.

After a number of measurements have been made at different stages, the discharge curve should be drawn. The location of each of the observed points with respect to the curve should then be studied in detail to determine whether or not the stage-discharge relationship has been changing. These points should be studied in chronological order and consideration given to the effect of either the constancy or the rate of change of the stage at the time each measurement was made. If a measurement made during a rising stage falls on or to the left of the curve, or a point obtained during a falling stage plots on or to the right of the curve, or a point obtained during a constant stage plots on either side, then either such measurements were in error, the curve was wrongly drawn in the first place, or the control has changed. Even though no evidence is found of such a change after several years of records, vigilance should be relaxed only by degrees. Instances have been known where a control showed no signs of change for a period of 5 yr and then changed quite radically in a short time.

If the control is found to be changing, the next step is to find the rapidity with which it changes because the best method of

obtaining records under such conditions depends upon that rate. When such changes occur slowly or only at infrequent intervals during time of flood, the simplest method of getting daily-discharge records is to apply the daily gage heights to the prevailing discharge curve until the discharge measurements indicate sufficient change in the stage-discharge relation to require a new curve. Enough measurements are then made to properly define the new location of the discharge curve that is then used until gagings

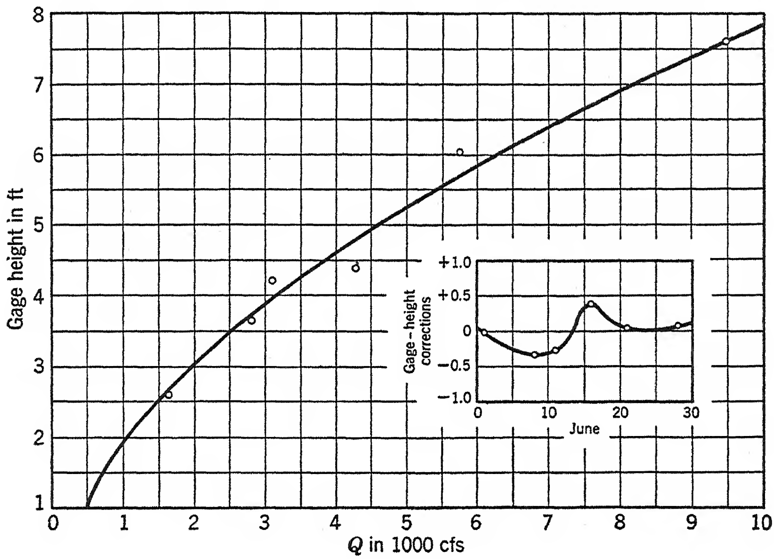

FIG. 129.

indicate another change in the control. The gaging station on Fall Creek at Ithaca, New York, is one of this type. On an average of about once each year the control changes enough to require a new rating curve.

At many stations, especially on streams in the plains of the Middle West, the control is almost constantly changing. The Stout method of obtaining daily-discharge records is perhaps the most commonly used in such cases. Its use will be illustrated by an example.

In Table 28, Column 2, is shown for a given month the recorded daily gage heights. In Column 3 are the results of the discharge measurements that were made during this period. In Fig. 129 is

shown the discharge curve as determined from these measurements. The exact manner in which this curve is drawn is not of vital importance. In Column 4 are shown in bold-faced type the corrections

TABLE 28

Date (1)	Recorded Gage Height (2)	Discharge (3)	Gage-Height Correction (4)	Corrected Gage Height (5)	Q (6)
June 1	7.62	9470	**−0.03**	7.59	9470
2	7.58		− .09	7.49	9240
3	7.50		− .16	7.34	8920
4	7.36		− .21	7.15	8520
5	7.11		− .25	6.86	7930
6	6.80		− .29	6.51	7250
7	6.45		− .32	6.13	6530
8	6.04	5760	− **.34**	5.70	5760
9	5.40		− .33	5.07	4780
10	4.75		− .31	4.44	3800
11	4.21	3110	− **.28**	3.93	3110
12	3.77		− .20	3.57	2650
13	3.48		− .09	3.39	2440
14	3.66		+ .15	3.81	2950
15	3.97		+ .33	4.30	3590
16	4.39	4290	+ **.39**	4.78	4290
17	5.01		+ .34	5.35	5180
18	4.83		+ .21	5.04	4690
19	4.52		+ .14	4.66	4110
20	4.07		+ .09	4.16	3400
21	3.65	2810	+ **.06**	3.71	2810
22	3.34		+ .05	3.39	2440
23	3.12		+ .04	3.16	2170
24	2.93		+ .03	2.96	1960
25	2.77		+ .04	2.81	1800
26	2.63		+ .05	2.68	1670
27	2.51		+ .06	2.57	1560
28	2.60	1660	+ **.07**	2.67	1660
29	2.66		+ .09	2.75	1740
30	2.62		+ .12	2.74	1730

that must be applied to the gage readings taken at the time of the discharge measurements in order that the discharge values as obtained from the curve will be the same as the measured discharges. These values are plotted in Fig. 129, and the gage-height correction curve is drawn through these points. For the intervening days between measurements the gage-height corrections shown in

Column 4 were obtained from this gage-height correction curve. These corrections applied to the recorded gage heights shown in Column 2 give the corrected gage heights in Column 5, which are then applied to the discharge curve shown in Fig. 129 to obtain the daily discharges shown in Column 6.

Discharge measurements are taken during subsequent months at intervals whose necessary frequency at each different gaging station can be determined only by experience. The same discharge curve as shown in the figure can be used for subsequent periods as long as the measured discharges continue to fall on both sides of the curve. If, however, it becomes apparent that another curve will fit the points better than the original curve, such a curve should be drawn in and used.

Plotting and Extending the Stage-Discharge Curve

After a number of discharge measurements have been made at a station, the results should be plotted on ordinary cross-section paper as shown in Fig. 118. It is customary to plot discharges as abscissas and gage heights as ordinates. Ordinarily difficulty will be encountered in drawing in a complete and satisfactory discharge curve with the first set of measurements for several reasons. In all probablity most of the gagings are made within a rather limited range at or somewhere near the average stage of the river. The curve usually has its greatest curvature in the low stages where there may be no measurements. It will, therefore, be necessary to extend the curve downward for the low stages and upward for the flood stages.

By breaking the discharge measurements into their two component parts and plotting the mean velocity and area curves therefrom on the same drawing and to the same gage-height scale as that of the discharge curve, it is often possible to discover errors in the measurements. In order that this may be done, however, it is essential that the water surface at the metering section and at the gage fluctuate in similar amounts and also that the cross section at the metering section be practically permanent. If these conditions prevail, the mean velocity curve will normally be found to be concave upward, and the area curve will be concave downward. If there is ponded water at the section at the stage of zero discharge, the mean velocity curve will reverse and be concave downward in the low stages. The data from which the area curve

is drawn can be determined accurately by means of level and rod for all stages. It is then a matter of extending the velocity curve only. According to the Manning formula the velocity in the stream is

$$V = \frac{1.486}{n} r^{2/3} s^{1/2} \tag{3}$$

Oftentimes $S^{1/2}/n$ becomes practically constant for the higher stages after all the rapids have become drowned out, and this formula may then be written

$$V = Kr^{2/3}$$

With the cross section and area known for all stages, the hydraulic radius can also be found for all stages. By taking various values of V from the known portion of the mean velocity curve and the corresponding values of r, values of K can be computed for the range in stage for which the mean velocity curve is known. By plotting these values of K against gage height, a curve is obtained that should approach a vertical line as an asymptote in the higher stages. By so extending this line, values of K should be obtained quite accurately, which when combined with their respective values of $r^{2/3}$ and A will give values of discharge that may be used for extending the discharge curve for all higher stages.

Stevens Method of Extension. Stevens[1] has devised a procedure that is commonly known as the $A\sqrt{D}$ method which is a variation of the one above described. For streams that are relatively wide and shallow, the mean depth, D, which is found by dividing the cross-sectional area, A, by the width of stream measured at the water surface, does not differ greatly from the hydraulic radius, r, which is A divided by the wetted perimeter. Therefore by substituting D for r the Chezy formula for flow in open channels may be written in the form $Q = C\sqrt{s} \times A\sqrt{D}$. For the higher stages $C\sqrt{s}$ sometimes becomes constant, in which cases values of Q plotted against $A\sqrt{D}$ form a straight line. Both A and D are functions of gage height and values of $A\sqrt{D}$ may be obtained for all stages of the river and plotted against gage height as shown in Fig. 130. By plotting the measured discharges against the values

[1] J. C. Stevens, A Method of Estimating Stream Discharge from a Limited Number of Gagings, *Eng. News*, July 18, 1907.

of $A\sqrt{D}$ as obtained from this curve, corresponding to the gage heights at the time of the measurements, a straight line may be obtained provided that $C\sqrt{S}$ is a constant for this range of stage. Even if it is not a straight line for the entire range, it will probably be a flat curve approaching a straight line in the higher stages. By extending this curve upward the discharge can be found for the value of $A\sqrt{D}$ corresponding to any desired gage height, as illustrated in Fig. 130. In some cases it may be found that instead

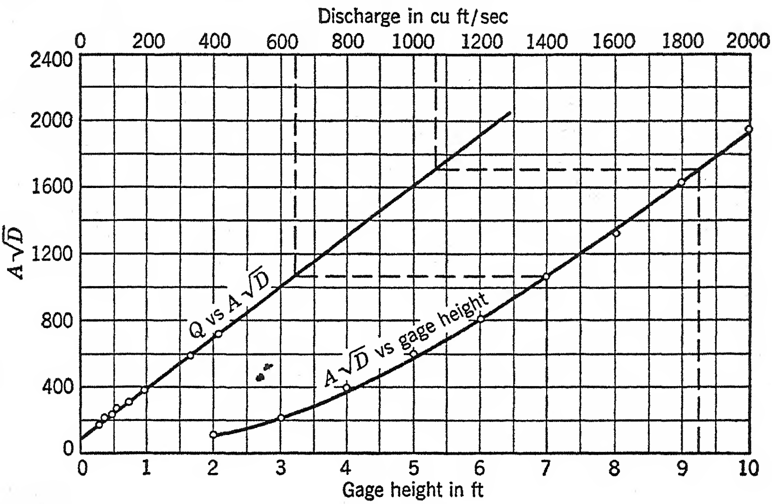

FIG. 130. Discharge curve for Huron River near Whitmore Lake, Mich. Extended by the $A\sqrt{D}$ method.

of using ½ as the exponent of D, some other value such as ⅔ will produce better results.

Logarithmic Method of Extension. If the cross section of a stream at the site of the gage is, or even approximates, a uniform section to which can be roughly fitted either a segment of a circle, or of a parabola, or a rectangle, or trapezoid, then the logarithmic method can be used advantageously. The relation between discharge and gage height can be expressed by the equation,

$$Q = C(G - a)^n \tag{4}$$

in which Q is the discharge in cubic feet per second, G is the gage height in feet, a is the gage height corresponding to zero discharge, and C and n are constants for any station.

Equation 4 may be written in the logarithmic form,

$$\log Q = n \log (G - a) + \log C \tag{5}$$

which is the equation of a straight line whose slope is expressed by n and whose intercept on the discharge axis is equal to $\log C$.

Ordinarily the gage height corresponding to zero discharge is unknown and must be determined, for if $\log Q$ is plotted against $\log G$ for any value of a other than zero, a curve will be obtained that will possess no advantage in extension over that obtained by ordinary plotting. There are two methods of determining the value of a without making a field investigation.

The simpler and more direct of these two was first suggested by Dr. T. R. Running, Professor Emeritus of Mathematics at the University of Michigan, and is illustrated in Fig. 118. In the application of this method three values of discharge are selected from the known portion of the curve, one value near the upper end of the segment, another near the lower end, with the intermediate value so chosen that the three form a geometric series. In Fig. 118 these values are represented by a, b, and c, having values respectively of 50, 150, and 450. Through a and b vertical lines are drawn, and through b and c horizontal lines are drawn intersecting the verticals at d and e. Through d and e a line is drawn that will intersect a line through a and b at f, the latter point being at the elevation of the gage height corresponding to zero discharge. This method is based upon the assumption that the lower portion of the discharge curve including the points a, b, and c is a parabola. In most cases the curve approaches a parabola near enough to give reasonably accurate results.

The gage height of zero discharge may also be determined by trying various values of a until values of $\log Q$ plotted against values of $\log (G - a)$ form a straight line. Instead of plotting logarithms of the values on coordinate paper, it is more convenient to plot actual values on logarithmically ruled paper. Although the actual measured values of Q and gage height may be used in this plotting, better results are usually obtained by selecting a number of points from the measured range of the discharge curve that seem to represent the average trend of these values. A series of such values taken in this manner from the discharge curve shown in Fig. 118 are plotted for a number of assumed values of a in Fig. 131. It will be seen that for a equal to 1 ft the curve is concave upward,

whereas for a equal to 2 ft it is concave downward, thus indicating that the value of a lies between one and two. In the example shown, a value of 1.4 ft results in a straight line. Having obtained a straight line, one can extend this line, and then values of discharge at higher stages may be read directly, or the values of C and n, the constants in equations 4 and 5, may be determined and the

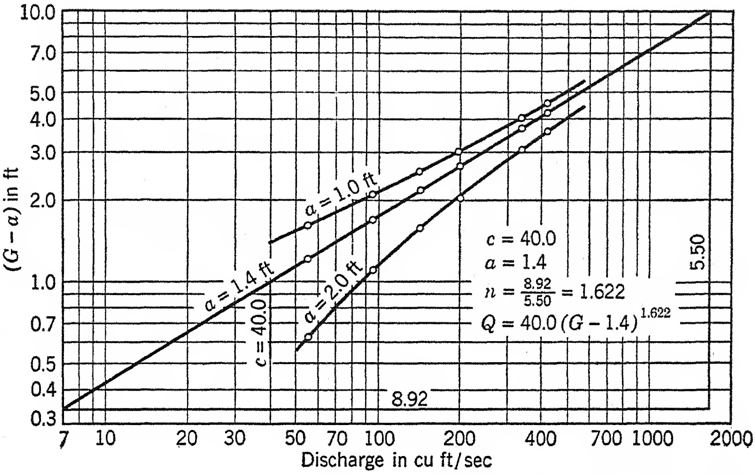

Fig. 131. Discharge curve for Huron River near Whitmore Lake, Mich. Extended by the logarithmic method.

higher values of Q obtained algebraically. C is the value of Q at the point on the straight line where $(G - a)$ is equal to one, and n is the tangent of the angle that the line makes with the $(G - a)$ axis.

Slope Stations

In the velocity-area stations discussed so far, it has been assumed that the gage is so located above a control that a single relationship exists between constant gage height and discharge. Occasionally, however, it is necessary to establish a gaging station in a location that is affected by backwater. Examples would be locations above dams where flow is controlled by various gates, or above the confluence of two rivers. Under such conditions the slope of the energy gradient no longer remains nearly constant, but it may differ greatly even for the same stage. Thus it is no longer possible to obtain a single stage-discharge relationship, but the third variable, slope, must be considered.

The slope is obtained by using two gages some distance apart. If the gages are placed at the same datum, a measure of the fall

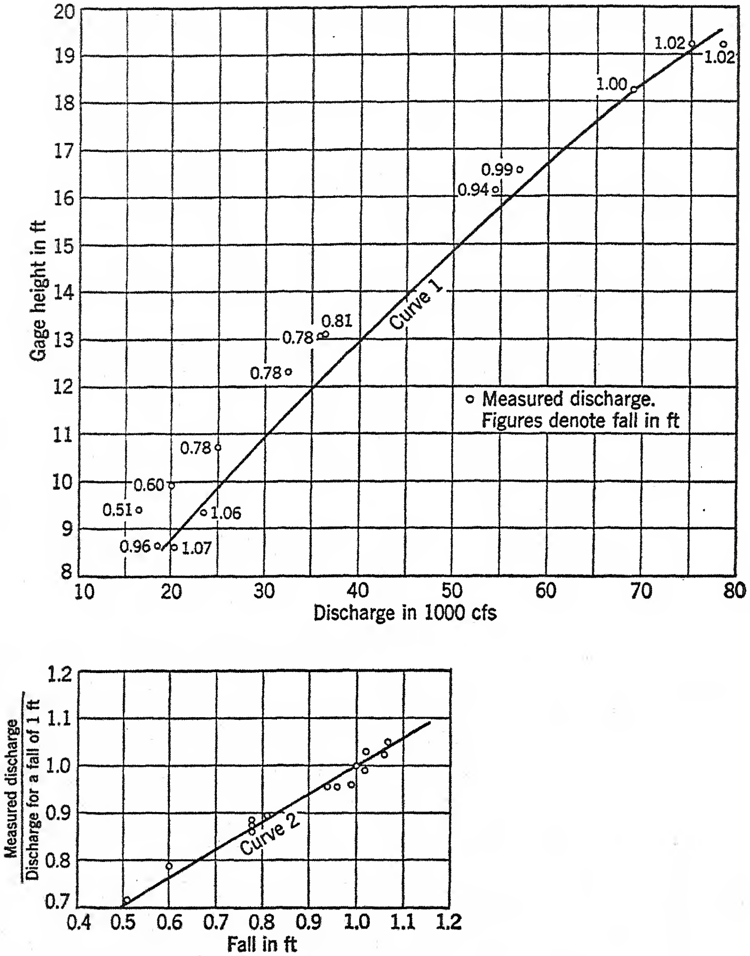

FIG. 132. Relation of stage to discharge and of discharge ratio to fall for Tennessee River at Chattanooga, Tenn. Data were taken from Plate 9, *U. S. Geological Survey Water-Supply Paper* 888.

in the energy gradient may be obtained by adding the corresponding $V^2/2g$ to each gage reading and then subtracting one energy gradient elevation from the other. Often the two values of $V^2/2g$

TABLE 29

1 Q_m cfs	2 Fall ft	3 Gage Height ft	4 Q_c cfs	5 $\frac{Q_m}{Q_c}$
20,300	1.07	8.60	19,300	1.050
18,500	0.96	8.65	19,400	0.953
23,500	1.06	9.35	22,900	1.025
16,500	0.51	9.40	23,100	0.714
20,000	0.60	9.90	25,400	0.787
25,000	0.78	10.70	29,000	0.862
32,500	0.78	12.30	36,800	0.883
35,800	0.78	13.05	40,900	0.875
36,500	0.81	13.10	41,000	0.890
54,500	0.94	16.15	57,000	0.955
57,000	0.99	16.55	59,300	0.961
69,000	1.00	18.25	69,200	0.997
75,000	1.02	19.20	76,000	0.987
78,500	1.02	19.20	76,000	1.035

are so nearly alike that the fall becomes equal to the difference in the two gage readings.

Several techniques employed by the U. S. Geological Survey for utilizing records from slope-gaging stations are described in *U. S. Geological Survey Water-Supply Paper* 888. An example of one of these procedures, the "constant fall" method, will be given here. In Columns 1, 2, and 3, Table 29, are shown a number of values of measured discharge, Q_m, fall, and gage height. Values of Q_m are plotted against gage height, usually for the upper gage as before, but with each point the value of fall is noted as shown in Fig. 132. It will be seen that these points are well scattered. However, some order may be noted among those points having similar values of fall. Therefore, some particular value of fall is arbitrarily selected for which to draw a discharge curve. This is called a "constant fall" discharge curve and is illustrated by Curve 1, Fig. 132, which was drawn for a fall of 1 ft. It will be noted that points having a fall greater than 1 ft are to the right of the curve whereas those having a fall less than 1 ft are to the left. For any gage height the corresponding discharge measured from the "constant fall" curve is called the "constant fall discharge" and is designated as Q_c. The next step is to determine Q_c from the curve for the gage height corresponding to each plotted point. These values are shown in Table 29, Column 4.

Values of Q_m/Q_c are then computed (Column 5) and plotted against corresponding values of fall as shown by Curve 2, Fig. 132. Usually it is desirable to check on the adequacy of Curves 1 and 2 before proceeding to use them for determining discharge. This check is accomplished by converting each value of Q_m to Q_c by dividing it by the value of Q_m/Q_c taken from Curve 2. When plotted, these values of Q should fall on or near the "constant fall" curve. If they do not appear satisfactory a second curve should be drawn and the process repeated. These curves are used to convert recorded values of gage height and fall to discharge in the following manner. The value of Q_c for the recorded gage height is read from Curve 1. For the corresponding measured fall, the value of Q_m/Q_c is read from Curve 2. The product of Q_c and Q_m/Q_c will then be the discharge corresponding to that pair of gage readings.

The Measurement of Peak Flood Flows

The determination of the maximum discharge occurring during flood periods often introduces special difficulties even on streams where gaging stations are already established.

There are many reasons why few current meter measurements are made at or near the time of peak flow. Often the U. S. Geological Survey and the various engineering organizations that normally do such work are otherwise occupied at the time and sometimes it is impossible for them to make measurements at all the locations on the various flooding streams within the relatively short duration of the flood. Transportation difficulties may make it impossible to reach the various metering sections, and sometimes it is impossible to operate a current meter during flood periods because of floating debris or bridge washouts.

If the maximum gage height is determined at an established gaging station, a good estimate of the discharge may be obtained by extending the rating curve to this gage height. Various methods of extending such curves were described on page 401. In the absence of a gaging station, the discharge may be obtained at a constriction in a river by applying the basic energy equations along with estimated energy losses in the manner described for control meters on page 363. Good approximations of discharge over embankments, over low dams, and through culverts can be made on the basis of experimental work on similar structures. At falls it is often possible to apply critical-depth formulas to obtain the discharge. In all cases

it is necessary to know at least one water-surface elevation. In the absence of a gage, water-surface elevations must be obtained from high-water marks. In the event that such marks do not exist near the location where they would be used, it may be possible to utilize one some distance up- or downstream by developing the water-surface profiles to the desired point. This procedure may also be used to tie old high-water marks in with newly established gaging stations.

The "slope-area method" is often used to estimate peak discharges where no gaging station exists. This method is based on an open-channel formula for determining the velocity of flow that in combination with the measured areas gives the discharge. The first step in applying this method is to select a reach of river having a relatively uniform channel and clearly defined high-water marks. The length of the reach, the difference in elevations of the high-water marks at the ends of the reach, and the cross-sectional area at each end are measured. The average hydraulic radius is determined from the areas. If an assumed value of kinetic energy is added to the elevation of the water surface at each end of the reach, the difference between these values, divided by the length of the reach, gives the average slope of the energy gradient. A value of roughness coefficient is then determined and the velocity and discharge computed by means of an open-channel formula such as the Manning formula. It is recommended that a number of careful measurements of s, r, and v be made for this reach of river at different discharges to permit the determination of values of n in the Manning formula. These values may then be plotted against water-surface elevations. An extension of this curve to the elevation during the flood may be expected to give a reasonably accurate value of the coefficient. This is only a first trial answer, it being necessary to check the values of kinetic energy previously assumed.

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